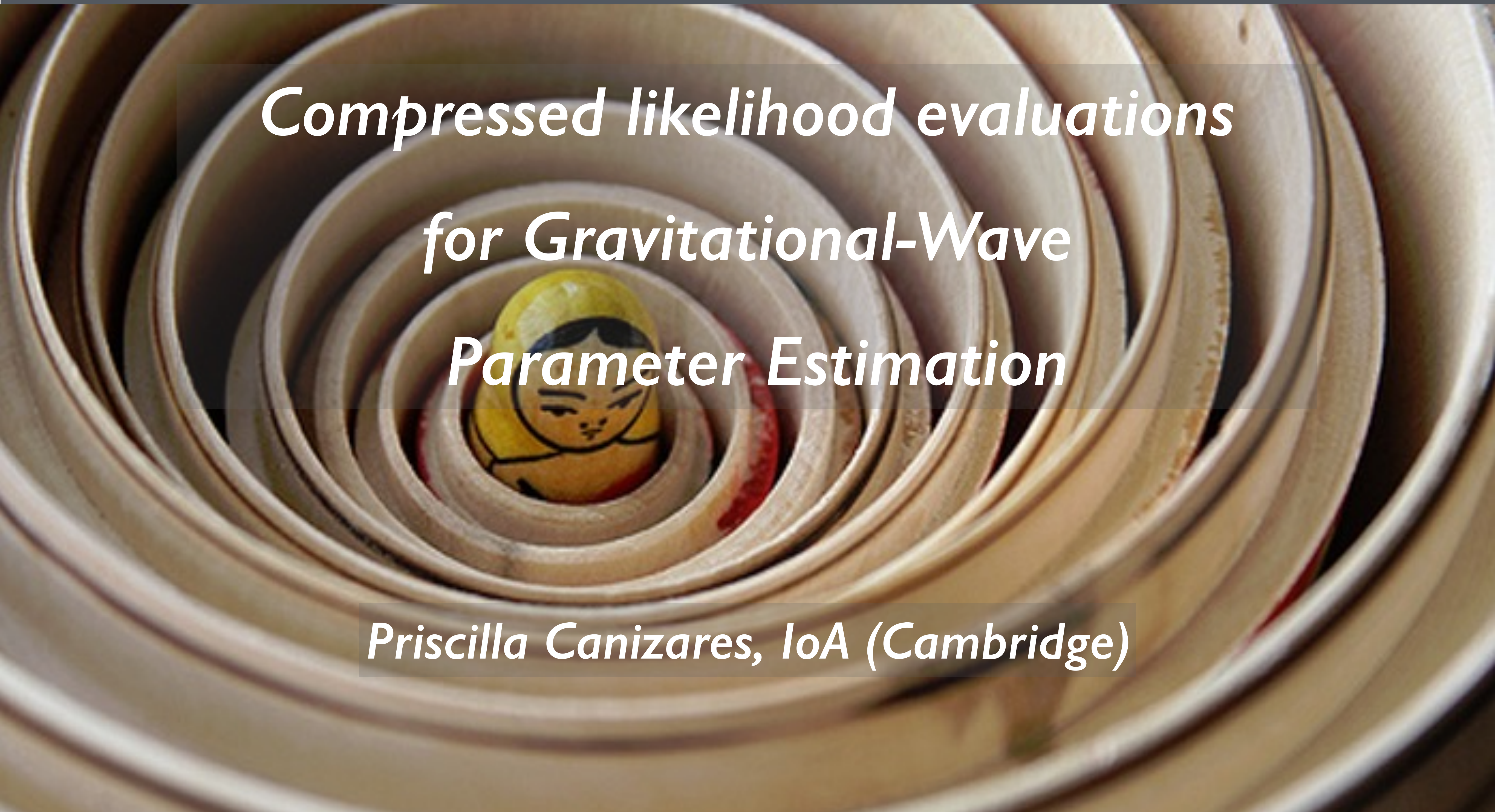


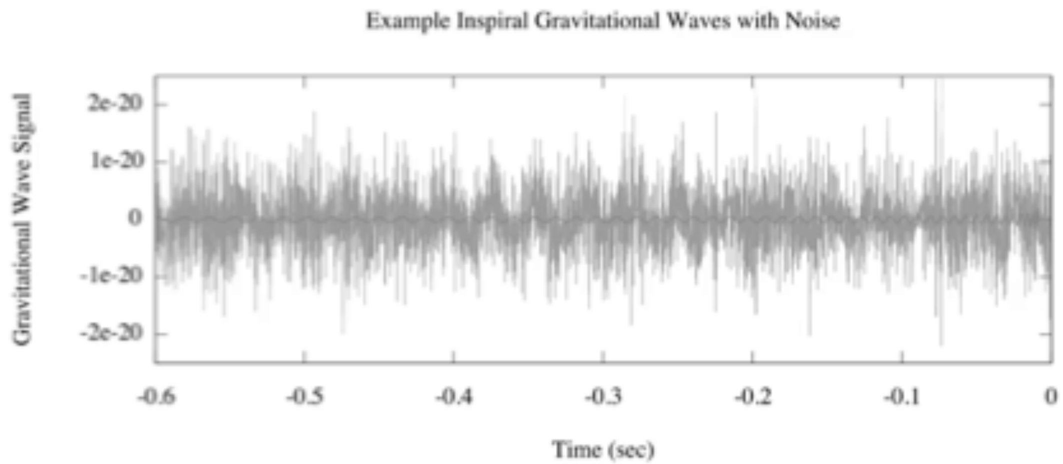
Compressed likelihood evaluations for Gravitational-Wave Parameter Estimation

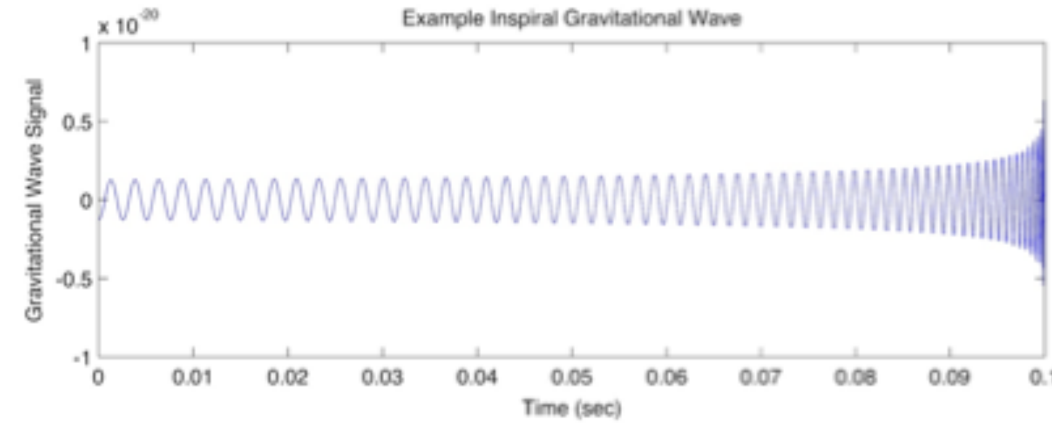
Priscilla Canizares, IoA (Cambridge)

GW Analysis

$$s(f) = h^e(f) + n(f)$$

$$h(f, \lambda) \quad \lambda := \{\lambda_i\}_{i=1}^m$$





- The detector signal is correlated against single GW template (match filtering)

$$\langle s|h \rangle = \sum_{k=1}^{M} \omega_k s^*(f_k) h(f_k)$$

- Integration by quadratures, i.e. evaluations of weighted inner products. Quadrature rule given by the quadrature points and weights

$$\{f_k, \omega_k\}_{k=1}^M$$

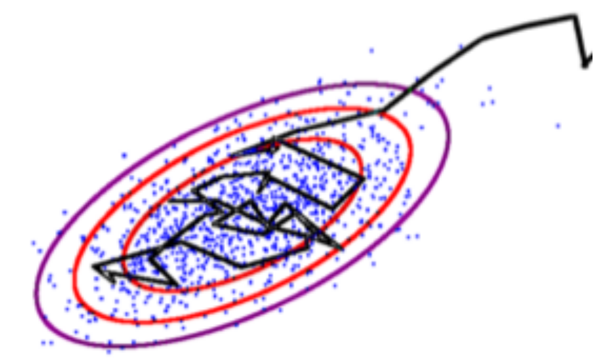
- Correlation cost scales with the observation time (larger M) and the dimension of the parameter space (many evaluations of $\langle .|. \rangle$)

GW Analysis

- Once we have a detection, we need to extract the physical parameters of the system. Ex. Markov chain Monte Carlo (MCMC) — requires repeated evaluations of the likelihood $P(s|h)$ across the parameter space

$$P(s|h) \propto exp^{(\langle n(f)|n(f)\rangle)}$$

$$n = s(f) - h(f, \lambda)$$



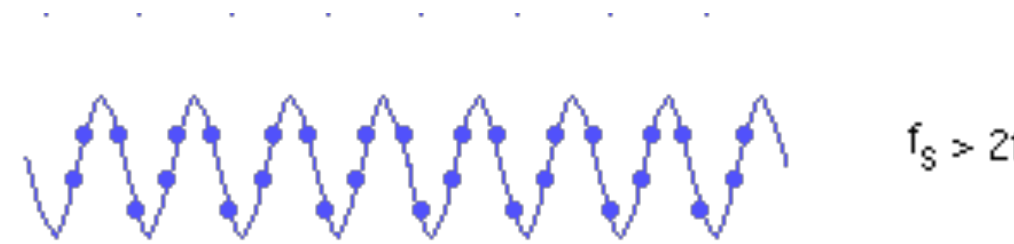
- MCMC techniques are computationally expensive: Depends on the # of sampling points and dimensionality of the waveform space

- Need of numerical tools to handle and analyse GW data in feasible times.
- On going efforts to improve the efficiency of Bayesian inference methods include *[see e.g. J. Veitch et al 2014]* a suitable choice of waveform parameterisation and better proposal distributions.

Compression of the GW model without loss of information - fewer computational operations.

Compressed sensing

- Classically to store and reproduce signals they are sampled at **fixed intervals.**



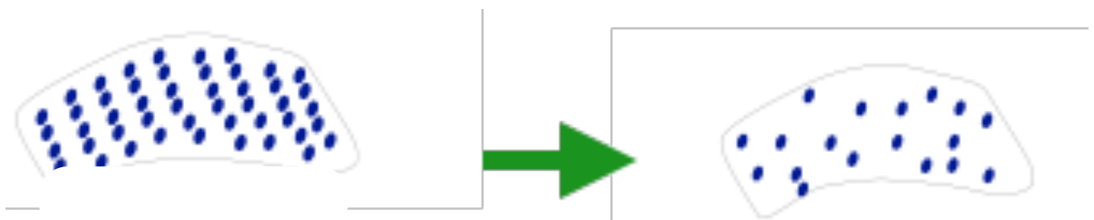
- Compressed sensing (CS) *[E. J. Candes et al 2006, D.L. Honoho 2006]* major development in applied mathematics of the last decade.
  - It allows one to get around the classical sampling limit and **recover signals from fewer measurements.**
  - The optimal sampling strategies depend on t**he sparsity of the signal and its structure** *[B. Adcock et al 2014].*
  - New developments and strategies are based on empirical evidences — mathematical justification built ad hoc.

Reduced Order Modelling — Key facts

Find a Reduced basis [Greedy algorithm]

- Input: Set of training templates evaluated at a sample of (training) points

- Output: The GW [reduced] basis (RB) and the associated points in parameter space



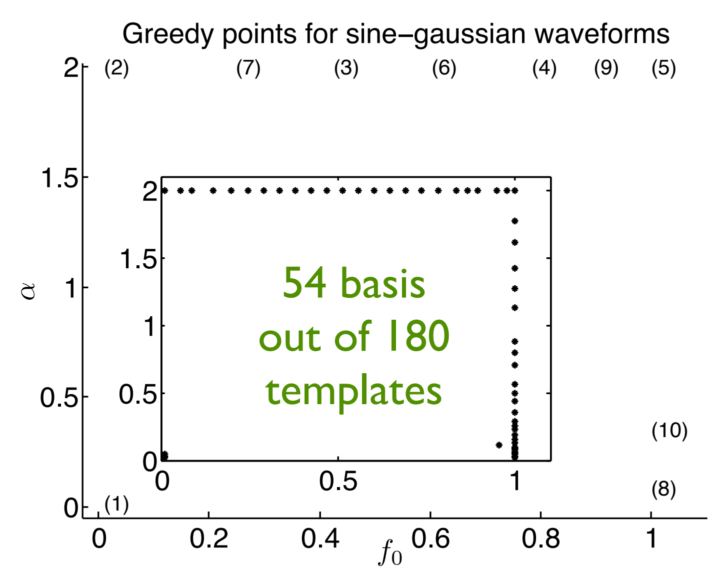
$\{h(f_k, \lambda_i)\}_{i=1}^{i=D}$

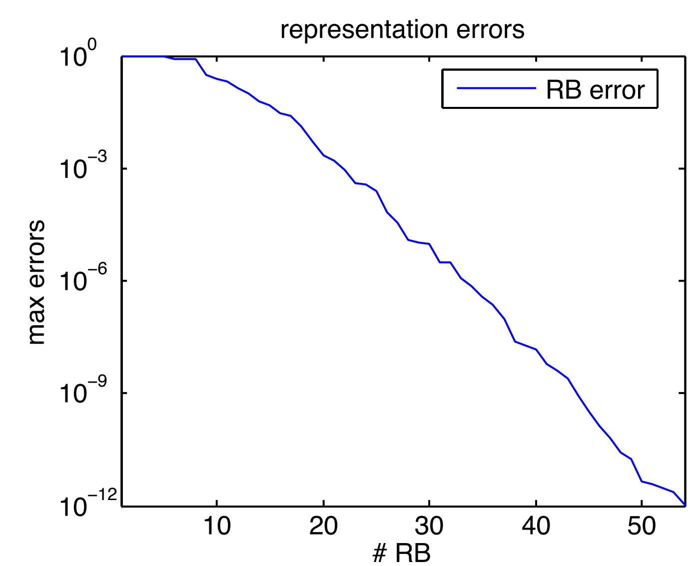
$k = 1, ..., M$

$\{e_i(f)\}_{i=1}^{i=n}$ $\{\lambda_i\}_{i=1}^{i=n}$

$$h(f,\lambda) \cong \sum_{i=1}^{n} c_i(\lambda) e_i(f)$$

- **Greedy algorithm ensures exponential convergence with the number of basis templates**





[Canizares et al 2013]

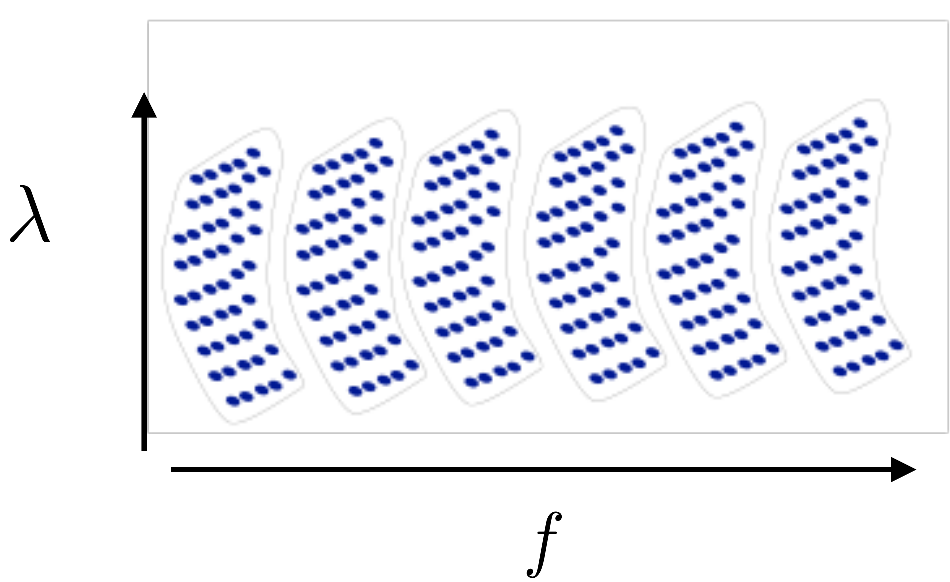
Reduced Order Modelling — Key facts

Find Empirical interpolation points [Greedy algorithm]

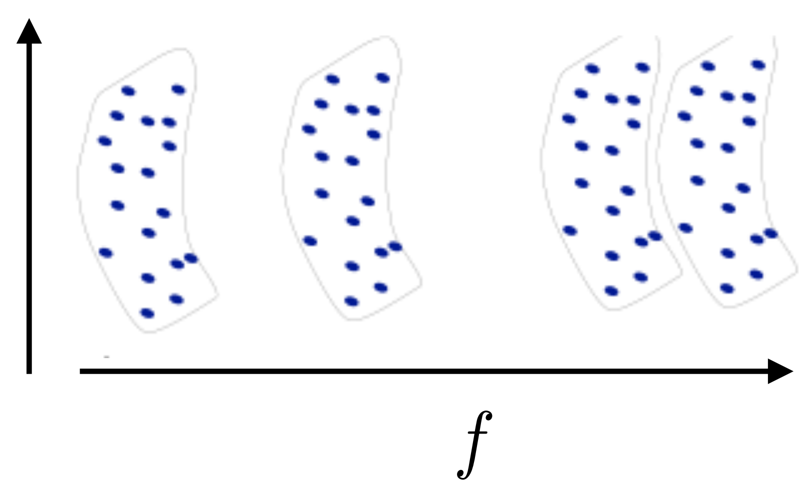
- Input: Set of GW-basis functions $\{e_i\}_{i=1}^{n}$ and sampling points $\{f_i\}_{i=1}^{M}$
- Output: Subset of sampling points $\{F_i\}_{i=1}^{n} \subset \{f_i\}_{i=1}^{M} \;|\, n \ll M$

☆ The set of EIM points is nested and hierarchical,

$\{h(f_k, \lambda_i)\}_{i=1}^{i=D} \quad k = 1, ..., M$



$\{e_i(F_j)\}_{i=0}^{i=n} \quad j = 1, ..., n \quad n << D, \;\; n << M$



ROM Parameter Estimation Recipe

- **Step (1) Construct reduced basis**: Find a set of templates that can reproduce every template in the model space to a certain specified precision. OFFLINE

OFFLINE

- **Step (2) Find empirical interpolation points**: Find a set of points at which to match templates onto the basis.

- **Step (3) Construct signal specific weights**: Compute the weights to use in the quadrature rule once data has been collected. w_k

startup

$$\langle s|h(\lambda)\rangle = 4\Re \sum_{k=0}^{M} s^*(f_k)h(f_k,\lambda)$$
$$\simeq 4\Re \sum_{k=0}^{n} \omega_k h(F_k,\lambda)$$

$$\omega_k = \sum_{k=0}^{M} s^*(f_k)\vec{e}(f_k)\mathbb{R}\ \Delta f$$
$$\mathbb{R}_{ij} = e_i(F_j)$$

The cost of evaluating integrals scales lineally as the # of RBs m

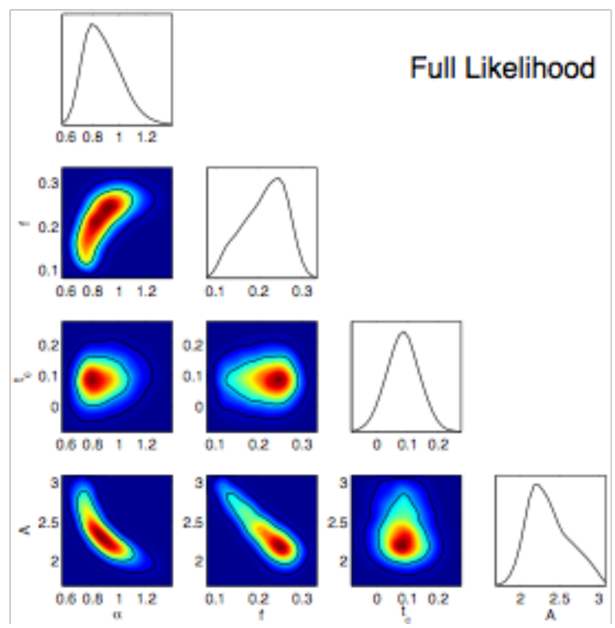
- **Step (4) Carry out parameter estimation**: Evaluate likelihood/posterior over parameter space using ROQ rule and, e.g., MCMC. ONLINE

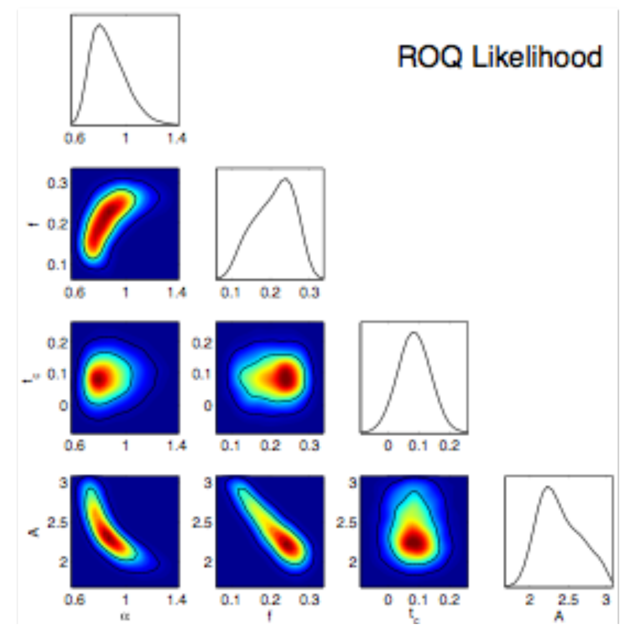
[Canizares et al 2013]

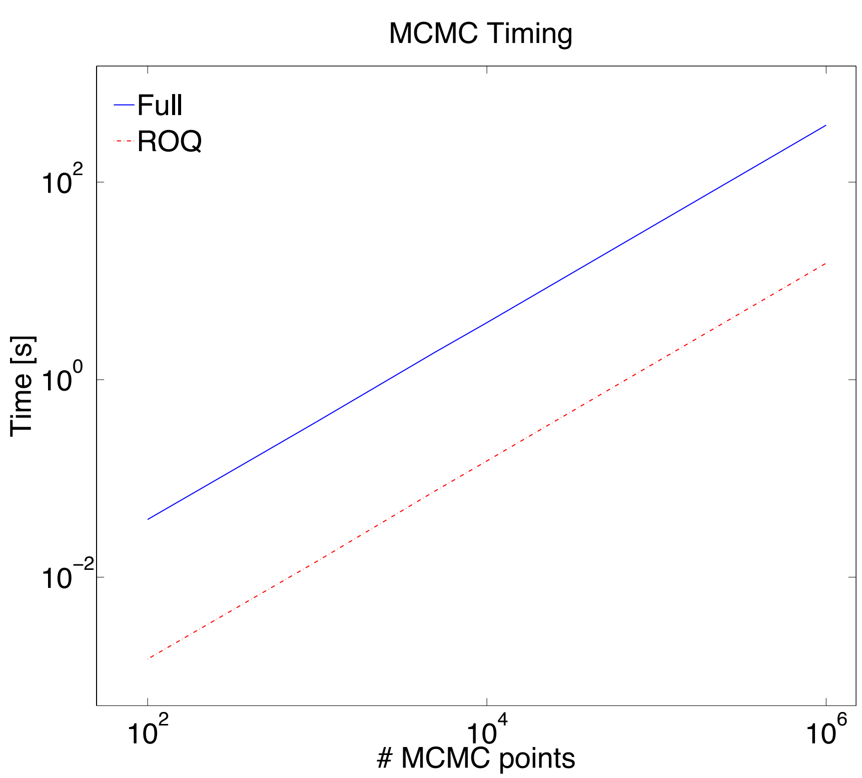
Compressed Likelihood

- Parameter estimation using ROM.

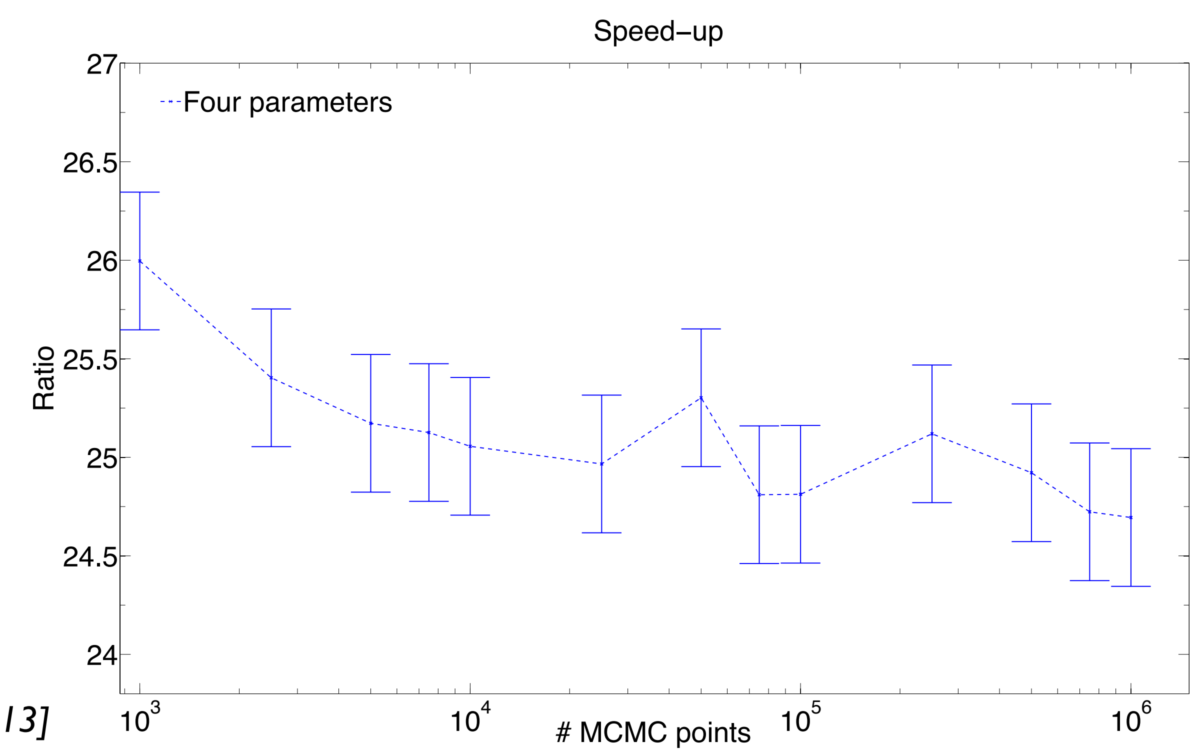
Burst waveform $\widetilde{h}(f,\lambda) = i2\sqrt{2\pi}\alpha e^{(i2\pi t_c - 2\pi^2\alpha^2(f_0^2+f^2))}\sinh\left(4\pi^2\alpha^2 f_0 f\right)$







[P. Canizares et al 2013]

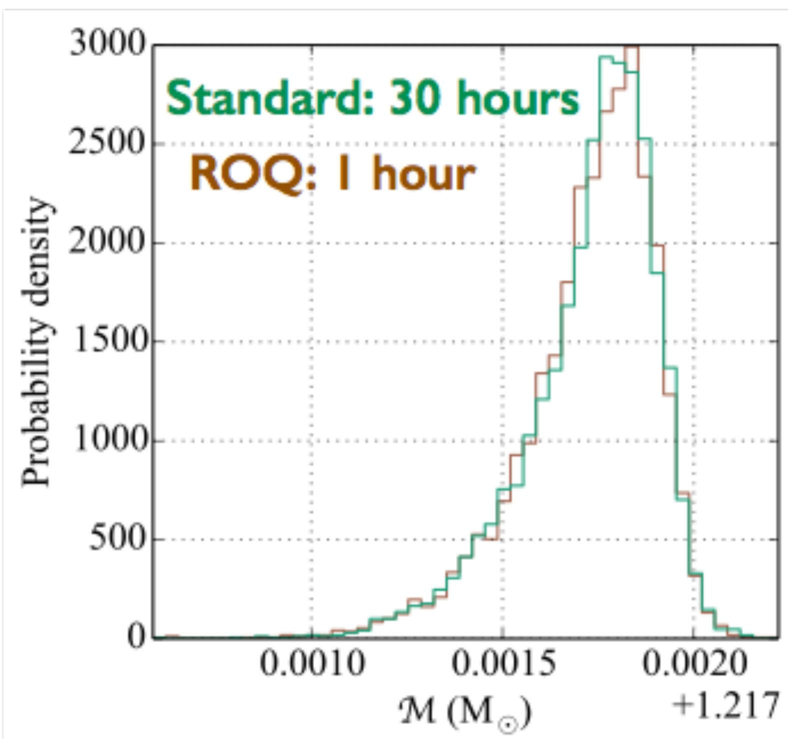


Compressed Likelihood

- Parameter estimation using ROM

TaylorF2 waveform (BNS) $h(f,\lambda) = A(\lambda) f^{-7/6} e^{i\psi_{3.5}^{F2}(\lambda)}$

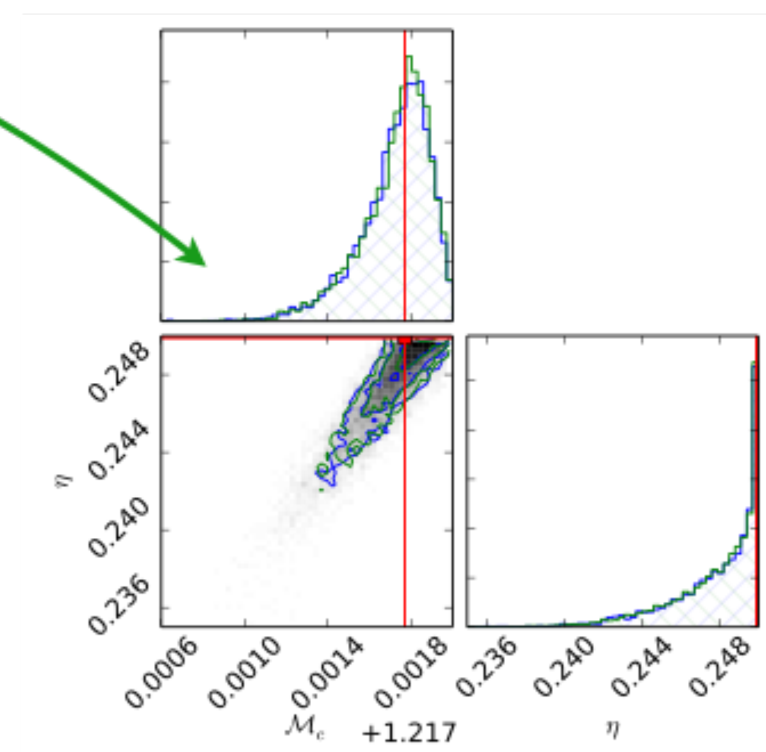
- **Speedup: 30 times faster!**



- **Comparison of both methods for recovering the values of intrinsic parameters**

	$\mathcal{M}_c\,(M_\odot)$	η	$m_1\,(M_\odot)$	$m_2\,(M_\odot)$	SNR
injection	1.2188	0.25	1.4	1.4	11.4
standard	$1.2188_{1.2184}^{1.2189}$	$0.249_{0.243}^{0.250}$	$1.52_{1.41}^{1.66}$	$1.30_{1.18}^{1.39}$	12.9
ROQ	$1.2188_{1.2184}^{1.2189}$	$0.249_{0.243}^{0.250}$	$1.52_{1.41}^{1.66}$	$1.30_{1.19}^{1.39}$	12.9

- **PDF for chirp mass and symmetric mass ratio.**
- **In red the injection values**



- **Fractional likelihood error**

$$\Delta \log \mathcal{L} = 1 - \left(\frac{\log \mathcal{L}}{\log \mathcal{L}_{\rm ROQ}}\right) \lesssim 10^{-6}$$

[P. Canizares et al 2014].

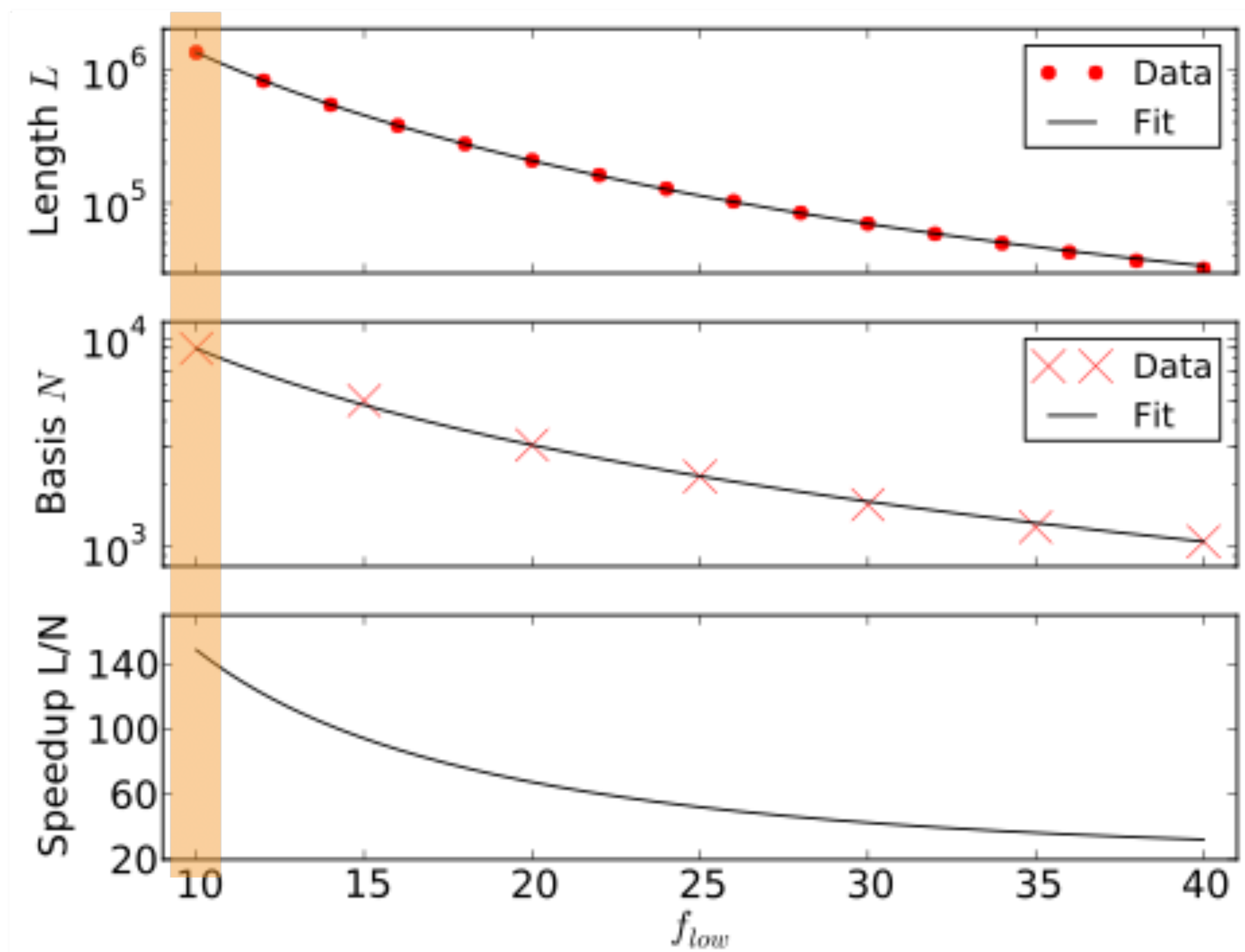
Reduced Order Modelling

$f^c_{high} \sim 1020Hz \qquad T_{obs} \sim 2000s$

Runtimes of 3 months @ one Petabyte reduced to 1 day!!

cientific collaboration

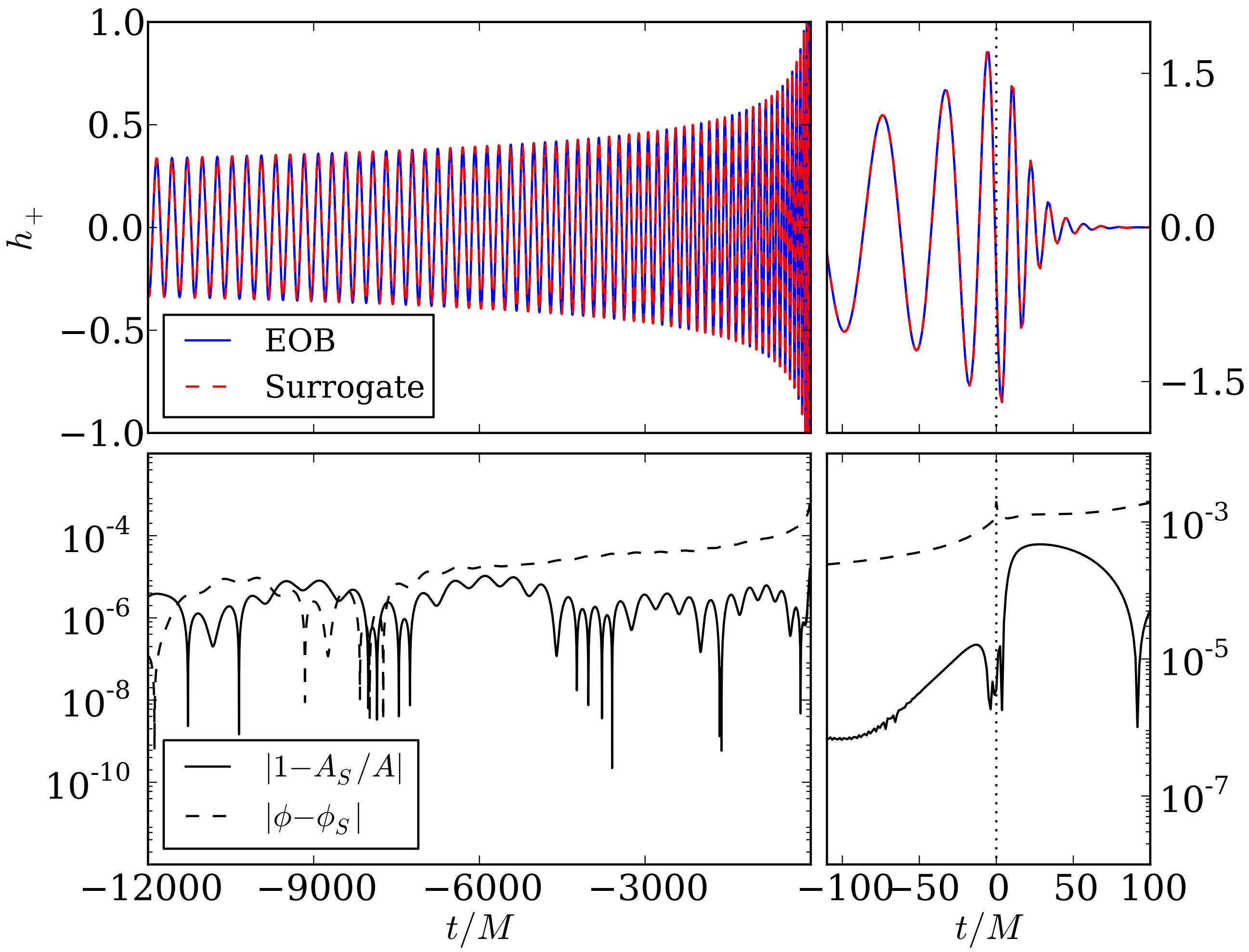
rary (LAL)

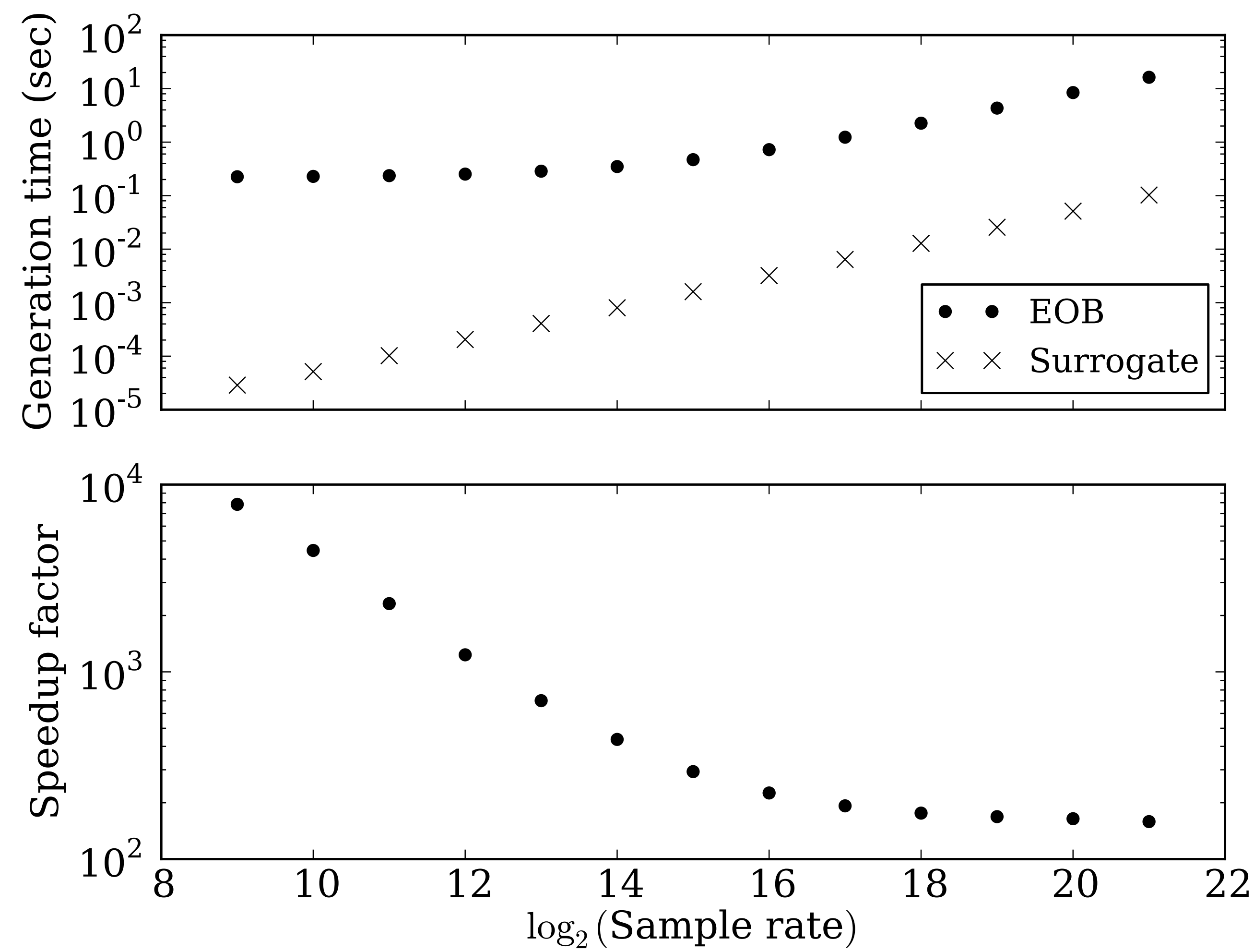


Canizares, et al (2014)

Further Developments

- Surrogate models: quick and accurate way to generate waveforms *[S. E. Field et al 2014, see also Pürrer 2014]*
  - Based on ROM builds a waveform (surrogate) model for arbitrary parameters
  - No need of close form waveforms





S. E. Field et al 2014,

Further Developments

- **Likelihood transformation** techniques accelerates MCMC convergence but increases the evaluation time. Using **surrogate models** this cost can be reduced *[R . H. Cole and J. R. Gair 2014]*
- New proposed approach **to account for model uncertainties** leading to systematic errors *[C. J Moore and J. R Gair 2014]*
- **Controlled compression scheme** that trade detection sensitivity for computational savings

See Chua's Poster!

Outlook

- Compressed sensing is an a promising line of research towards development efficient pipelines to generate and analyse GW in feasible times.

- Recent works show that Reduced Order Modelling has the potential to speedup gravitational-wave analysis by orders of magnitude without loosing accuracy.

- The cost of generating GW templates can be dramatically decreased using surrogate models.