Exploring the binary black hole parameter space with Gaussian Process Regression

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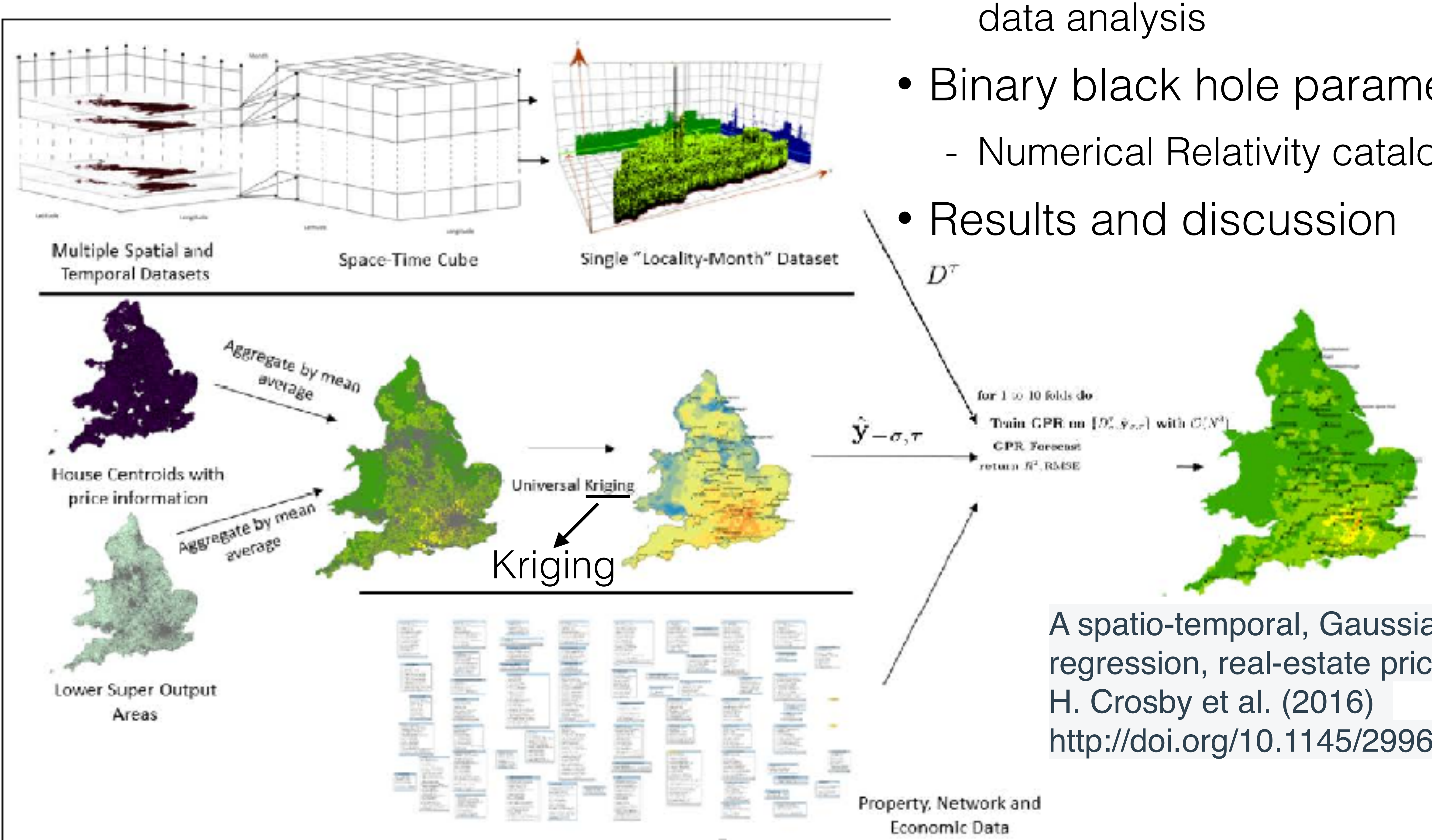
Outline

- Gaussian Process Regression
  - Applications to gravitational wave data analysis
- Binary black hole parameter space
  - Numerical Relativity catalogue
- Results and discussion



A spatio-temporal, Gaussian process regression, real-estate price predictor, H. Crosby et al. (2016) http://doi.org/10.1145/2996913.2996960

Need for a surrogate model

- Numerical Relativity (NR) simulations are computationally very expensive
- Covering the whole parameter space (eg. for binary black hole mergers) is impractical without some means to interpolate between waveforms
- Traditional fitting methods tend to use a best fit (maximum likelihood) approach
- Bayesian approach propagates the uncertainties through the fitting process
- This can highlight lack of knowledge about particular regions of the parameter space

Gaussian Process Regression

- Gaussian process regression (GPR) is a Bayesian, non-parametric solution to this problem.
  - Make minimal assumptions about the form of the underlying function
  - Provides uncertainty (in fact a full posterior prob. dist.) on its "fit"
- Gaussian processes are a Bayesian approach to multidimensional interpolation and regression.
- GPR is therefore a good way of keeping track of uncertainty within an interpolant model.

Gaussian Process Regression

- Consider a set of data, *D*, such that $\mathcal{D} = \{(x_n, y_n), n = 1, \ldots, N\}$
- To fit some underlying function to the data, we typically do

$$y = \sum_i w_i \phi_i(x) \rightarrow y = \vec{\Phi}\vec{w}$$

where $\phi_i(x)$ is a set of test functions and w_i is each function's corresponding weight

- The weights are chosen randomly from a Normal distrubution with 0 mean and variance $\vec{\Sigma}_w$

$$p(\vec{w}) = \mathcal{N}(\vec{w}|\vec{0}, \vec{\Sigma}_w)$$

- To obtain the posterior on the best fitting function, we must marginalise over the weights

$$\underbrace{p(\vec{y}|\vec{x})}_{\text{posterior}} = \int_{\vec{w}} \overbrace{\delta(\vec{y} - \vec{\Phi}\vec{w})}^{\text{likelihood}} \underbrace{p(\vec{w})}_{\text{prior}} d\vec{w}$$

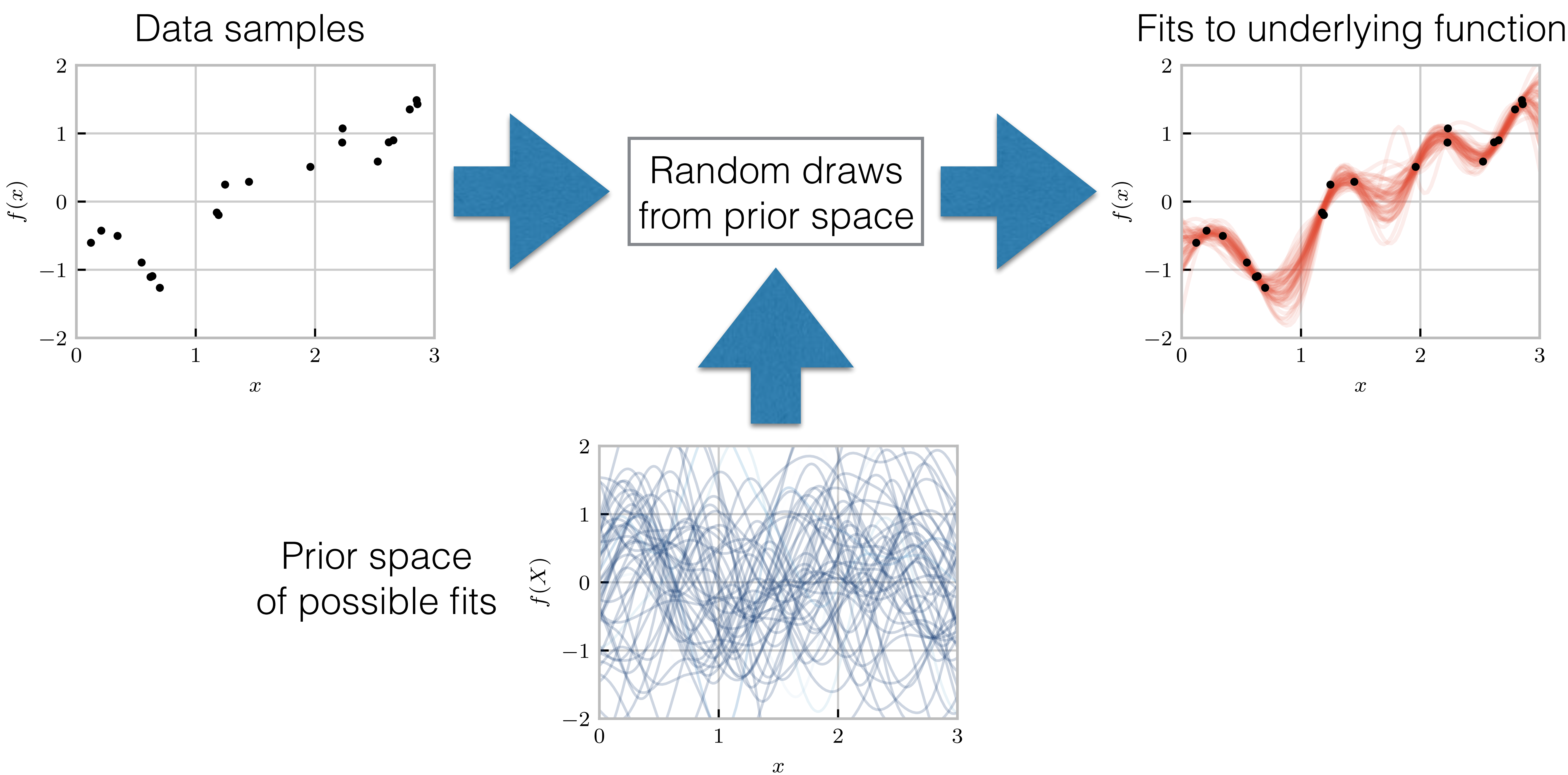
- This reduces to a normal distribution given by

$$p(\vec{y}|\vec{x}) = \mathcal{N}(\vec{y}|\vec{0}, \vec{K})$$

where ***K*** is the covariance matrix

$$[K] = k(x^n, x^{n'})$$

Example


Data samples
Random draws
from prior space
Fits to underlying function
Prior space
of possible fits
$f(x)$
$f(X)$
x
2
1
0
−1
−2
0
1
2
3

Gaussian Process: prediction

- Since we have $p(\vec{y}|\vec{x}) = \mathcal{N}(\vec{y}|\vec{0}, \vec{K})$ then if we want to predict a value *y** at a new parameter choice, *x**, then we have

$$p(\vec{y}, y^*|\vec{x}, x^*) = \mathcal{N}(\vec{y}, y^*|\vec{0}, \vec{K}^+) = \mathcal{N}\left(\vec{y}, y^*|\vec{0}, \begin{bmatrix} K & K^* \\ K^{*T} & k \end{bmatrix}\right)$$

- This becomes

$$p(y^*|x^*, \mathcal{D}) = \mathcal{N}(y^*|K^{*T}K^{-1}y,\ k - K^{*T}K^{-1}K^*)$$

- The covariance between different pairs of x locations can be written down in terms of a distance between these pairs; we define a distance function

$$d^2(x_1, x_2) = \sum_{a,b} \Lambda_{ab}(x_1 - x_2)^a(x_1 - x_2)^b$$

- Then, the covariance matrix is a (squared-exponential) function of these distances

$$k_{\mathrm{SE}}(d; \Lambda) = \exp\left(-\frac{d^2}{2\Lambda^2}\right)$$

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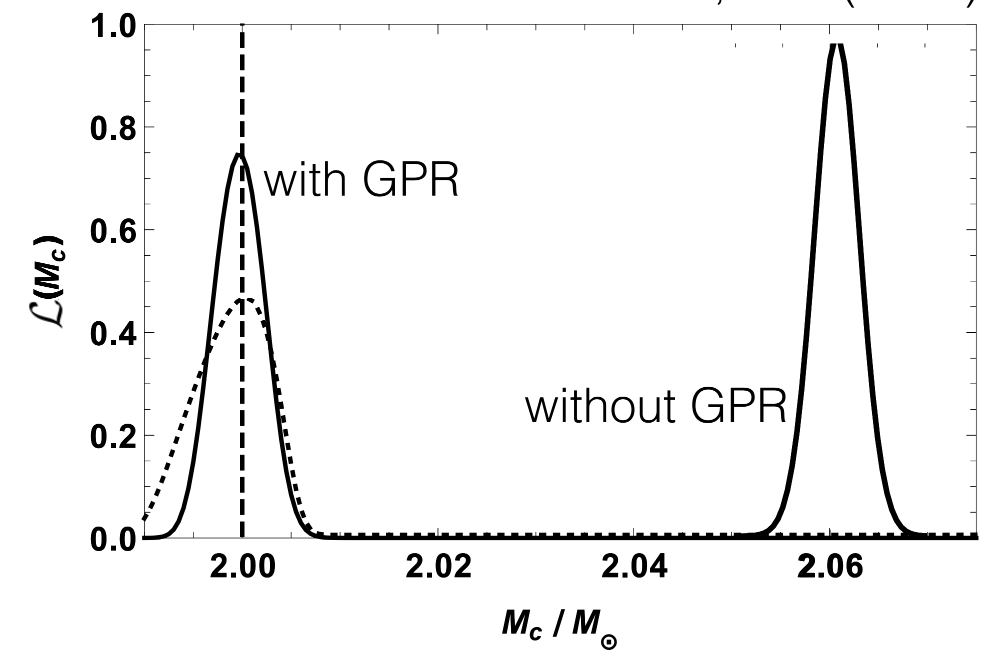
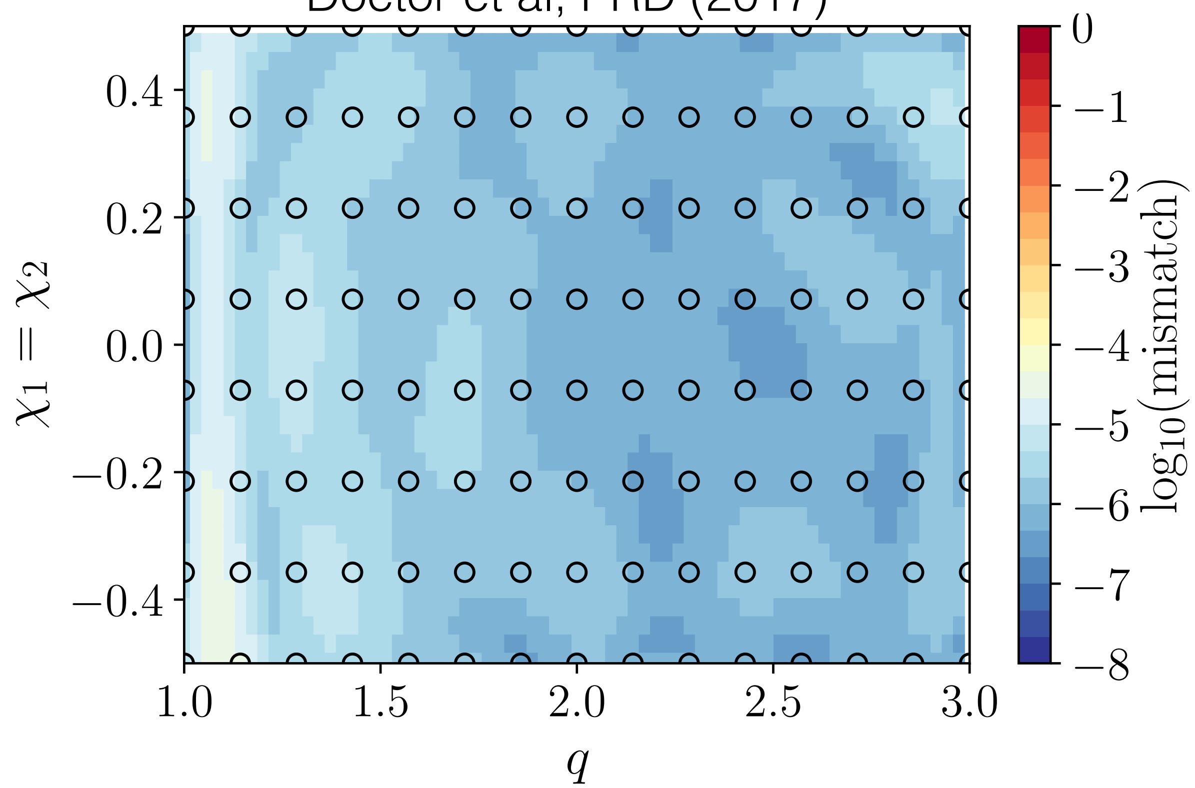
Gaussian Process Regression for gravitational wave data analysis

- Moore and Gair (2014) first demonstrated the use of GPR for gravitational wave data analysis
  - Physical Review Letters 113 (2014)
- This has since been followed by other work; Eg.
  - Moore, Berry, Chua & Gair, PRD 93 (2016)
  - Doctor, Farr & Holz, PRD 96 (2017)
  - Huerta et al., PRD 97 (2018)

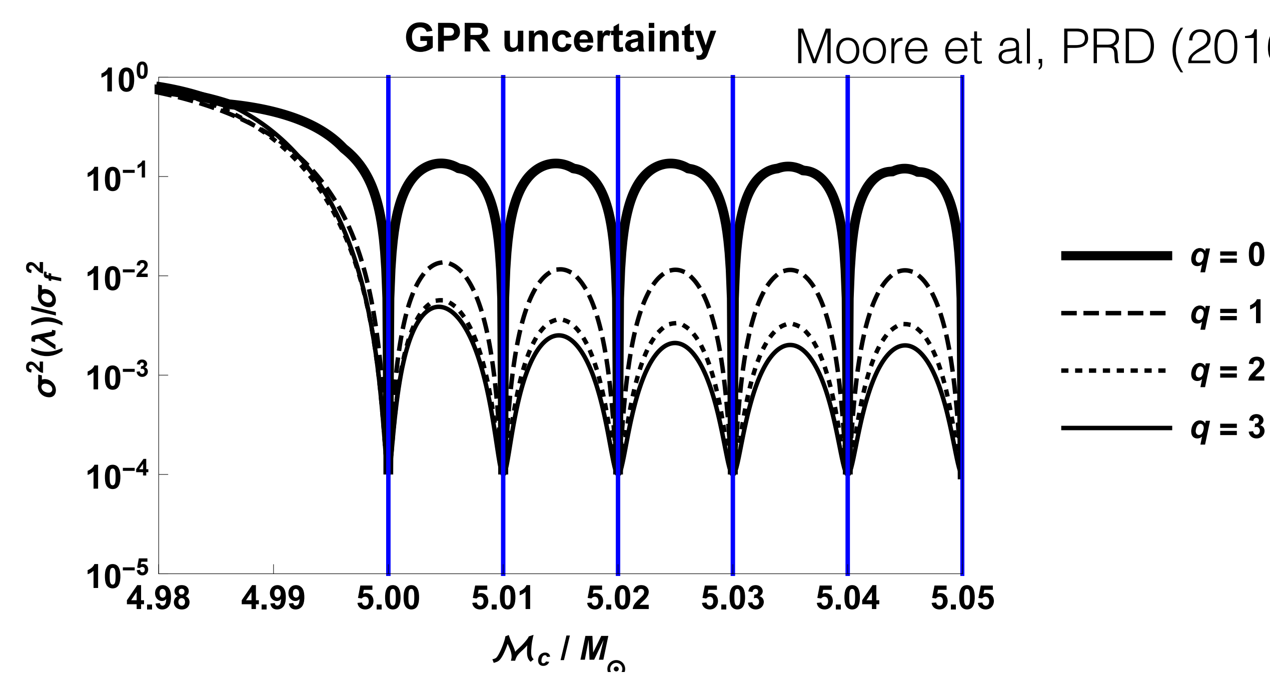
edited from Moore & Gair, PRL (2014)



Doctor et al, PRD (2017)



Moore et al, PRD (2016)



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Numerical Relativity catalogue

- GPR described here uses binary black hole (BBH) merger and ringdown waveforms from NR simulations performed by the Georgia Tech group
  - includes non-aligned spins and precession
- The catalogue had a total of 417 waveforms: 309 used for training GPR and 108 used for testing/validation
- For our GPR, each waveform, $h(t)$, is defined by 10 parameters:

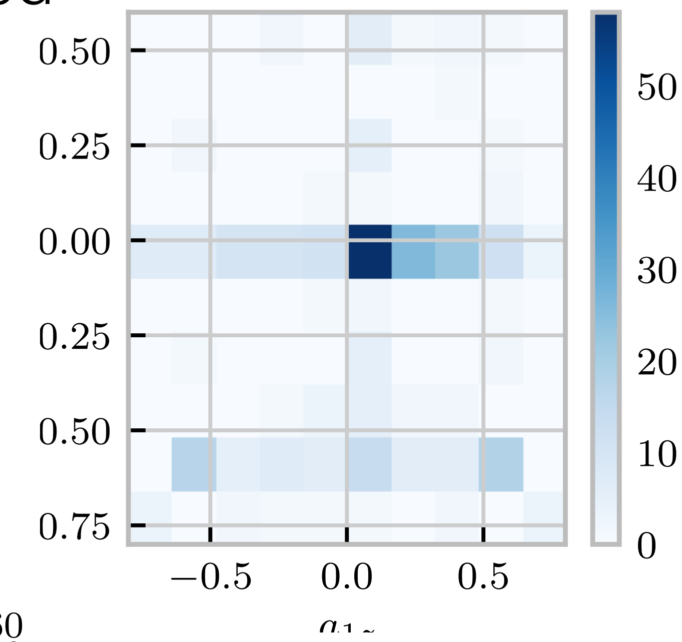
mass ratio: q
BBH1 spin: a_{1x}, a_{1y}, a_{1z}
BBH2 spin: a_{2x}, a_{2y}, a_{2z}
orbital spin: L_x, L_y, L_z

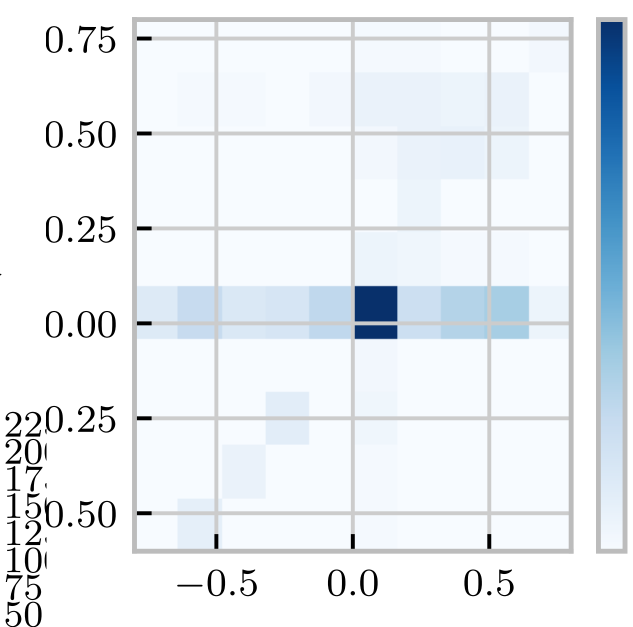
- The GPR is run over these parameters plus time, t, (total of 11 parameters) to estimate the strain amplitude, h, for each choice of these parameters

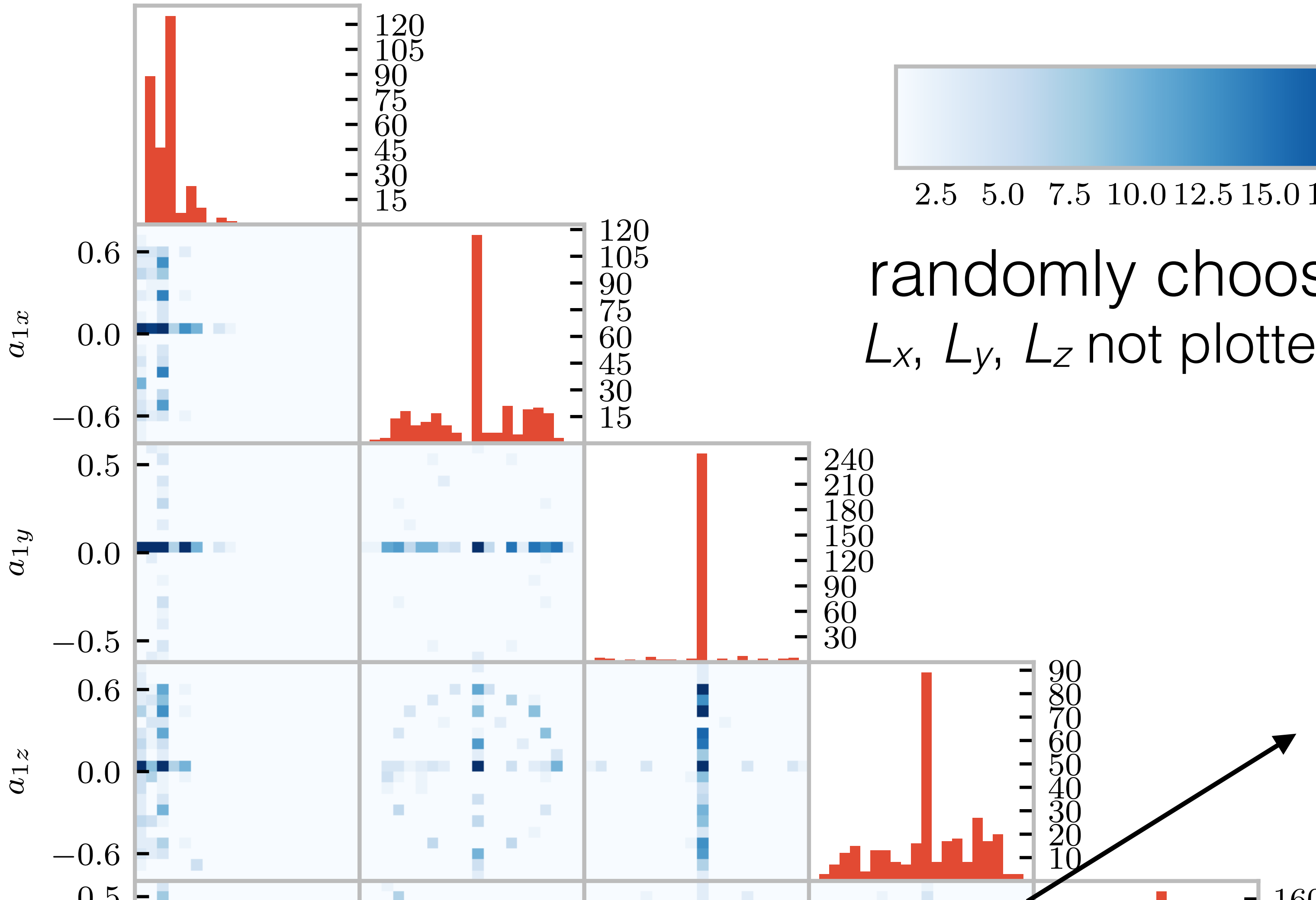
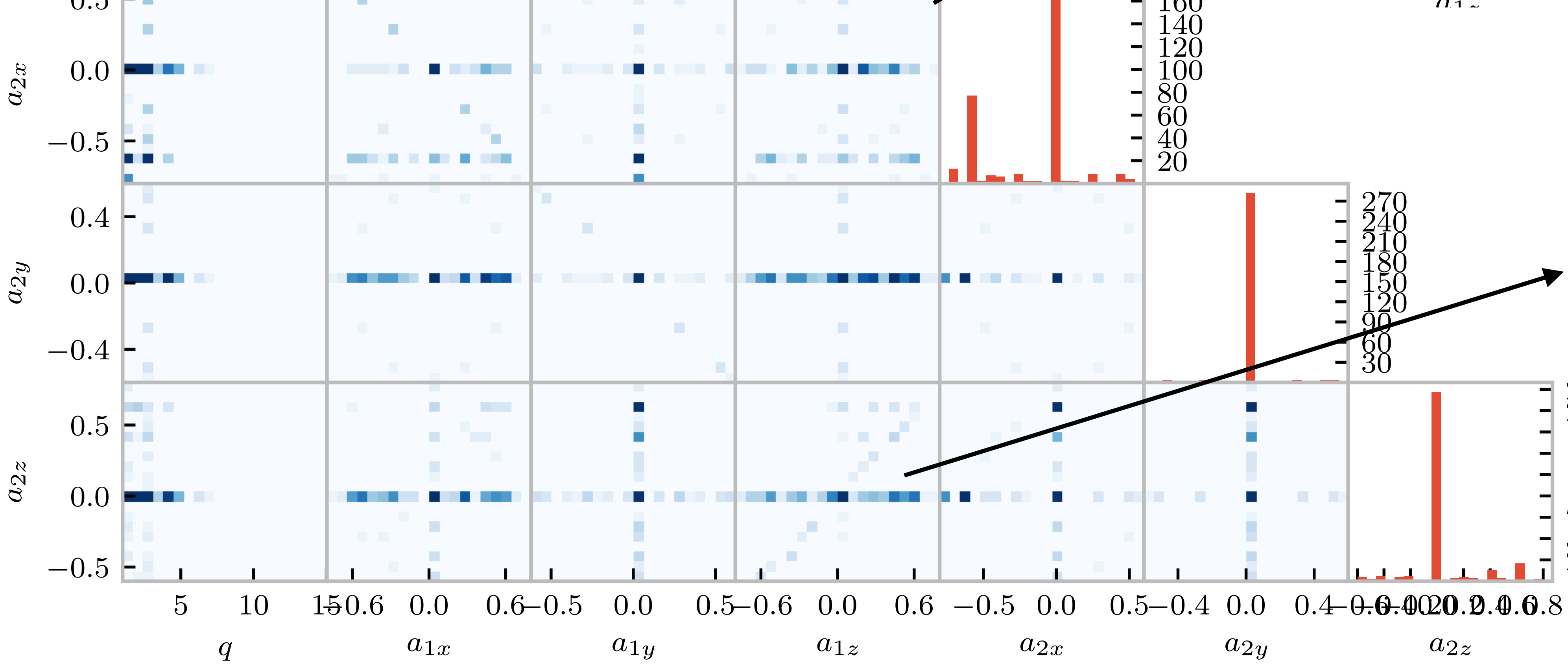
Training data

randomly choose 309 waveforms to train

L_x, L_y, L_z not plotted



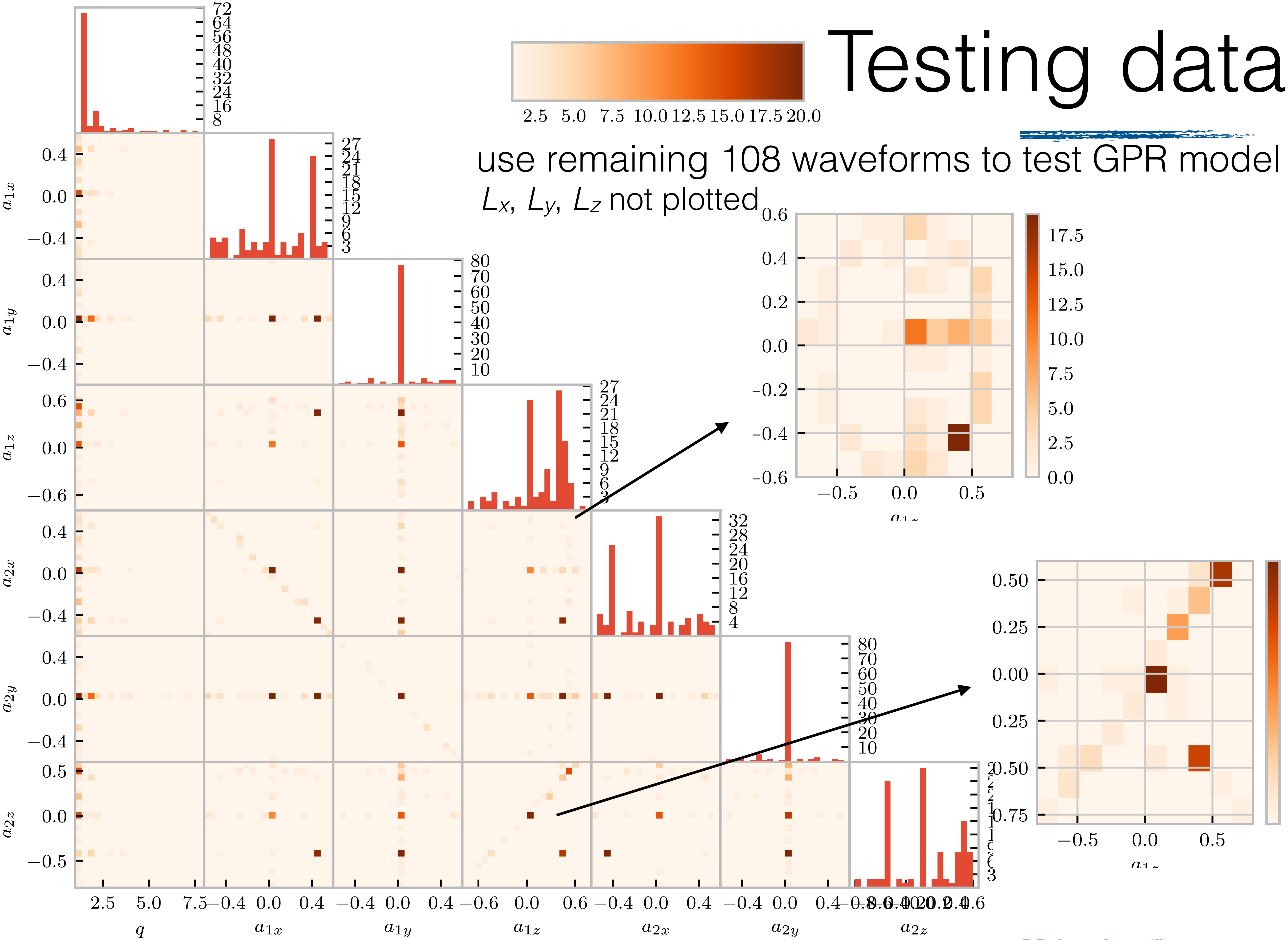


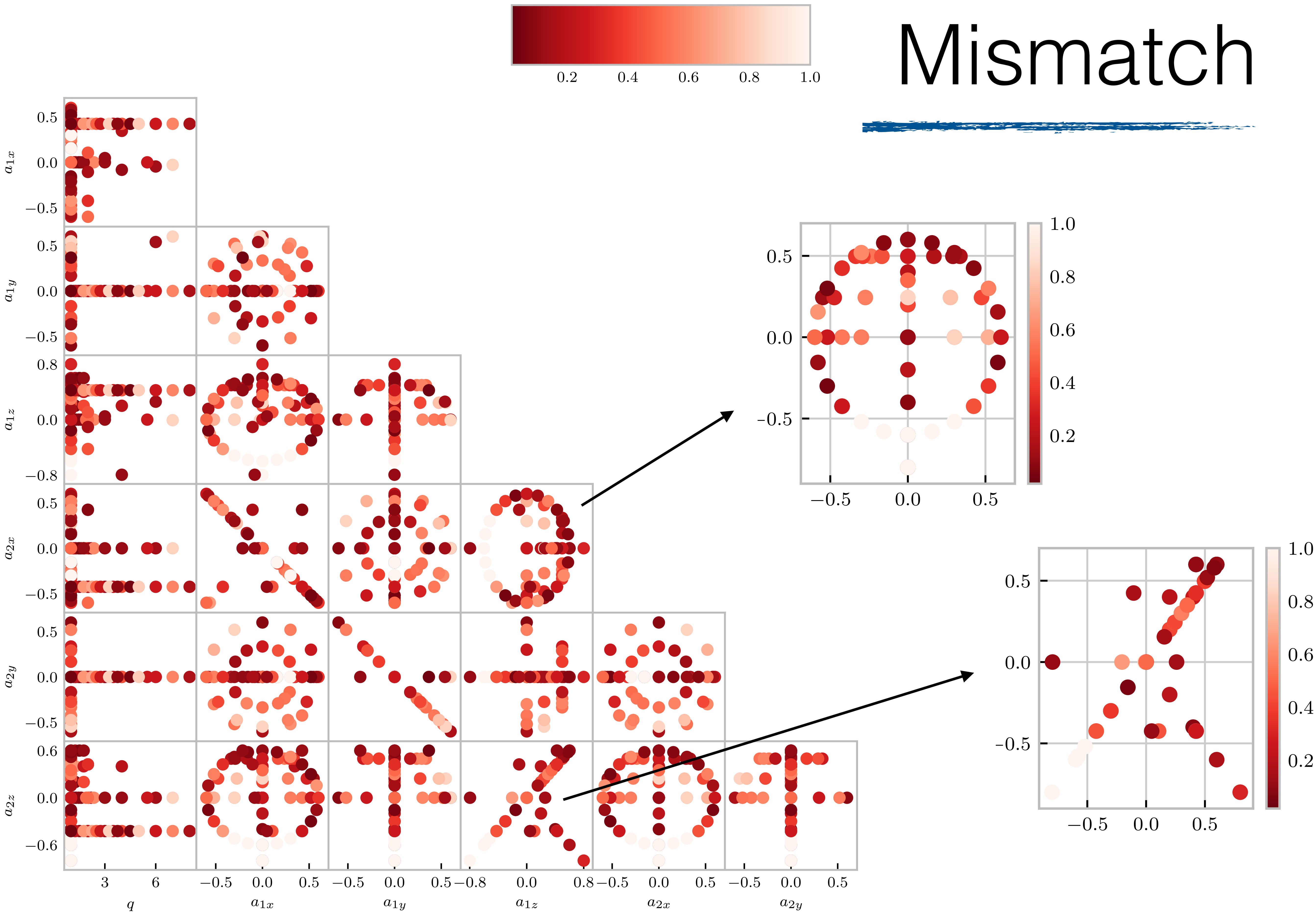
Testing data

use remaining 108 waveforms to test GPR model

L_x, L_y, L_z not plotted



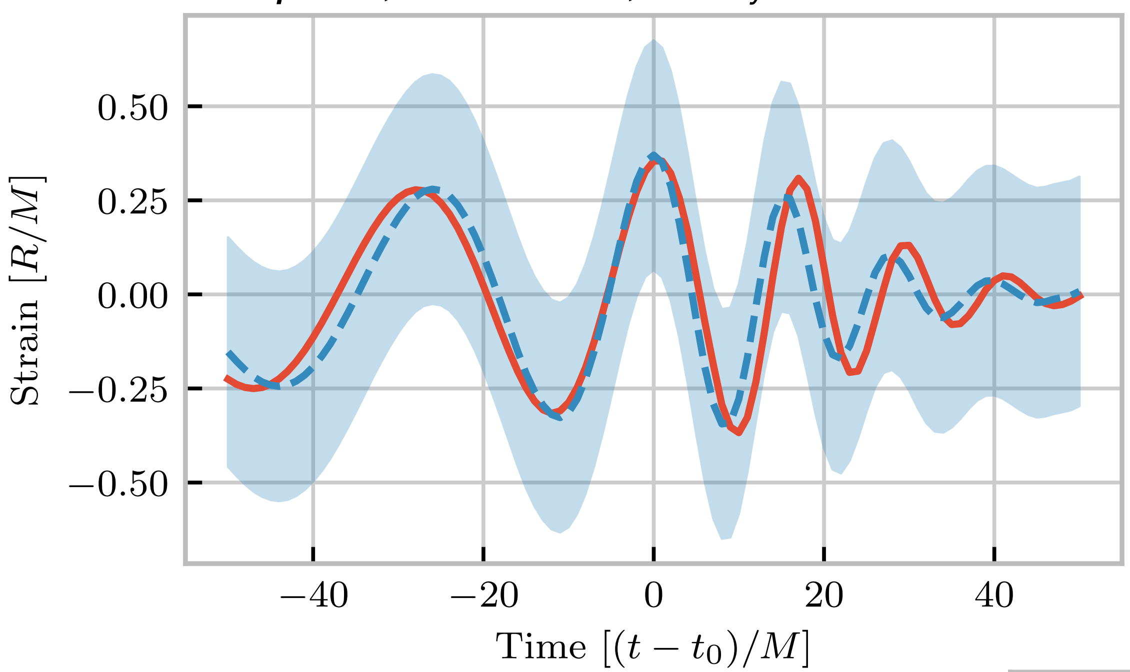
Mismatch



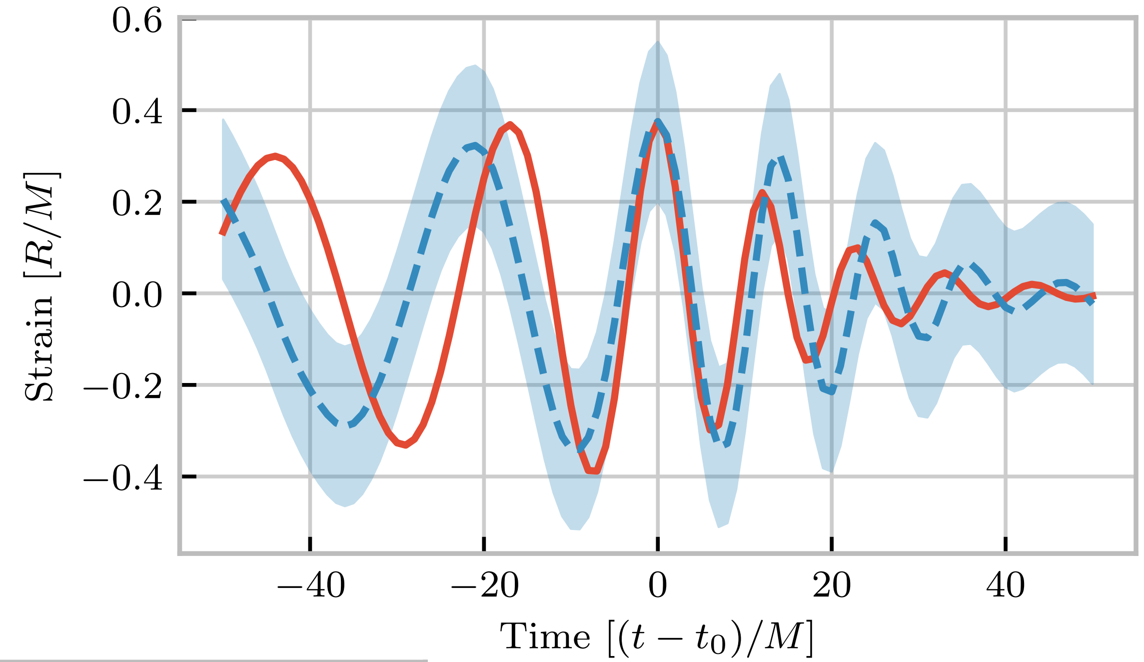
Waveform generation

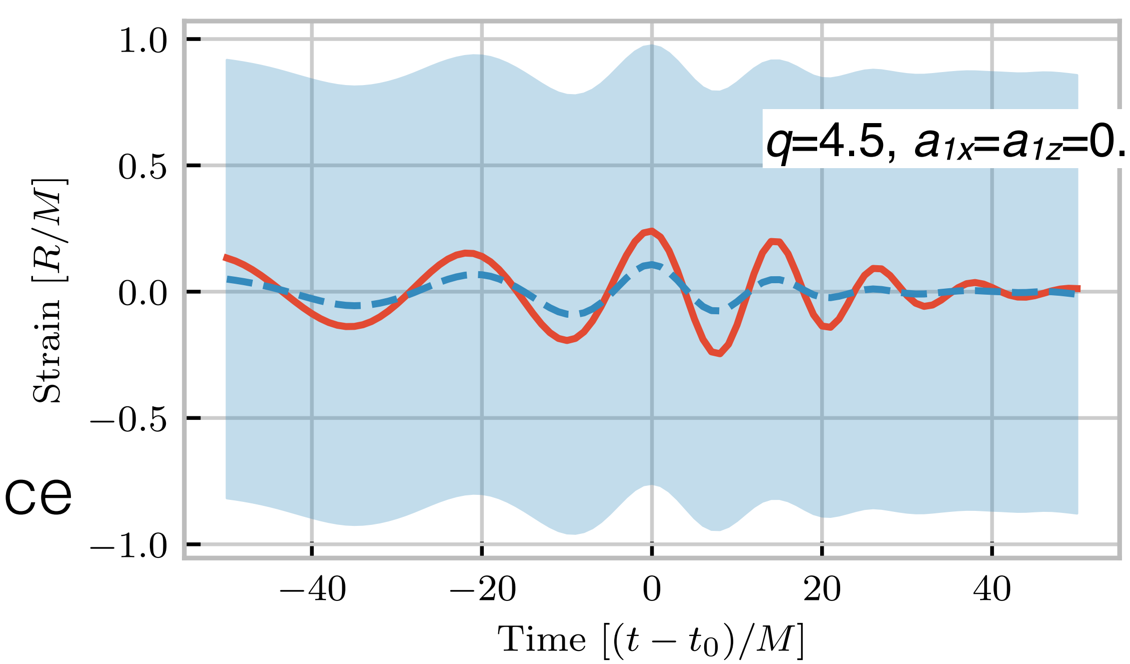
- Since we can predict the strain amplitude for any choice of parameters, we can generate waveforms for any where in the spinning, non-aligned parameter space defined by the Georgia Tech NR catalogue

q=1.6, L_z=0.9107, L_x=L_y=a_1=a_2=0.0



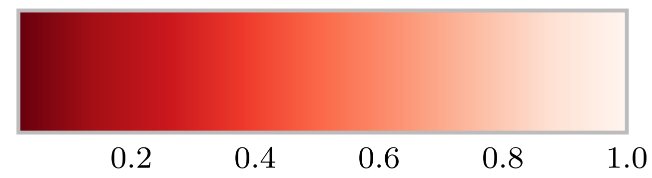
q=1.0, a_{1z}=a_{2z}=0.35, L_z = 0.9073, L_x=L_y=0.0

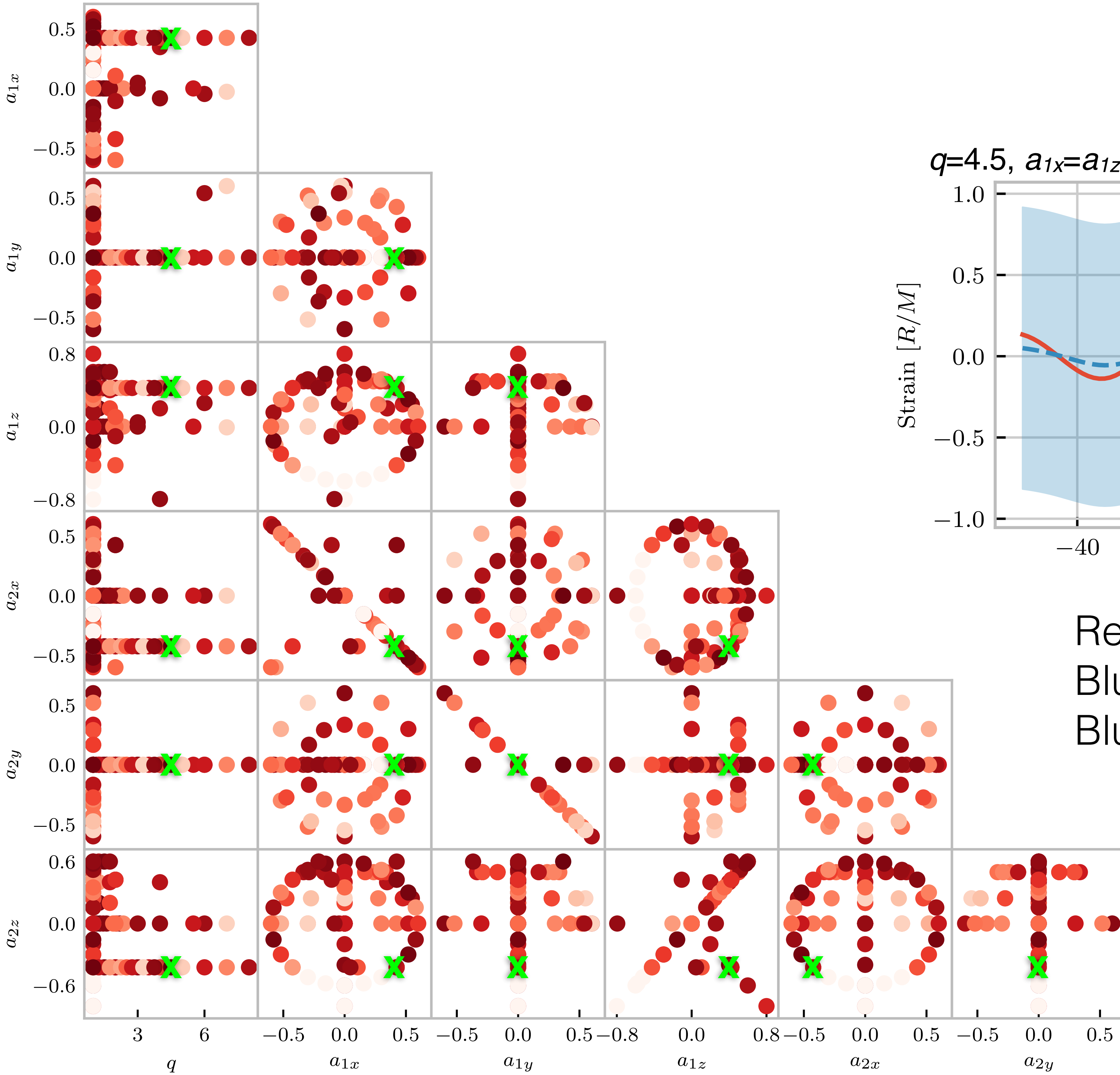


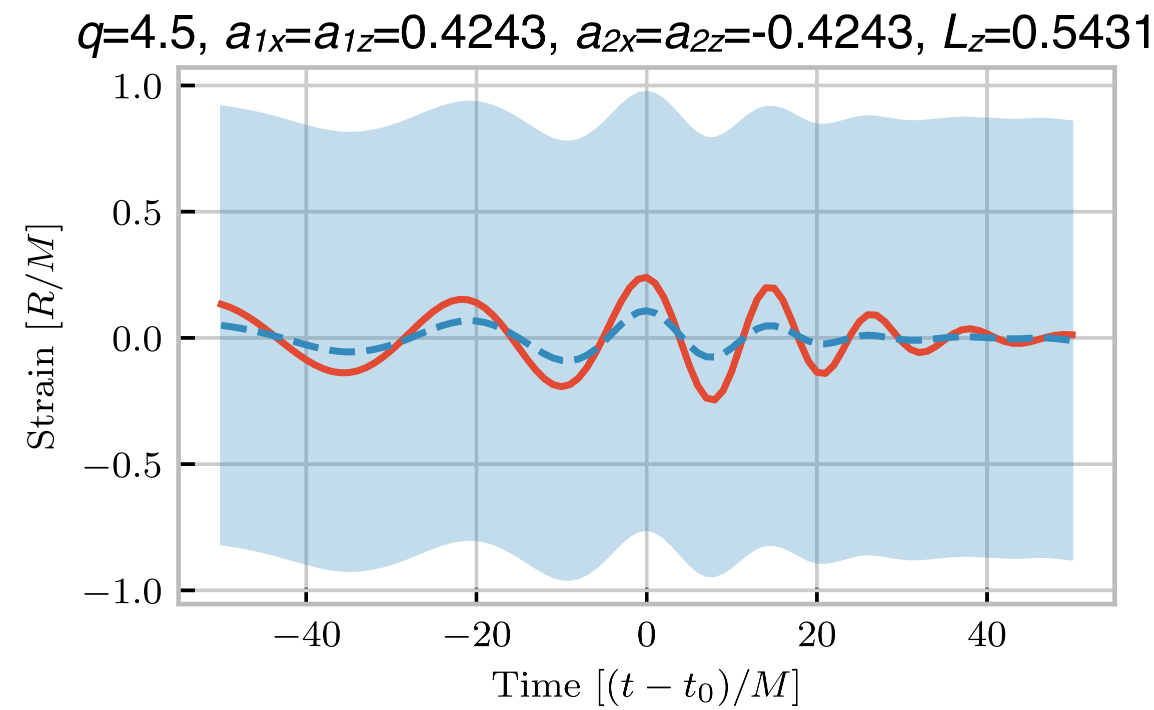


q=4.5, a_{1x}=a_{1z}=0.4243, a_{2x}=a_{2z}=-0.4243, L_z=0.5431

Red - NR waveform
Blue dashed - mean fit
Blue shaded - 90% confidence







Red - NR waveform
Blue dashed - mean fit
Blue shaded - 90% confidence

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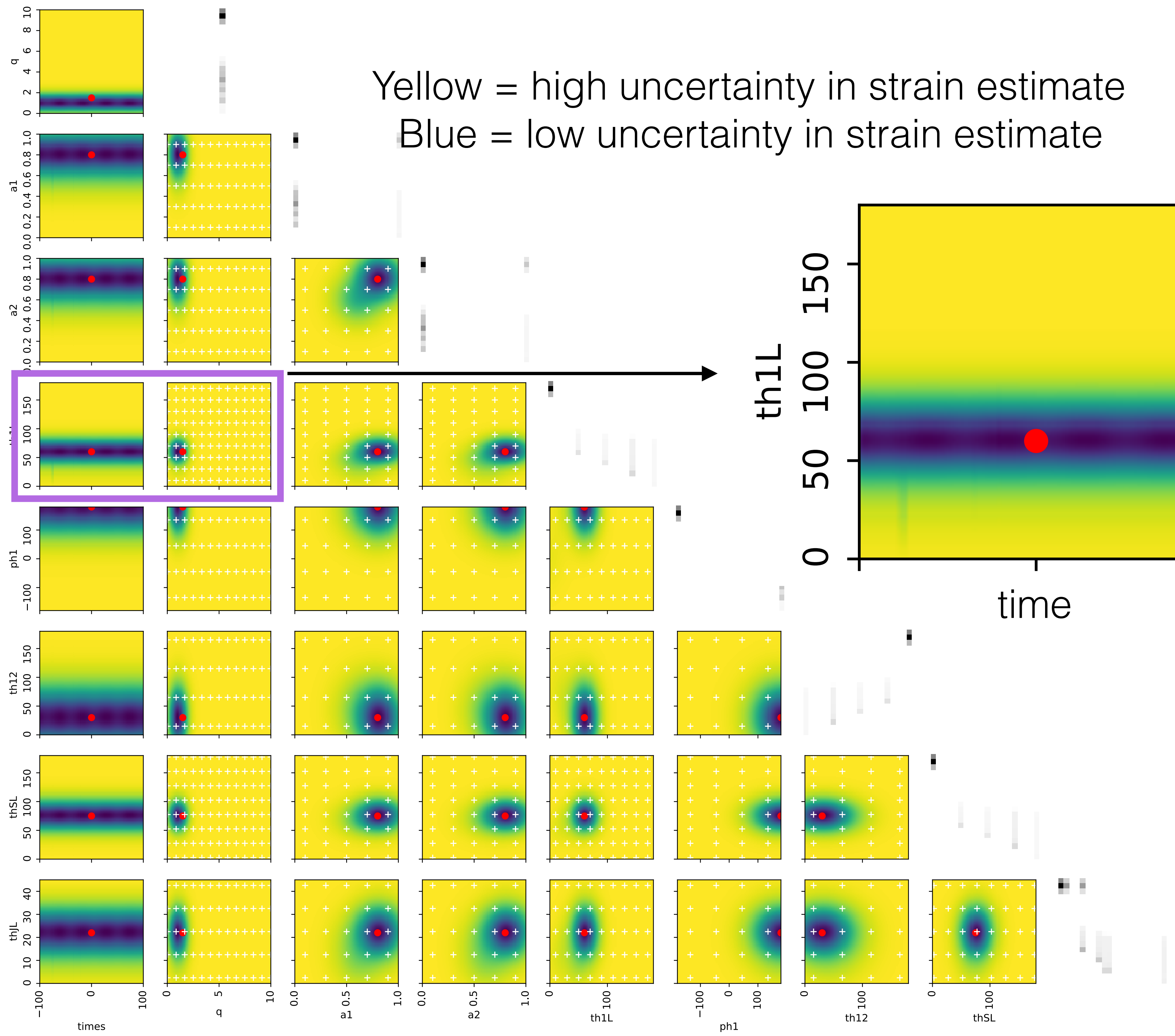
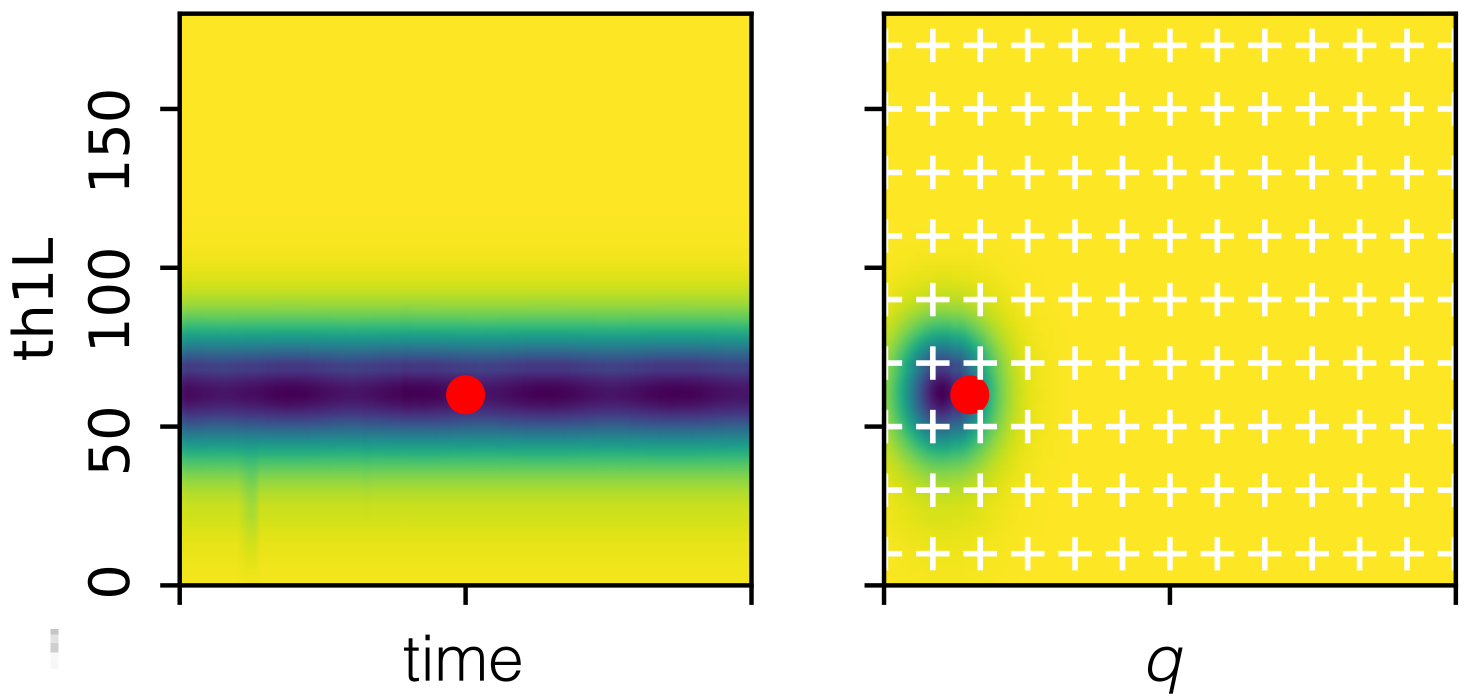
Knowledge of the parameter space

Yellow = high uncertainty in strain estimate
Blue = low uncertainty in strain estimate





Summary & discussion

- We have developed a Gaussian Process Regression scheme which is trained directly on numerical relativity waveforms for BBH merger and ringdown, using all 11 degrees of freedom for each waveform
- We can use the GPR to generate h(t) waveforms for any combination of parameter values
  - good initial results, further optimisation ongoing
- We can also use the GPR to characterise the uncertainties across the entire parameter space defined by NR catalogue
- GPR outputs can also inform where to place additional simulations to optimise knowledge of parameter space for minimal number of NR simulations
  - once GPR is trained and validated, request GAtech generate NR waveforms at predict new locations for comparison
- Also applying GPR to other gravitational wave related topics