Fundamental Physics and the Nature of Reality – Part 2
Special and General Relativity

Norman Gray

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Preamble

This block is concerned with special and general relativity, and some bits of it are, I'm aware, rather hard going. What I want to concentrate on, however, are not any of the many details, but on the ideas underpinning these details. These underpinning ideas are, I think, the material in the section on special relativity, and the idea of taking a geometrical approach to general relativity.

1 Special Relativity

Special Relativity really *is* special. It fundamentally challenges our view of the world, and yet the mathematical knowledge required to explain it is negligible, in comparison to other areas of physics. I am going to present a *full* account of special relativity (SR), up to length contraction and time dilation. This will include a couple of equations, but they will include nothing beyond multiplication and addition; furthermore, there is nothing of great intrinsic importance in those equations, and you can safely skip over them. The point I want to emphasise is that, although the material in this block is rather demanding, the axioms of SR and the logic that springs from them are intelligible without any hugely sophisticated mathematics.

1.1 The axioms

I'm going to start off by setting the scene and describing the important terms we're going to use. Then I'm going to describe the two axioms of SR – the only actual *physics* I'm going to introduce in this block – and describe our common-sense view of the world in these new terms. Then I'm going to come to the central part of the block, and describe how the two axioms of SR inevitably upset our notions of time, space and simultaneity.

1.1.1 Inertial frames and the equivalence principle

The notion of an *inertial frame* is central to SR, so it's important to be clear what it is. At heart, it means a 'point of view' where Newton's laws hold, but the point can be well made with examples. Suppose you're sitting in a train which is accelerating out of a station. A ball placed on a table in front of you will start to roll towards the back of the train, rather than staying put, as balls on tables normally do. This observation only makes sense from the point of view of someone on the station platform, who sees the ball as stationary, and the train being pulled from under it. The station is an inertial frame (but see Sect. B.1), and the accelerating train carriage, where things happen such as balls being unaccountably accelerated, is not. Similarly, if you are perched on a spinning children's roundabout, and toss a ball to someone on the opposite side, it'll veer off to one side. This, again, is only sensible from the point of view of someone standing watching all this go on, who sees the ball go exactly where it should, but the person it's aimed at turn out of the way. The playpark is an inertial frame, the spinning roundabout is not. In both cases, you can tell if you're the one in the non-inertial frame: in the first case you feel yourself pushed back into the train seat, and in the second case, your perch on the roundabout stops you flying off, and you feel yourself thrown towards the outside by centrifugal 'force', and held in your place only by the force exerted on you by your seat.

Acceleration and force are intimately connected with the notion of inertial frames – an inertial frame is one which isn't accelerated in any way. From that, you would be correct to conclude that once the train has stopped accelerating, and is speeding smoothly on its way (we imagine a perfectly smooth track), it becomes an inertial frame again; if you closed your eyes, you wouldn't be able to tell if you were on a moving train or at rest in the station. Anything you can do whilst standing on a station platform (such as juggling, perhaps), you can also do whilst racing through that station on a train, irrespective of the fact that, to the person watching the performance from the platform, the balls you're juggling with are moving at a hundred miles an hour, or so.

Now we can grasp the significance of the first axiom of SR, the *Equivalence Principle*: 'all inertial frames are equivalent for the performance of *all* physical experiments'. This is a much more general statement than one about juggling on trains: it says that no matter what you do, be it a mechanical experiment like juggling or an experiment with wires and magnets, you won't be able to tell that you're in one inertial frame rather than another – you can't tell if you're moving. Furthermore, it says that it makes no sense to pick one frame, say the frame of the solar system or of the fixed stars, and say that *that* frame is stationary in some absolute sense, and that everything else is moving. The Equivalence Principle is also known as the Relativity Principle.

1.1.2 The constancy of the speed of light

The other axiom is that 'the speed of light has the same numerical value in all inertial frames'. That seems innocuous at first, but it puts a bomb under our commmon-sense notion of how things move around.

Imagine throwing a ball at, say, 20mph from a car moving at 60mph. That ball leaves you at 20mph, but anyone watching this performance from the roadside will see it move at 80mph. Now imagine shining a torch through the windscreen of this speeding car: the light will leave you at the 'speed of light', denoted c (which is $299792458\,\mathrm{m\,s^{-1}}$, by the way, or about 186 000 miles per second). But the person at the roadside who also measures the speed of the light from the torch will measure, not $c+60$mph, but c exactly. And this is not just because you're moving at such a tiny fraction of the speed of light – the same would happen if you were moving at half the speed of light, or 99% of the speed of light. It is worth emphasising this: if you were moving at half the speed of light, and you shone a torch forward, so that the light left you at the speed c, someone watching you move would measure the speed of the light from the torch not as $1.5c$, but as precisely c. As you can imagine, this radically upsets our notions of speed, distance, and time.

There's no way that I can justify either of these axioms to you – they're not a consequence of anything – they are simply very surprising facts of experience in the universe we live in. There's nothing *necessary* about this – a universe where Newton's laws are true is (obviously) perfectly conceivable, and is mathematically consistent. However such a universe would be very different from ours with, just for a start, very different laws of electromagnetism (light), and so it could be that it is not a universe we could live in.

1.2 Time dilation

This section is where we investigate the consequences of the axioms I've described above. Imagine you're on a train which happens to be charging through a station[1]. You throw a ball into the air and catch it again: how would you describe this? You'd say that the ball started off in your left hand, followed a parabolic path (like anything thrown), landed in your right hand, and that it took one second (say) to do it all. Now imagine you're on the platform watching this go on: how would you describe it now? You'd say that it started off at the start of the platform, landed a good way down the platform, and took one second to do it. These two perceptions agree that the ball follows a parabolic path (different parabolae, yes, but parabolae nontheless – this is the Equivalence Principle at work), but they disagree on how far the ball travelled in flight. That disagreement is easily explained: from the point of view of the observer on the platform, the ball was travelling very quickly, since it had the train's speed as well as its own, so of *course* it covered more ground before it landed again.

None of this is mysterious. I've made it mysterious by the elaborate way in which I've described it. But I've described it that way to pull this perfectly normal situation into line with the next step, the *light clock*.

1.2.1 The light clock

The light clock (see Fig. 1) is an idealised timekeeper, in which a flash of light leaves a bulb, bounces off a mirror, and returns. If the mirror and the flashbulb are a distance L apart, so that the round trip is a distance of $2L$ and I, standing by the light clock, time the round trip as t' seconds then, since distance is equal to speed times time, and since the speed of light is the constant c, it must be true that

$$2L = ct' \tag{2.1}$$

(note that t' here is the time on my watch, standing by, and moving with, the clock).

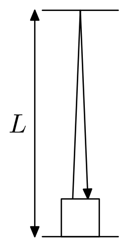

Figure 1

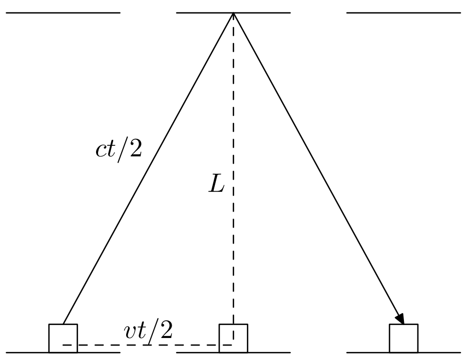

Figure 2

Now imagine the light clock sitting on the train going through the station, as you watch it from the platform. The clock is in motion, at a speed v, while the light crosses the clock, so that when the light bounces off the mirror, the clock is further down the platform that it was when the light started, and by the time it gets back to the bulb, the whole thing is further down the platform still (see Fig. 2). If, standing on the platform, you time all this and find that it takes a time t for the light to make the round trip, then (remembering again that distance is speed times time) the clock will have moved a distance vt down the platform.

[1]. . . and I really mean it's shifting. For any of this to be noticeable the train would have to be travelling at a significant fraction of the speed of light and you'd have to have *very* fast reactions. You might prefer to imagine that the speed of light has somehow been slowed down for us.

How far has the light travelled? We know it travelled at a speed c (the second axiom tells us that, no matter where or how the light was travelling – in a clock sitting beside us on the platform, or a clock zooming though the station in front of us – we would *always* measure its speed to be the same number c), and you timed its round trip at t seconds (this is a number on your watch, standing on the platform). Using no mathematics more sophisticated than Pythagoras' theorem, it turns out that we must have

$$t' = t\sqrt{1-\frac{v^2}{c^2}} \qquad (2.2)$$

(see Sect. B.2 for some details).

Now, the important thing about this equation is that it involves t', the time for the clock to 'tick' as measured by the person standing next to it on the train, and it involves t, the time as measured by the person on the platform, and *they are not the same.*

How can this possibly be? Why is this different from the perfectly reasonable behaviour of the ball thrown down the carriage? The difference is that when you watched the ball from the platform, you saw it move with the speed it was given plus the speed of the train – in other words, the person on the platform and the person on the train had a perfectly reasonable disagreement about the speed of the ball, which resulted in them agreeing on the time the ball was in flight. However, both of them *agree* on the speed of the light in the light clock, as the second axiom says they must. Something has to give, and the result is that the two observers disagree on how long the light takes for a circuit.

So at least one of the clocks is broken? They're both in perfect working order. They only work properly when they're stationary? No, the Equivalence Principle tells us that there's no sense in which either of them is 'more stationary' than the other, so that the clocks work in exactly the same way whether they're moving or not. No...

> *Both clocks are perfectly accurately measuring the passage of time. Time is flowing differently for the two observers.*

A useful image I find is to imagine a clock like some design of ship's log (is this the right name?), which is some type of propellar arrangement towed behind a ship which records how much water the ship has moved through. Analogously, a clock is a device which records how much time the clock has been dragged through.

1.3 Simultaneity

At this point you might be rather unsure of your footing. If time behaves in this bizarre way, what else goes wrong? Well...

1.3.1 More train-spotting

Imagine standing in the centre of a train carriage, with suitably agile friends at either end: Fred (at the Front) and Barbara (at the Back). At a prearranged time, on your carefully synchronised watches, you fire off a flashbulb and your friends note down the time showing on their watches when the flash reaches them. Comparing notes afterwards, you all find that it took some time for the flash to travel from the middle of the carriage to the end, and that your friends have noted down the same arrival time on their watches, time '3', say (not seconds!)[2]. In other words, Fred's and Barbara's watches both reading '3', are *simultaneous* events.

[2]These watches are obviously not calibrated in seconds. For a flash of light to travel 10m, would take roughly 3×10^{-8} s – three hundredths of a millisecond.

But if this train is moving through a station as all this goes on, and you look from the platform into the carriage, what would you see from this point of view? You would see the light from the flash move both forward towards Fred and backwards towards Barbara, but remember that you would *not* see the light moving forwards faster than the speed of light – its speed would not be enhanced by the motion of the train – nor would you see the light moving backwards at less than c. Since the back of the train is rushing towards where the light was emitted, the flash would naturally get to Barbara first. If, standing on the platform, you were to take a photograph at this point, you would get something like the upper part of Fig. 3. Barbara's watch reads '3'. But at this point, the light moving towards Fred cannot yet have caught up with him: since the light reaches Fred when *his* watch reads '3', his watch must still be reading something less than that, '1', say. In other words, Barbara's watch reading '3' and Fred's watch reading '1' are simultaneous events in *your* inertial frame on the platform.

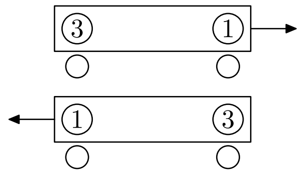

Figure 3

What is going on here? Are these events simultaneous or not? What this tells us is that our notion of simultaneity is rather naïve, and that we have to be very careful exactly what we mean when we talk of events as being simultaneous. The only case where two events are quite unambiguously simultaneous is if they take place at exactly the same point in space. That's why we could say without hesitation that the light reached Barbara when her watch read '3', because that's what she had seen and noted down.

1.3.2 Length contraction and time dilation

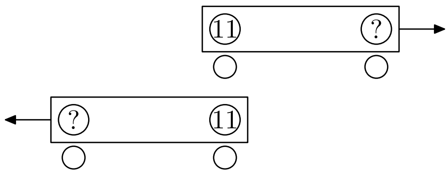

Figure 4

We're not finished with the trains, yet. Imagine now we're standing on the platform and see now two trains go past. We've cunningly arranged the speeds, timetable and flashbulbs so that we can get the photograph in Fig. 3, where the light has reached both rear observers and neither front one. Now pause a moment, and take another photograph when the two rear observers are beside each other, this time getting Fig. 4.

After all this hectic fun is over, everyone calms down, ambles together, and compares notes. Barbara (standing at the back of the top carriage) could remark "I saw the front of the other carriage pass us when my clock was reading '3' " (this is perfectly correct, as you can confirm by looking at the 'photograph' in Fig. 3). At which Fred would say "But the back of the carriage passed me at time '1' – it must have been well past me at time '3' ". From this they, and we, can quite correctly remark that the carriage they observed moving past them was measured to be shorter than their own. They have measured the length of a moving carriage, and found that it is shorter than a similar carriage (their own) which they can measure at leisure when stationary. This is *length contraction*.

Fred then says "I looked through the window at the clock at the back of the other carriage, and I noticed that it was reading '3', when mine was reading '1' – it was two seconds fast". Barbara says "Well, I saw that *same* clock a bit later [in Fig. 4], and it was reading '4', just like mine – it wasn't fast at all." They know that their own clocks were synchronised throughout the encounter (they can make sure that their clocks are synchronised at some point, and they know that they both go at the same rate), so they can only conclude (correctly) that the clock they both saw was going more slowly than theirs were. Time in the other carriage is passing more slowly than in their own, the same phenomenon as we saw with the light clock, which led up to the mathematical expression of this in Eqn. (2.2) above.

The extraordinary thing is that Barbara and Fred's counterparts in the other carriage would come to precisely the same conclusions. Because this setup is perfectly symmetrical, they would measure Barbara and Fred's clocks to be moving slowly, and their carriage to be shorter. There is no sense in which one of the carriages is *absolutely* shorter than the other[3].

1.3.3 Is there anything I can hold on to?

At this point you may be feeling rather seasick. People tend to find relativity rather disorienting, as more and more pillars of their dynamical intuition are kicked away. You can end up in the situation where you trust none of your steps at all, and find yourself unable to move at all, for fear that the whole edifice will come tumbling down.

Distinguish between what you know, and what you intuit. In fact, rather little of what you know has changed: it comes down to not much more than the addition of relativistic velocities (loosely '$c+0.5c=c$') and the relativity of simultaneity ('not everyone agrees that two events are simultaneous'), both of which are fairly direct consequences of the second axiom, in Sect. 1.1.2. Unfortunately, both of these eat away at our intuition of how moving objects behave.

Look at what has *not* changed, however. For example, in our discussion of the trains above, it was still true that the backward-moving light flash hit the rear of the carriage before the forward-moving one hit the front because, reasonably enough, the carriage rear was moving *into* the flash. It was still true that the order of events at a single point in space is absolutely fixed. It was still true that when Fred and Barbara looked at their own watches those watches told them the local time accurately, and when they looked at the nearby watches on the other train (which are, in principle, at the *same* point in spacetime as Fred and Barbara), they could reasonably measure what they saw there.

In this focus on (local) measurement, we can see the influence of the philosophical *positivism* which influenced Einstein in his development of SR. This focus can also help us understand what is going on. If we concentrate on working out what the participants would measure – that is, on what they would see happening at their own point in spacetime – rather than on what we intuitively expect them to see, then when we find ourselves puzzled, saying 'that couldn't possibly happen, because…', we have someplace to start.

1.4 Making sense of all this

Hold a ruler in front of you. How long is it? A foot, of course. Now hold it at an angle: how long is it now? It's still a foot long, but it doesn't look like it – because it's at an angle, we see it as being shorter. Our intuitive knowledge of the geometry of three-dimensional space tells us that the thing really is the same length as it always was, but because it's been turned in space, its projection onto our field of view is less, and so this observed shortening is merely an artefact of the geometry of the situation.

Special Relativity tells us that we do not in fact live in the three-dimensional world we thought we did. We live in a four-dimensional world, albeit one with an unfamiliar geometry, and time is one of the directions. That isn't just wordplay: movement is something with a temporal as well as a spatial element, and when we move, we 'rotate' in this four-dimensional space. When others see us move, they experience the analogue of the foreshortening of the ruler and they naturally see our time, and our spatial extent, change.

[3] Note also, by the way, that there is nothing fundamental about the presence of this third party, taking photographs. They are there simply to provide us with some photographs – all the effects here can be quite adequately demonstrated with only the sets of observers on the moving trains

1.5 Summary of Special Relativity, and its notion of 'Space-time'

Before we go on to discuss the generalisation of relativity, I should like to pull together some of the features of the above discussion of *special* relativity, to highlight some aspects that will be useful to us in discussing its more general partner.

Special Relativity deals heavily with *inertial reference frames.* Put simply, these are reference frames which are not accelerating in any way, so that their movement remains the same both in speed and in direction. That means that an object placed at rest in an inertial frame will remain at rest, and an object given an initial velocity will keep that velocity indefinitely. Above, I repeatedly used a station platform and a train moving at a constant velocity as examples of inertial frames: I want to emphasise now what I only mentioned above, that these are not *strictly* inertial frames. If you hold a ball out in front of you and let go, it will not stay in place, but will accelerate towards the ground. As long as we confined ourselves to horizontal movement, we could ignore this subtlety (we could ignore the force of gravity) but now we will have to be more precise. See also the comments in Sect. B.1.

Special Relativity is the consequence of two axioms. The first is the *equivalence principle* for SR: the principle that all inertial frames are equivalent, that the laws of physics are just as valid when you're moving as when you're stationary. The second is that the speed of light in a vacuum, c, is always measured to have the same numerical value, irrespective of how quickly we, or the source that emitted it, are moving. Contrary to our Newtonian expectations, its speed is not enhanced by the motion of the flashbulb that emitted it.

That led to the *light clock.* Someone standing by the clock can measure, with a clock, how long the light takes to bounce to the mirror and back. Someone else, watching the clock go by, can watch that *same* to-and-fro motion, and can use their clock to measure how long the light takes to bounce back, and can measure how far the clock travels in that time. Since the light travels a greater distance for the second observer but, from the second axiom, both observers measure its motion to have the same speed, the time that the second observer measures must be greater than the time that the co-moving observer measures. From the first axiom, we know that the clocks in the two inertial frames are both accurately measuring the passage of time, and we are forced to conclude that the second, 'stationary', observer will have quite genuinely observed less time flow by in the clock's frame than in her own.

A consideration of what the folk on two passing trains would see of each others watches led us to the general conclusion that our measurements of distances and times between events in frames which are moving relative to us, are *different* from the distances and times between those same events, as measured by folk sitting in that moving frame.

2 General Relativity and the geometry of space-time

We can make some sense of the behaviour we have seen by looking at the analogous situation where a line of observers are examining a pole at some angle to them (see Fig. 5). The observers can measure the 'length', l, of the pole by measuring the distance between the two observers who can see one end of the pole directly in front of them; and they can measure the 'depth', d, of the pole by having the same two observers measure the distance to the end of the pole that they can see. Both these measurements l and d have *some* physical significance, but as long as the observers insist on using l and d as their measurement of 'the length of the pole', then they will get different answers for the pole's length depending on where they are standing when they make the measurements. This is an undesirable situation, which is only resolved when we make use of our knowledge of geometry to construct the invariant length, s, of the pole, where Pythagoras tells us that

$$s^2 = l^2 + d^2. \tag{2.3}$$

No matter how the two observers are arranged, it is a geometrical fact that the paired values of l and d that they measure will lead to the *same* value for the invariant length s.

Now consider measuring the separation of two events such as a banger going off here at this time, and one going off there at that time. If we insist on using the distance between the bangers, l, or the time between the explosions, t, as our measure of the 'separation' of the two events, then we will measure different values for the 'separation' depending on how fast we are moving relative to the events (or, equivalently, how fast they are moving relative to us). It is only our knowledge of Special Relativity that allows us to resolve the conflict and calculate the *invariant interval*, s, separating the events, which is related to the time and distance separating them by

$$s^2 = (ct)^2 - l^2. \tag{2.4}$$

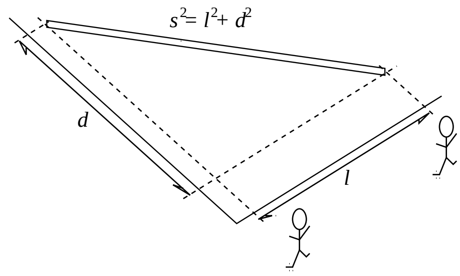

Figure 5: Observers make different measurements of the 'length' of the pole.

No matter how fast we are moving relative to the two bangs, it is a fact that the values of l and t that we measure will lead to the *same* value for the invariant interval s.

The similarity between these two situations is more than a coincidence – we can very profitably approach SR in geometrical terms. In this view, space and time are not fundamentally different things. We do not move in time through the three-dimensional world of our intuitive experience: instead we exist in a four-dimensional world in which time and space are merely different 'directions'. This four-dimensional world is called *spacetime*.

Geometry in spacetime is not the same as the Euclidean geometry we are familiar with. In Euclidean geometry, we have familiar theorems like Pythagoras' theorem, Eqn. (2.3), the familiar statement that "the square on the hypotenuse is equal to the sum of the squares on the other two sides". In this new four-dimensional world, the analogue of Pythagoras' theorem, and the invariant we can cling to in our dizzy new circumstances, is Eqn. (2.4). It is clear that this is similar to Pythagoras' theorem; it is equally clear that the difference is hugely significant.

This is the first hint of the fundamental role that geometry has in our understanding of the nature of spacetime.

2.1 The geometry of a rotating frame

Our consideration of the mechanics of motion at high speed – Special Relativity – has been confined to high *constant* speeds, unaffected by gravity. When we generalise from this to motion near large masses, or in accelerating frames, our first question has to be 'what can we still hold on to?' I'll try to answer that by considering a particular kind of accelerated motion: rotation.

It may not be immediately clear to you that rotation at a constant speed involves any acceleration at all. To stay on a roundabout, however, you definitely need to hold on – the roundabout is obviously exerting some pull on you which, in another context, you would naturally associate with an acceleration.

Newton's first law says that in the absence of acceleration, objects move with a constant velocity. In physics, the term 'velocity' includes both speed and direction, so this law is formally stating what you would expect to see, watching a puck move across an ice rink: the puck moves at a constant speed, in a straight line. The only thing that disrupts this motion is if some other agency, such as a hockey stick, or some roughness on the ice, or a draught from the side, accelerates it away from this natural motion.

So, if you were to loosen your grip on the roundabout, you would resume your natural motion, and fly from the roundabout in a straight, tangential, line. Your grip on the roundabout is accelerating you away from the motion you would have in its absence: motion at constant velocity.

2.1.1 Co-moving inertial frames

Suppose you are holding on to the outer rim of a turning roundabout, just as someone runs straight past it. If they have chosen their speed to match yours, then there will be a short time when you and they are moving at the same speed, in the same direction. From the point of view of someone perched on the centre of the roundabout, therefore, there will be no difference between rulers and watches held by you and by the runner (see Fig. 6). They can therefore use SR to discuss the motion of the runner, and apply the results to you, temporarily moving at the same speed. The inertial frame attached to the runner is called a *co-moving* inertial frame. Taking over our results from SR, the person perched at the centre of the roundabout will measure your ruler (and the runner's) to be shorter than their own, and will measure your watch to be showing less time passing than their own (do remember that we are implicitly assuming that the runner and the roundabout are moving at relativistic speeds here).

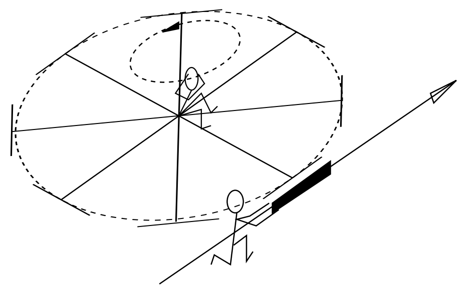

Figure 6: Running past a roundabout

We have come to a remarkable conclusion. All the rulers throughout an *inertial* frame are the same length, and all the clocks move at the same rate, but if we imagine a frame turning with the roundabout – an *accelerated* frame – we see that rulers and clocks have systematically different lengths and rates at different points. This has an interesting consequence. . . .

2.1.2 Geometry is distorted

If you draw out a circle on the ground, measure its diameter, and measure its circumference, perhaps by laying a ruler round the edge, you will find that the ratio of the circumference of the circle to the diameter will be $\pi = 3.1415\ldots$. What if you were to do the same thing on the roundabout?

Use your ruler to mark out a circle on the roundabout with a radius of, say, five ruler-lengths. Now measure its circumference by repeatedly laying the ruler along the circle you've marked. Someone standing by the roundabout, watching all this going on, will agree about the diameter of the circle, but will see these circumferential rulers whip past them at speed, and so will measure them as being shorter than a similar ruler of their own. Naturally, therefore, it will take *more* rulers to mark out the circumference than it would if the roundabout were stationary[4]. In other words, the ratio of the measured circumference to the diameter – the number of ruler lengths it takes to circumnavigate the circle, divided by the number it takes to cross it – will be *greater than* π. Geometry on the roundabout does not obey Euclid's axioms. This is not an illusion – the frame attached to the rotating roundabout is a different shape from one that isn't accelerating.

This is a good place to emphasise that a foot-rule remains a foot-rule, even when the space it's sitting in expands. If all the atoms that make up the rule were free particles, sitting quietly in space-time, then when that space expanded, the atoms would remain in position with respect to the space, and expand with it. However, the atoms are *not* free particles, and are instead held together by forces (largely electromagnetic) which mean that the particles remain in position with respect to each other. Analogously, imagine a group of ants sitting on a balloon which is being inflated: as

[4] You can think of this more directly by imagining a spinning spoked wheel. As it is spun up to relativistic speed, the spokes will not change in length, but the rim will burst apart, as it will no longer be long enough to reach all the way around the circumference. If the wheel is 1 m across, then the rim will still be 3.14 m long, but the circumference of the circle has *expanded* to become longer than this.

the balloon expands, the ants will move further and further away from each other; but if they sat in a circle and held hands (how picturesque!), they would be dragged over the surface of the expanding balloon, with the result that the circle would remain its original size.

2.2 The equivalence principle, GR, and curvature

The equivalence principle in GR is just as central as the corresponding principle in SR. However, it is difficult to discuss the equivalence principle in detail without becoming enmeshed in subtleties which, though they are very important, easily distract from the main line of the argument, which aims to show how the framework on which SR is built is incompatible with the existence of gravitating matter.

2.2.1 The nature of the problem

The argument is as follows:

1. Briefly, the equivalence principle is: in a small laboratory, physical systems in a gravitational field behave the same way as in an accelerated frame.

2. The behaviour of a free particle at loose in an accelerated frame is obviously independent of the composition of the particle. The equivalence principle tells us that the same *must* be true for a free particle in a gravitational field.

3. Any frame attached to a freely falling particle behaves like an inertial frame, and we can *define* an inertial frame as being one attached to a particle in free-fall.

4. Thus two inertial frames (such as those attached to a particle falling down a liftshaft on earth, and one cruising through outer space, or two of our free-falling lifts on opposite sides of the earth) can be *accelerating* relative to each other.

In point 1, the limitation to a small laboratory is important. In a laboratory which is large enough, in time or in space, you will always be able to tell whether your laboratory is accelerating, or is sitting near a planet. But the difference between these phenomena is a 'large' scale effect, and *locally*, the two effects are equivalent.

Point 2 is known as the *universality of free fall*, and translates into the profound observation that all masses, of whatever material, fall in a gravitational field with *exactly* the same acceleration. Galileo was the first to emphasise this (and was why he was reputedly imperilling passers-by by flinging cannonballs from the leaning tower of Pisa), and it is an extraordinary, and completly inexplicable fact. That is, the gravitational mass of a particle, which tells you how much it is attracted to the Earth, and the inertial mass, which tells you how susceptible it is to being pushed, are exactly the same. This direct consequence of the equivalence principle has been elaborately verified.

Free fall means motion under the influence of gravitation alone. Since gravitation affects all matter equally (the universality of free fall), the particles in a plummetting lift will not be accelerated relative to each other. Things will stay where they are put, so that the lift acts as an inertial frame, hence point 3.

Point 4 is the problem. In SR, *all* inertial frames move with constant velocity relative to each other. They cannot accelerate. Here is a straightforward contradiction with SR, created by the presence of gravitating matter.

2.2.2 The nature of the solution

It is important to be clear exactly where the problem is.

When we are dealing purely with local inertial frames, for example the frame attached to a rocket cruising through space, or the frame attached to a lift plunging towards destruction, we can use Special Relativity to discuss motion in those frames. It is an indirect consequence of the equivalence principle that, for a particle in free fall, the geometry of spacetime is *locally* that of SR (which, if you fancy a bit of name-dropping, is called Minkowski space).

That word ‘locally’ is the important one. We have seen that Special Relativity (or, more precisely, the notion of globally applicable inertial frames) fails in the presence of matter because that matter causes inertial frames attached to freely falling particles to accelerate relative to one another. Although SR correctly describes the local space-time of a freely falling frame, it is the job of GR to tie these local inertial frames together and so to tell you how to move from one to the other.

In other words, we already know the essential local properties of spacetime, namely that described by Special Relativity. General Relativity describes the *global* properties, with acceleration and matter included.

2.2.3 The curved geometry of spacetime

It should not come as any surprise, after the various mentions I’ve made of geometry, that GR produces a solution to the problem in *geometrical* language, using the notion of curvature.

You are probably more familiar with non-Euclidean geometry than you think! Draw a circle and a triangle on a piece of paper. If you measure the circumference and diameter of the circle, they will have a ratio of π, as everyone knows. If you look at the internal angles of the triangle, they will add up to 180°, as Euclid says they should.

Now do the same thing on a globe: use a piece of string to draw a circle with one pole as its centre, and draw a triangle with one vertex at one pole and the other two at the equator[5]. If you compare the circumference of your circle with its diameter, you will find that the ratio is *less than* π, and if you add up the internal angles of your triangle, they will come to *more than* 180°. The sphere is intrinsically curved, and the geometry of shapes drawn on its surface does not obey Euclid’s axioms: this is a non-Euclidean surface.

When we talk of curved spacetime, this is what we mean, except that the curvature is a feature of four-dimensional spacetime of three spatial dimensions plus time, rather than the two-dimensional surface of the sphere.

The principal application of these ideas is in Cosmology.

2.3 Summary

In this section I have discussed how we used what we had learned of special relativity to investigate the structure of an accelerating frame – a rotating roundabout – and we found that, merely because some parts of the frame (the rim) were accelerating with respect to other parts (the axis), the surface of the roundabout was not flat, in the sense that some familiar geometrical laws did not apply on the surface.

I then attempted to describe just what it is that goes wrong with SR when we include acceleration and the presence of gravity. In brief, any frame attached to a particle in free-fall behaves like an inertial frame (in fact, this is the *definition* of an inertial frame in GR); but this means that two inertial frames (perhaps two falling lifts on opposite sides of the earth) can be *accelerating relative to each other*, which is contrary to a basic assumption that SR makes about inertial frames.

GR comes to the rescue, by saying that SR accurately describes the spacetime *local* to a particle in free-fall (this is a version of the equivalence principle for GR), but that we need GR to tie these various inertial frames (ie, these various freely-falling lifts) together.

[5] How do you draw a triangle on the surface of a sphere, if a triangle is three points connected by straight lines? In curved geometries, just as in Euclidean geometry, a straight line is the shortest distance between two points, so you can get your straight line by tying a piece of string to the two points you want to connect, and simply pulling it taut.

GR does this by describing a curved spacetime, which is locally that of SR (in the same way that the surface of the Earth is locally flat). The amount of curvature, at a particular point in spacetime, is governed by the amount of mass there.

3 GR and Cosmology

In this section, I'm going to talk about how we describe the structure of spacetime on large scales, by using the notion of *curvature*. In Sect. A, I'm going to discuss two particular spacetimes, namely the spacetime that will develop round a single large mass, which leads us to the notion of a *black hole*; and a possible spacetime for a complete universe, which leads us to the notion of an expanding universe, a Big Bang, and Hubble's constant. Almost all the details of what we are moving on to discuss are very technical, so we shall have to be content with a description of the main results.

3.1 Curvature

We can get an idea of what it means to say that a 'space' is 'curved' by looking at curvature on familiar two-dimensional surfaces, where we have some intuitive notions to guide us. We will be guided by geometry.

On a plane, as we know, Euclid's geometry is valid. This means that, for example, we can find parallel lines (straight lines which never meet), the internal angles on a triangle add up to $180°$, and the ratio of the circumference of a circle to its diameter is π. This is naturally just as true for a group of ants on a piece of paper as it is for us. Confined to the (two-dimensional) surface of the paper, they could make measurements of the circles and triangles we draw, and reach the same conclusions. A sheet of paper is obviously *flat*: we can extend the notion of flatness, and say that a *space* is flat if Euclid's geometry is valid, and triangles drawn in space do what they should. The space we live in is, to an excellent approximation, flat.

What would our ants see if they were to make their measurements on the surface of a sphere, say (remember that they cannot see the sphere from above, or outside, so making measurements of the curvature of the surface they live in is the only way they can find out about their world)? They could draw large circles or triangles on their sphere, and measure the ratio of the circumference to the diameter, or add up the internal angles, and they would indeed find out that they were living on a curved surface. With a little mathematics, they could develop an quantitative measure of the 'amount of curvature' on their sphere, and might well come up with a number that corresponded to what we, looking at the sphere in our three-dimensional world, would call the radius of the sphere.

The important point is that the ants have a notion of curvature that does not depend on the sphere being embedded within an three-dimensional world. This will be very puzzling to the ants – 'But where does the surface curved *into*?'; 'Into the *third dimension*!'; 'The what?!'. In the same way, we can talk about curvature in three-dimensional space: if we were to measure the surface area and diameter of a ball in a curved 3D space, we would find that the ratio would, again, not be π, we would conclude that the space is curved, and we would be puzzled by questions 'where does the space curve *into*?' These notions of curvature also work in four dimensions, and in the more general geometrical spaces of special and general relativity, even though our intuition gives out.

The surface of a sphere is an example of a space with a constant curvature. Not all spaces or surfaces have constant curvature – think of the surface of an egg, for example – and the four-dimensional spacetime we live in is like that. How do we detect this curvature?

The curvature of spacetime affects us because it governs how matter moves. When matter is in free-fall, it moves along a *geodesic*, which is a 'straight line' in spacetime, in the sense that it is the shortest distance between two points[6], just as a straight line is the shortest distance in the spaces we're used to. In a spacetime such as that of SR, this 'straight line' (for which I will now use the term 'geodesic') in spacetime appears to us as a straight line in space, but a geodesic in a *curved* spacetime, appears as a curved line in space. We will come back to this in a moment.

What makes a curved spacetime curved? Matter. In the presence of (suitably large amounts of) matter, spacetime alters its shape, and becomes curved. Matter in free fall – a ball thrown in the air, perhaps – follows a geodesic through that curved spacetime, and we perceive this as a curved passage through space, and an uneven passage through time. In other words, we see the ball's path as a graceful (*curved*) parabola, and we see the ball accelerate towards the earth. We are *seeing* a geodesic in a curved spacetime.

In Fig. 7a, we see a freely-falling lift cabin: in the context of GR, this means that the lift cabin constitutes an inertial frame (remember that this means that the lift could be plummetting towards the ground, or it could be floating in outer space). A flash from a torch on the left hand wall, travels across the cabin in a straight line and hits the opposite wall at a marked spot. In Fig. 7b, we consider what would happen if this took place in a gravitational field. If the lift cable were cut at the instant the flash left the left-hand wall, then the cabin would revert to being an inertial frame, and we know from Fig. 7a that the flash would travel across the cabin and strike the marked spot. Since the lift is falling, as perceived (or measured) by the horrified observer who was about to step into the lift, we can see that the light would take a curved path, again as perceived by that observer. There is nothing about being in the lift cabin which *affects* the path of the light (it doesn't drag it along in any sense): this construction merely allows us to work out what the path of the light *would be*, irrespective of any nonsense about falling lifts and suddenly nervous lift passengers. That is, we have elegantly deduced that a beam of light would curve whenever it is in a gravitational field, and this is an effect which was famously confirmed by Eddington's observation of the bending of starlight when its path to us grazes the surface of the Sun. Even more directly than with the thrown ball, massless light's bent path is a very direct illustration of the curvature of the spacetime the light is moving straight through.

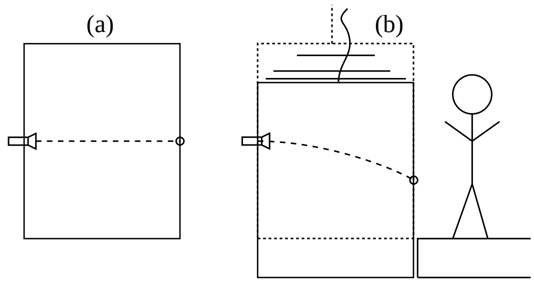

Figure 7: (a) A beam shone across an inertial lift cabin. (b) The same, measured from a non-inertial frame.

The amazing thing here is that gravity, in the sense of Newton's mysterious action at a distance[7], has to some extent 'disappeared'. In GR, there is *nothing* called 'gravity' – particles do not 'know' that a large mass is near them, they simply move as straight as they can through a curved spacetime, the curvature of which was created by the presence of that large mass. There is a slogan: 'Space tells matter how to move; matter tells space how to curve'. This is bizarre; it is also very elegant and very beautiful.

The picture of reality that this gives us is of a curved stage, on which all the rest of physics takes place. The stage is nonetheless affected by the actors on it, and the two are linked by (and only by) the mass of the actors.

As a final remark, it is important to separate what is mere mathematics in what I have said, and what is physics. The remarks about the geometry of curved spaces, and about the existence and properties of geodesics are mathematics: there is nothing debatable about them and, for me at least, nothing fundamentally interesting. The

[6] To be honest, the geodesic is actually the line between two points which has the *longest* distance, but this is merely an artefact of the odd geometry of spacetime.

[7] Newton, by the way, was never happy with the implications of his model of gravitation. There is nothing in his inverse-square law which corresponds to the *propagation* of gravitational force: if an arrangement of masses was changed, the effects would be felt throughout the universe *instantly*. This was in straightforward contradiction to the Cartesian belief that mechanical effects must proceed from their causes by impacts – local interactions. As a consequence of this, Newton believed his model to be incomplete, but neither he, nor anyone else for the next 250 years, managed anything better.

statements that free-falling matter follows geodesics, and that matter causes a difference in the spacetime in which it sits, are fundamentally different. These are *physical* remarks – statements about the universe which we could readily imagine to be false, in a way that we could not imagine '$2+2=4$' to be false. That they are not in fact false, hugely enriches our picture of the universe.

A Cosmology: The Schwartzschild solution and black holes

This appendix is rather supplementary to the descriptions of special and general relativity I've given above. In this course, I want to concentrate on fundamental physical ideas, rather than applications; but cosmology is such an important and useful application of GR, and one with so much popular interest, that it would be perverse to omit it entirely.

The big question now is, *how* is curvature related to spacetime? After some trial and error, Einstein produced what has become known as the Einstein Field Equation(s), which I might as well quote in full:

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = -\kappa T_{\mu\nu}$$

(there are *many* mathematical details hidden behind this deceptively simple line; but *The Waste Land* isn't an easy read either). The terms on the left are related to the curvature of a spacetime, and the term on the right gives the distribution of matter there (actually energy, which is the same thing, in relativity). This is by no means easy to solve, or even to understand, but solutions have been found in certain simple situations.

One of the first people to produce a solution to the Einstein field equations was Karl Schwartzschild, in 1916. This describes the spacetime round a symmetric, non-rotating, mass at rest in an otherwise empty universe. This (relatively) simple solution describes mathematically various effects I have hinted at before: if you were sitting close to the mass, an observer far away would see your watch move more slowly than his; free particles placed around the mass will accelerate in towards it, relative to the observer; and so on.

One of the interesting things about the Schwartzschild solution is that it makes clear the extent to which even *light* is affected by matter. Since light is massless, you might expect that it would be unaffected by gravity. However, light is in free fall as well, if it's moving only under the influence of gravity, and it follows a geodesic just as matter does, so if it's moving through a curved spacetime then its path will be curved. Near the Sun, this is a tiny effect, but it *is* observable.

This solution becomes interesting and exotic when you manage to squeeze all the mass in the problem into a tiny volume. If you were to compress the entire mass of the Sun into a sphere 6 km across, or an entire galaxy into a sphere the size of the solar system, you would create a *black hole*. As you go closer and closer to the centre of a black hole, spacetime becomes more and more distorted until, when you cross the *event horizon*, it is so distorted that there is no possible route back out of the horizon. Nothing that falls past the horizon, including light, can ever get out again, and has no option but to move to smaller and smaller radii from the centre, until they eventually disappear from the universe.

What would you see if you were falling into a black hole? You would notice nothing particularly odd, as your watch would carry on working as before, and so on. Even as you crossed the event horizon, you would notice nothing odd – there isn't any 'surface' there, for example – but once you had crossed the horizon you would be doomed, as there is nothing you could do to push yourself back past it again. Just about the only thing you would notice would be the 'tidal' forces due to the difference in 'gravity' between your head and your feet, say. If the black hole were a rather small one, made from our Sun, for example, the tidal forces near the event horizon would be sufficiently strong to pull you apart. If all the matter in our galaxy were compressed into a volume the size of our solar system, however, that would make a black hole with an event horizon large enough that you could fall across it without noticing anything amiss.

What would be seen by an observer watching you from a safe distance? As you fell in, they would see time, as measured on your watch, slow down; they would see you fade away as the light from you was shifted into red (losing energy as it climbs out of the gravitational well). But they would never actually see you cross the event horizon: as you got closer and closer to the horizon, they would see your watch – measuring the passage of time for you – get slower and slower, relative to theirs and, from their point of view, they would never see the time when you actually crossed the horizon.

A.1 The Robertson-Walker solution and expanding universes

The Robertson-Walker (RW) spacetime is another of the solutions to Einstein's equations. Whereas the spacetime in the Schwartzschild solution is unchanging, the RW spacetime evolves in time. Specifically, one of the possible solutions is for it to *expand* as time goes on. This, or one of its variants, is the spacetime most usually thought to reflect the large scale structure of our own universe, whilst the Schwartzschild solution is the one most often used used to model the spacetime round isolated bodies.

If the universe is expanding, it must have expanded *from* something. This 'something', commonly called the Big Bang, must have been a hugely dense collection of energy, expanding at a huge rate. Whether it was created like that, or whether it bounced back from a 'Big Crunch' – a collapsing universe – is, really, still an open question.

Our universe is pretty certainly expanding still. We can tell this because the light from distant galaxies is *redshifted*, meaning that it is made more red by those galaxies' recession from us in much the same way that a sound is moved to lower pitch by its source moving from us. We can tell how quickly galaxies are receding from us by measuring that redshift, and we can make estimates of how far away they are. The two measurements are related, in that a galaxy twice as far from us as another will be measured to be receding from us twice as quickly. This effect was first noted by *Edwin Hubble* at the beginning of this century, and the parameter that relates the velocity and distance is known as the *Hubble constant* (a misnomer, as it may not be constant in fact). It is of crucial importance to astronomy, as it has a very direct connection with the rate of expansion in the RW spacetime, and thus with the age of our universe. It is also, unfortunately, particularly difficult to measure, and current estimates can disagree by a factor of two.

A.2 The Big Bang

The Robertson-Walker solution allows us to 'look back in time'.

After establishing the properties of the universe now, we can unwind the evolution of the Robertson-Walker universe, and try to pin down the properties the universe must have had at various earlier times. Using physics that we understand fairly well, we can account for the evolution of the universe back to times only $10^{-43}\,\mathrm{s}$ after the creation of the universe. At the point when the Hubble expansion took over (around $t = 10^{-32}\,\mathrm{s}$) the universe was only $0.1\,\mathrm{m}$ in size. Since energy is conserved (one of the physical principles that survives even into this extreme situation), all the energy that is in the present huge universe will have been concentrated into a tiny volume, which consequently had a *huge* temperature.

As the universe expanded, it cooled down, and as it did so, it became cool enough for first protons and neutrons, then atomic nuclei, and finally atoms to exist (before that, the universe was filled with a gas of radiation and subnuclear particles). At around this time, the matter and the radiation in the universe stopped being as closely bound together as they had been before, and they continued their evolution separately. The matter in the universe started to clump together to form the heirarchy of galaxies, galaxy clusters, and superclusters, that we see today (the details of this ‘structure formation’ form one of the most important areas of modern cosmology). The radiation simply carried on expanding with the rest of the universe, and it is this radiation that we observe today, as the Cosmic Microwave Background.

The microwave background presents a great problem, however. No matter which direction we look in, the microwave background is the same – it is *homogeneous* – to a very high degree of accuracy. For this to be the case after all the expansion it has undergone, it must have been almost unbelievably homogeneous before. Related to this, a good deal of evidence points to our universe having just the right amount of mass to make its curvature, on the very largest scale, almost zero – the universe is flat. Again, this is so intrinsically unlikely that it calls for an explanation.

The currently believed explanation is that, before the Hubble expansion got under way, some small part of the tiny universe that was there suffered a period of *inflation*, during which it experienced a huge increase in size (something like 50 orders of magnitude), and the homogeneity of the universe at present is explained as a consequence of the (easily explained) homogeneity of a small part of a tiny universe.

This has been a particularly rapid account of the Big Bang, which I’ve included as an example of how GR can be applied. For further details, I’ll refer you to other popular accounts of the subject.

B Some of the details

When discussing relativity, one has to tread a fine line between a presentation which is chummy but inaccurate, and a presentation which is pedantically accurate but about as accessible as a conveyancing document.

I hope I have trodden a middle line, and produced an account which is accurate, but made accessible by carefully skimming subtleties. These subtleties are more likely to bite you, however, the more deeply you think about the topic. That bite can turn septic, and send you off into delirium.

This is why I have added this section. It contains rather more detailed discussions of some knotty points which crop up in the earlier part of these notes. It is not meant to be immediately intelligible, and I will not refer to it in the evening lectures, but I hope it will provide a resource when you are puzzling over the obscurities in this or some other account of relativity.

B.1 Inertial frames

Above, I used a station as an example of an inertial frame.

To be strictly correct, the station is *not* an inertial frame, as long as the force of gravity is present. This is not an issue until we start to discuss general relativity, however, and as long as we confine ourselves to motion on a flat surface, the platform is equivalent to an inertial frame.

Similarly, we should be careful when talking about throwing balls or juggling (as I do repeatedly) within an inertial frame (and to be correct should confine ourselves to discussion of clearly manifest forces such as springs or rockets). However, as long as we are talking about SR rather than GR; as long as all the relevant motion (of inertial frames) is horizontal; and as long as no-one throws the ball further than a hundred miles or so (!), denying ourselves any mention of projectile motion would achieve nothing beyond removing a vivid and natural example to focus on. If you really want to, you can remove gravity from the examples by imagining the events take place in a free-falling space capsule, with some suitably baroque arrangement of downward jets of air or rocket packs, to supply the forces.

I don't want to make a great big deal about inertial frames, but if you're still a little puzzled by them, the following remarks might help.

We need to understand first what a *reference frame* is, then what is special about an *inertial* (reference) frame, and finally what is different about the way that special and general relativity treat the notion of inertial frames.

A *reference frame* is simply a way of assigning a position to events. The scheme that is possibly most familiar to you is that of map references: every point on the earth can be specified by a latitude and a longitude. For example, the centre of Glasgow is roughly $55^\circ 52'$ north of the equator, and $4^\circ 18'$ east of the Greenwich meridian. The lines of latitude and longitude constitute a reference frame centred on, and fixed to, the centre of the earth. You could specify any point in the universe using those coordinates.

Similarly, the distance posts that lie along railway tracks give the distance to that point from the last station (or is it the last signal box?). This is another reference frame – you could specify any point by giving its distance along the track from Queen Street station, say. Reference frames need not be fixed to a stationary body, though. A train driver most naturally sees the world in terms of distances in front of the train. An approaching station can quite legitimately be said to be moving through the driver's reference frame.

You can generate an indefinite number of reference frames, fixed to various things moving in various ways. However, we can pick out some frames as special, namely those frames which are *not accelerating*, in a fairly obvious sense. In SR we can complete the definition by saying that the movement is horizontal, and we can ignore gravity. We have to do a little more work to make the definition precise, but this captures the essence of the notion as far as SR is concerned.

B.2 The light clock – calculating time dilation

In Sect. 1.2.1, I described how time as measured by an observer with the light clock was related to time as measured by an observer on the platform. Here, I want to walk through this brief calculation. If the following makes no sense to you, that doesn't matter. The maths really does not add anything. The point of adding this section is merely to demonstrate that the maths is far from sophisticated, and to satisfy the curiosity of anyone who is comfortable with the maths (I'm aware that me saying "this involved only simple maths" is like someone saying to me "but it only involves *very* simple Greek").

We know the light travelled at a speed c (the second axiom tells us that, no matter where or how the light was travelling – in a clock sitting beside us on the platform, or a clock zooming though the station in front of us – we would *always* measure its speed to be the same number c), and we timed its round trip at t seconds, so the light beam must have travelled a distance ct in the time that the clock itself travelled a distance vt. But from the figure, and using Pythagoras' theorem, we can write

$$\left(\tfrac{1}{2}ct\right)^2 = L^2 + \left(\tfrac{1}{2}vt\right)^2.$$

From Eqn. (2.1), we know that $2L = ct'$, or $L = ct'/2$, so

$$\left(\tfrac{1}{2}ct\right)^2 = \left(\tfrac{1}{2}ct'\right)^2 + \left(\tfrac{1}{2}vt\right)^2.$$

We can rearrange and simplify this to find

$$c^2 t'^2 = c^2 t^2 - v^2 t^2.$$

And we can divide both sides of this by c^2 and rearrange to find

$$t' = t\sqrt{1 - \frac{v^2}{c^2}}. \qquad (2.5)$$

C Quotations

Galileo on the Principle of Relativity

SALVATIUS: Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small flying animals. Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a wide vessel beneath it. With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin. The fish swim indifferently in all direction; the drops fall into the vessel beneath; and, in throwing something to your friend, you need throw it no more strongly in one direction than another, the distances being equal; jumping with your feet together, you pass equal spaces in every direction. When you have observed all these things carefully (though there is no doubt that when the ship is standing still everything must happen in this way), have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that. You will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still. In jumping, you will pass on the floor the same spaces as before, nor will you make larger jumps toward the stern than toward the prow even though the ship is moving quite rapidly, despite the fact that during the time that you are in the air the floor under you will be going in a direction oposite to your jump. In throwing something to your companion, you will need no more force to get it to him whether he is in the direction of the bow or the stern, with yourself situated opposite. The droplets will fall as before into the vessel beneath without dropping toward the stern, although while the drops are in the air the ship runs many spans. The fish in their water will swim toward the front of their bowl with no more effort than toward the back, and will go with equal ease to bait placed anywhere around the edges of the bowl. Finally the butterflies and flies will continue their flights indifferently toward every side, nor will it ever happen that they are concentrated toward the stern, as if tired out from keeping up with the course of the ship, from which they will have been separated during long intervals by keeping themselves in the air.

Galileo Galilei, 'Dialogue Concerning the Two Chief World Systems'

Newton on absolute space and time

I. Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external, and by another name is called duration: relative, apparent, and common time, is some sensible and external (whether accurate or unequable) measure of duration by the means of motion, which is commonly used instead of true time; such as an hour, a day, a month, a year.

II. Absolute space, in its own nature, without relation to anything external, remains always similar and immovable. Relative space is some movable dimension or measure of the absolute spaces; which our sense determine by its position to bodies; and which is commonly taken for immovable space; such is the dimension of a subterraneous, an aerial, or celestial space, determined by its position in respect of the earth. Absolute and relative space are the same in figure and magnitude; but they do not remain always numerically the same. For if the earth, for instance, moves, a space of our air, which relatively and in respect of the earth remains always the same, will at one time be one part of the absolute space into which the air passes; at another time it will be another part of the same, and so, absolutely understood, it will be continually changed.

From the Scholium to the definitions in 'Philosophiae Naturalis Principia Mathematica', Bk. 1 (1689), as translated by Andrew Motte (1729), revised by Florian Cajori (Berkeley: University of California Press, 1934). See `http://acnet.pratt.edu/~arch543p/readings/Newton.html`

Einstein on relativity, 1905

It is known that Maxwell's electrodynamics – as usually understood at the present time – when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena. Take, for example, the reciprocal electrodynamic action of a magnet and a conductor. The observable phenomenon here depends only on the relative motion of the conductor and the magnet, whereas the customary view draws a sharp distinction between the two cases in which either the one or the other of these bodies is in motion. For if the magnet is in motion and the conductor at rest, there arises in the neighbourhood of the magnet an electric field with a certian definite energy, producing a current at the places where parts of the conductor are situated. But if the magnet is staionary and the conductor in motion, no electric field arises in the neighbourhood of the magnet. In the conductor, however, we find an electromotive force, to which in itself there is no corresponding energy, but which gives rise – assuming equality of relative motion in the two cases discussed – to electric currents of the same path and intensity as those produced by the electric forces in the former case.

Examples of this sort, together with the unsuccessful attempts to discover any motion of the Earth relatively to the 'light medium', suggest that the phenomena of electrodynamics as well as of mechanics possess no properties corresponding to the idea of absolute rest. They suggest rather that, as has already been shown to the first order of small quantities, the same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good. We will raise this conjecture (the purport of which will hereafter be called the 'Principle of Relativity') to the status of a postulate, and also introduce another postulate, which is only apparently irreconcilable with the former, namely, that light is always propagated in empty space with a definite velocity c which is independent of the state of motion of the emitting body. These two postulates suffice for the attainment of a simple and consistent theory of the electrodynamics of moving bodies hased on Maxwell's theory for stationary bodies. The introduction of a 'luminiferous ether' will prove to be superfluous inasmuch as the view here to be developed will not require an 'absolutely stationary space' provided with special properties, nor assign a velocity vector to a point of the empty space in which electromagnetic processes take place.

Beginning of 'Zur Elektrodynamik bewegter Körper', Annalen der Physik, vol. 17 (1905). Translated as 'On the electrodynamics of moving bodies' in Lorentz, Einstein, Minkowski and Weyl, The Principle of Relativity. Trans: W Perrett and G B Jeffery. Dover, 1952

Minkowski on space and time

The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.

Opening of an address delivered at the 80th Assembly of German Natural Scientists and Physicians, Cologne, 21 September 1908. In Lorentz et al. (Ibid)