Detecting Gravitational Waves using a Pulsar Timing Array

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…and why we havn’t yet

Data problems:

- Noise models

Computational problems:

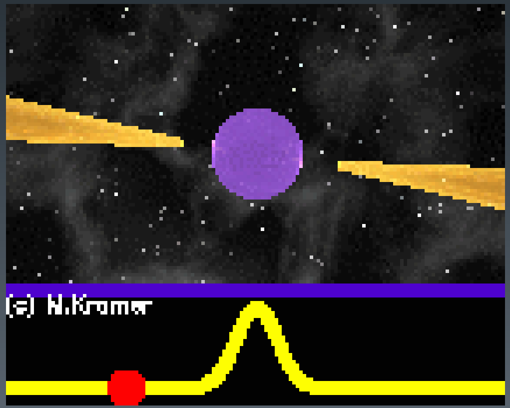
- High Dimensionality
- Big datasets

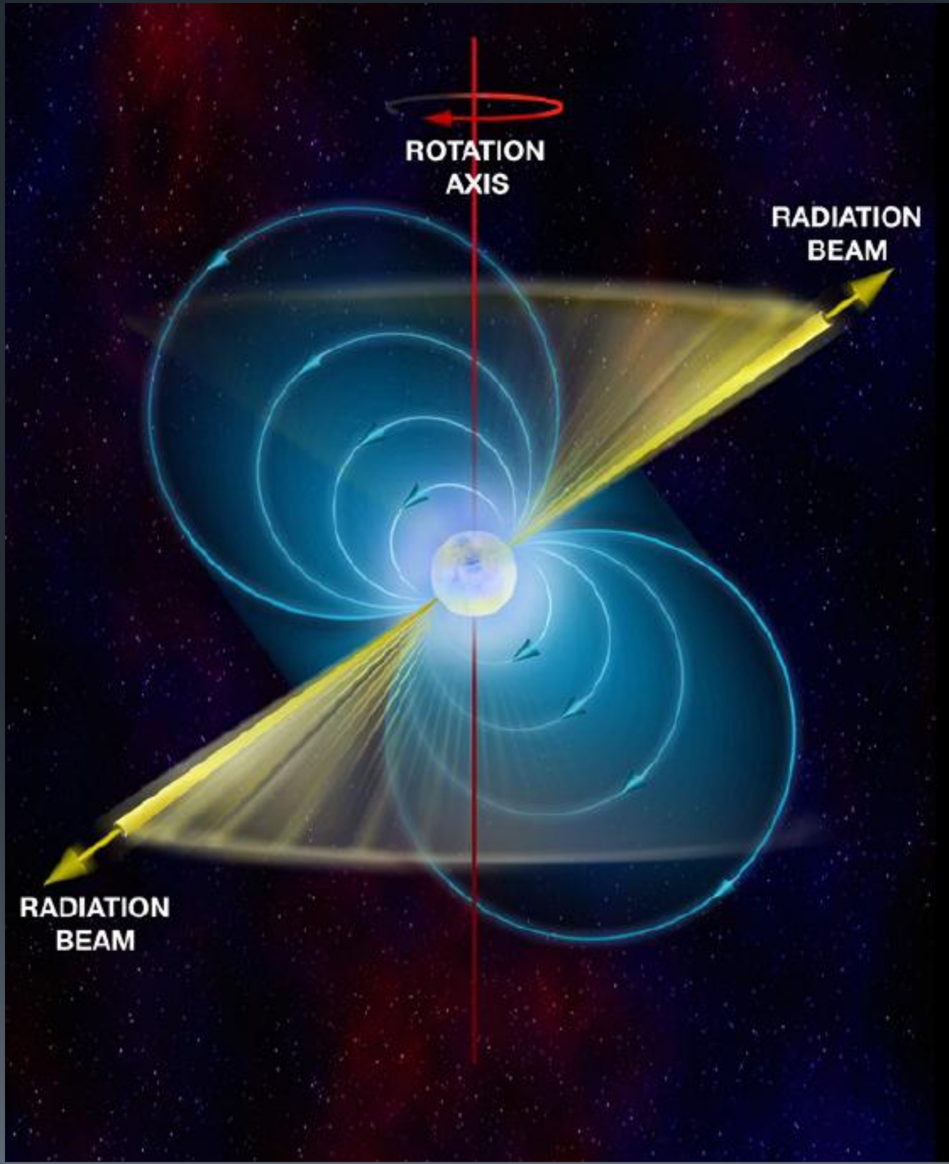
Astrophysical problems:

- Current limits getting interesting/depressing

Pulsar Timing

- Mass > our Sun
- 20km across
- Hundreds rotations/sec
- ‘lighthouse’ effect





Pulsar Timing

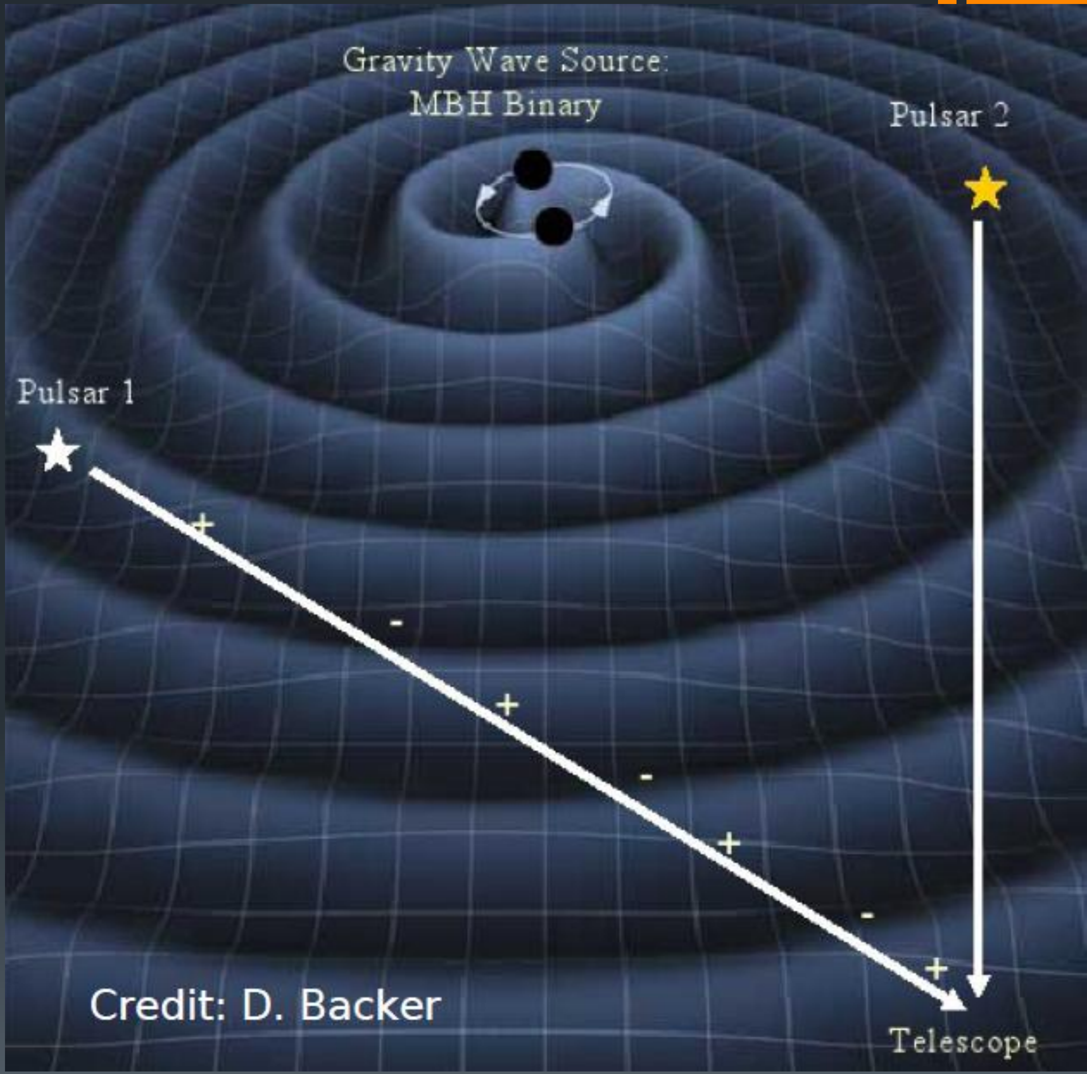
- Extremely precise astronomical clocks.

Spin period of PSR J0437-4715:
P = 0.005757451936712 59 s +/- 0.000000000000000002 s !

- Period of pulsar known to 1 part in 10^{15}

Pulsar Timing

- Sensitive to nHz GWs
- Earth-pulsar distance changed
- See deviation in arrival time of pulse

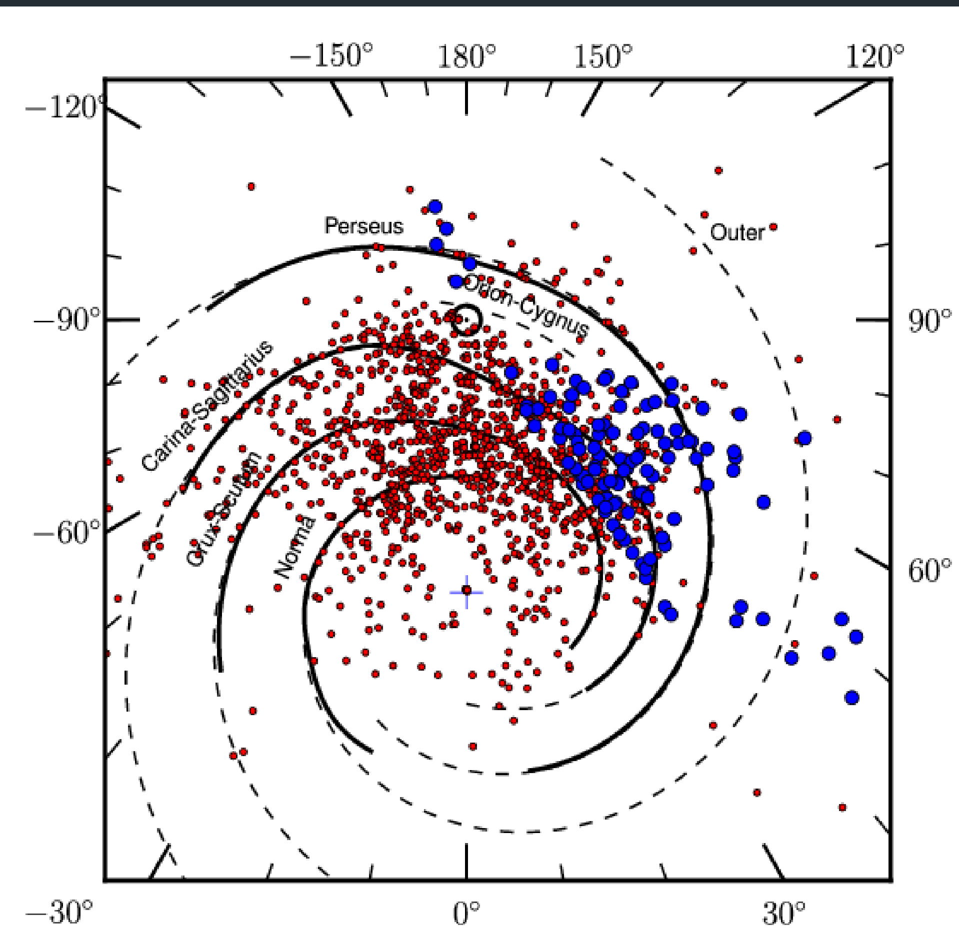


Pulsar Timing

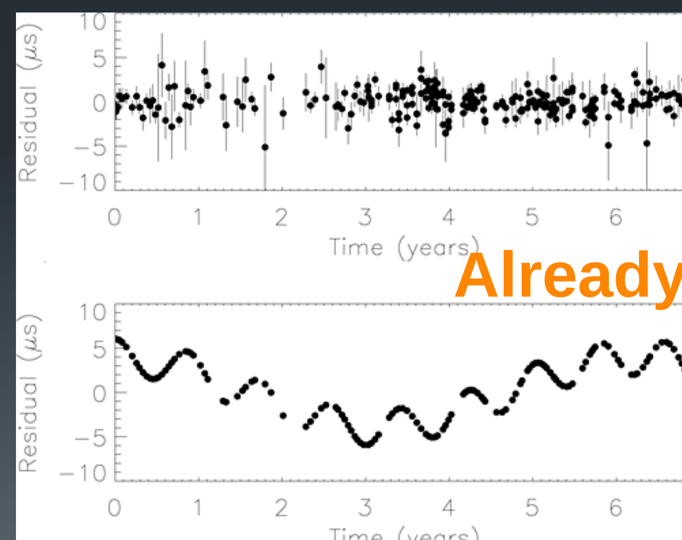
Also far away ~ 1kpc = 3x10^{19} meters

Change in path length from GWs:

~ few hundred meters
= 0.1 – 1.0 μs



Red: PSRCAT Pulsars |b| < 5° (1245) Blue: PALFA Pulsars (113)



Already There!

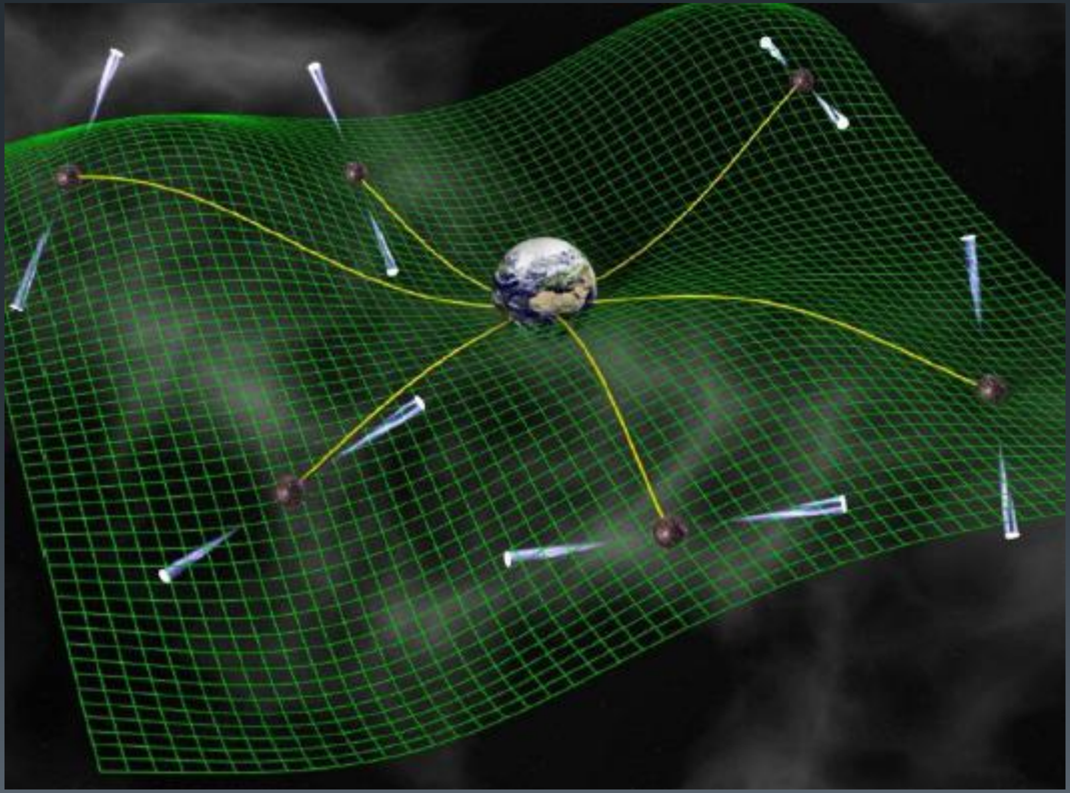
Jenet et al. 2004, ApJ, 606, 799

Pulsar Timing

Use a collection of pulsars: pulsar timing array

GW signal correlated
between pulsars

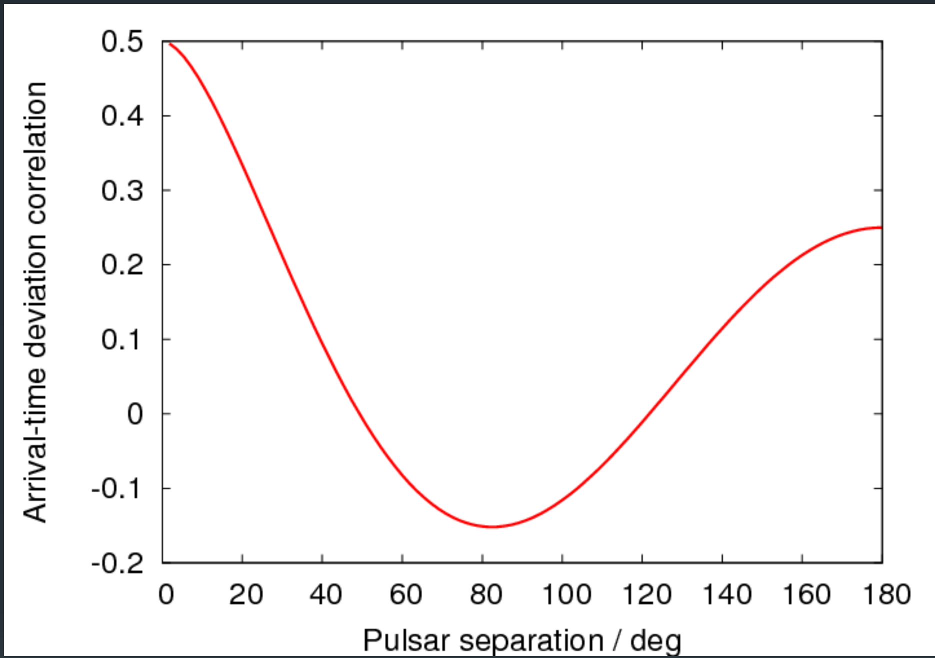
<u>Signal in Residuals</u>
Clock errors:
monopole
Ephemeris errors:
dipole
GW signal:
quadrupole

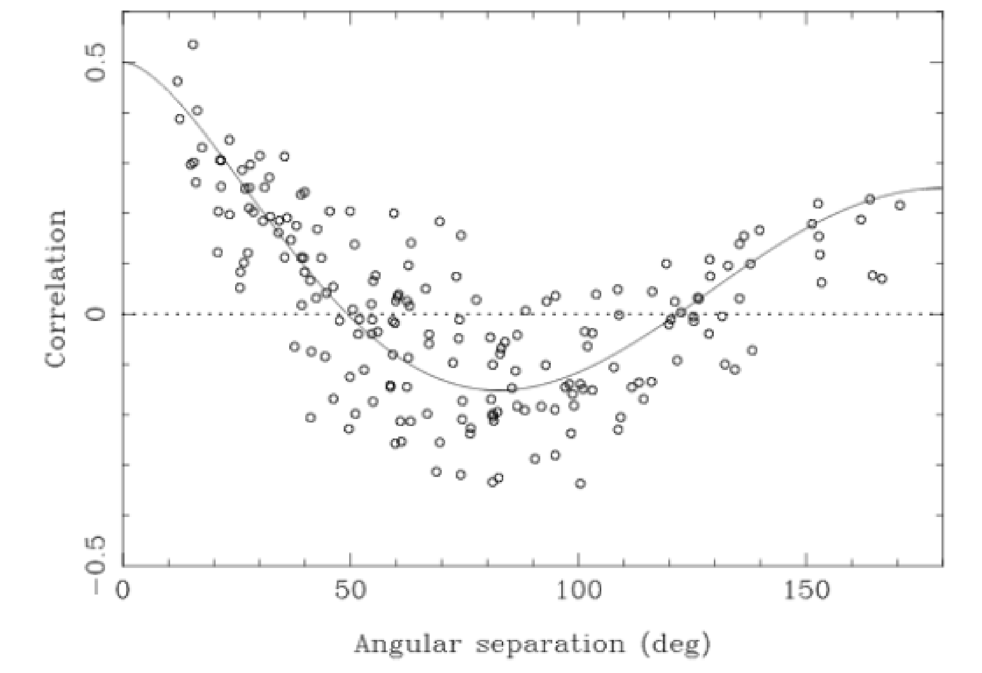


The Hellings-Downs Curve

For an isotropic background the angular correlation has an analytic solution

$$^{ab}\Gamma(\epsilon) = 3\left(\frac{1}{3} + \frac{1-\cos\epsilon}{2}\left[\ln\left(\frac{1-\cos\epsilon}{2}\right) - \frac{1}{6}\right]\right)$$





Smoking Gun of a real GW detection.

e.g. Hellings & Downs, 1983, ApJL, 265, 39; Jenet et al. 2005, ApJL, 625, 123

IPTA
Green Bank, West Virginia
Westerbork, The Netherlands
Effelsberg, Germany
Pune, India
Arecibo, Puerto Rico
Goostrey, United Kingdom
Nancay, France
San Basilio, Italy
Parkes, Australia

Some predictions..

20 pulsars

100ns white noise

Detection in: 5 years (e.g. Jenet et al 2004)

Current IPTA dataset:

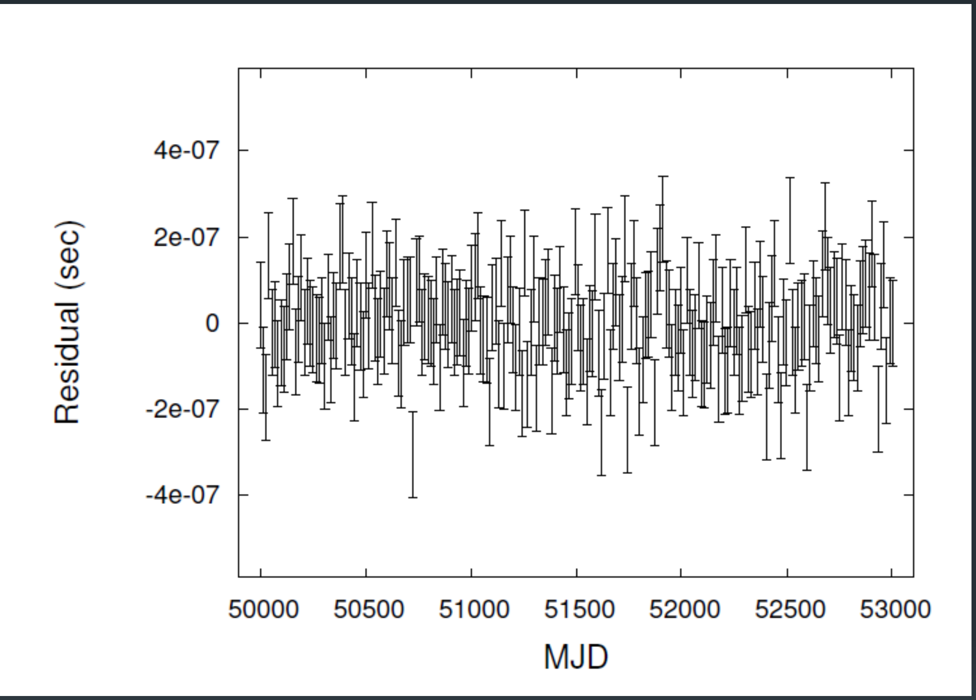
40 pulsars

20 years of data

Some < 100ns

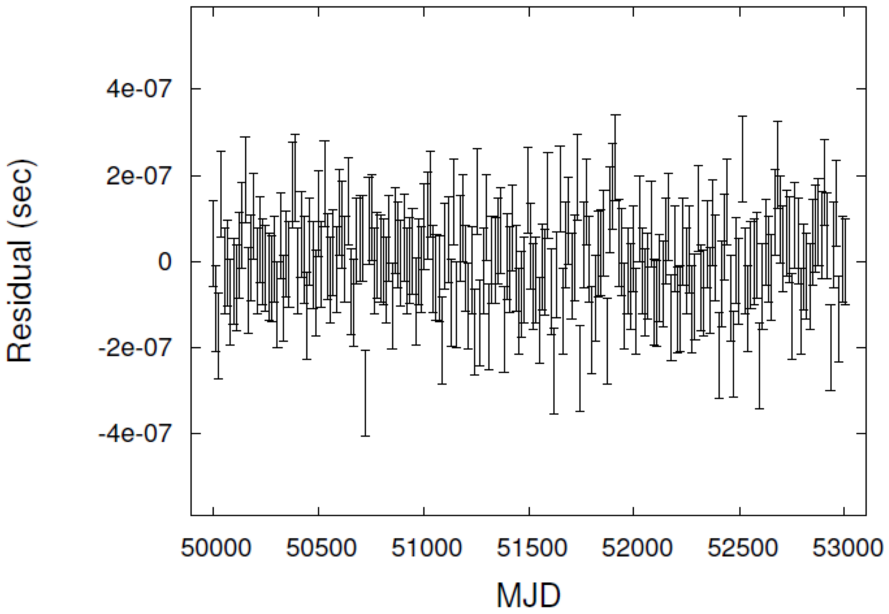
.. Where’s the detection?

Data challenges



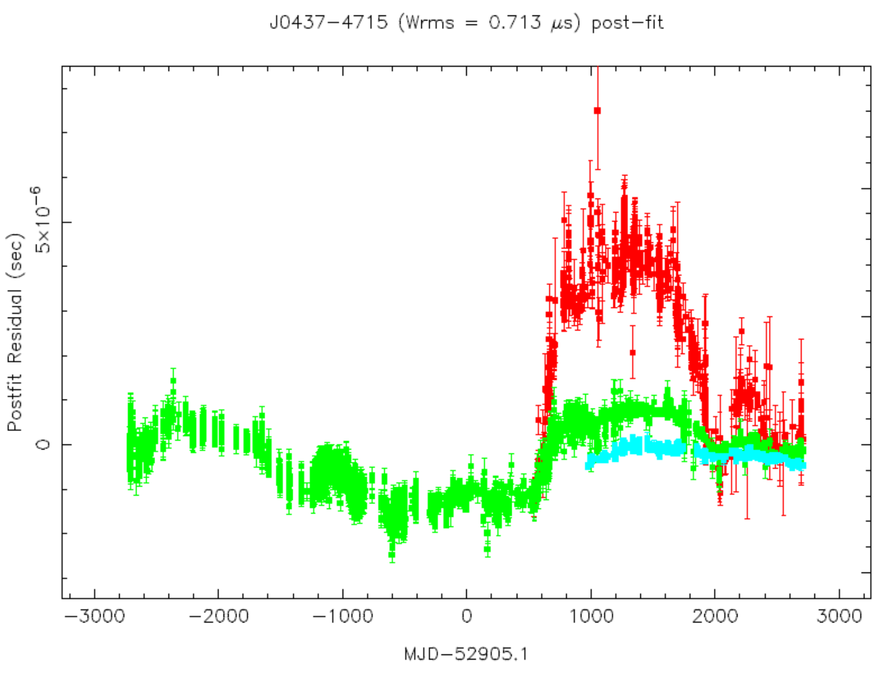
Residuals:
Subtract expected time of arrival from actual time.
<- 100ns white noise

Data challenges



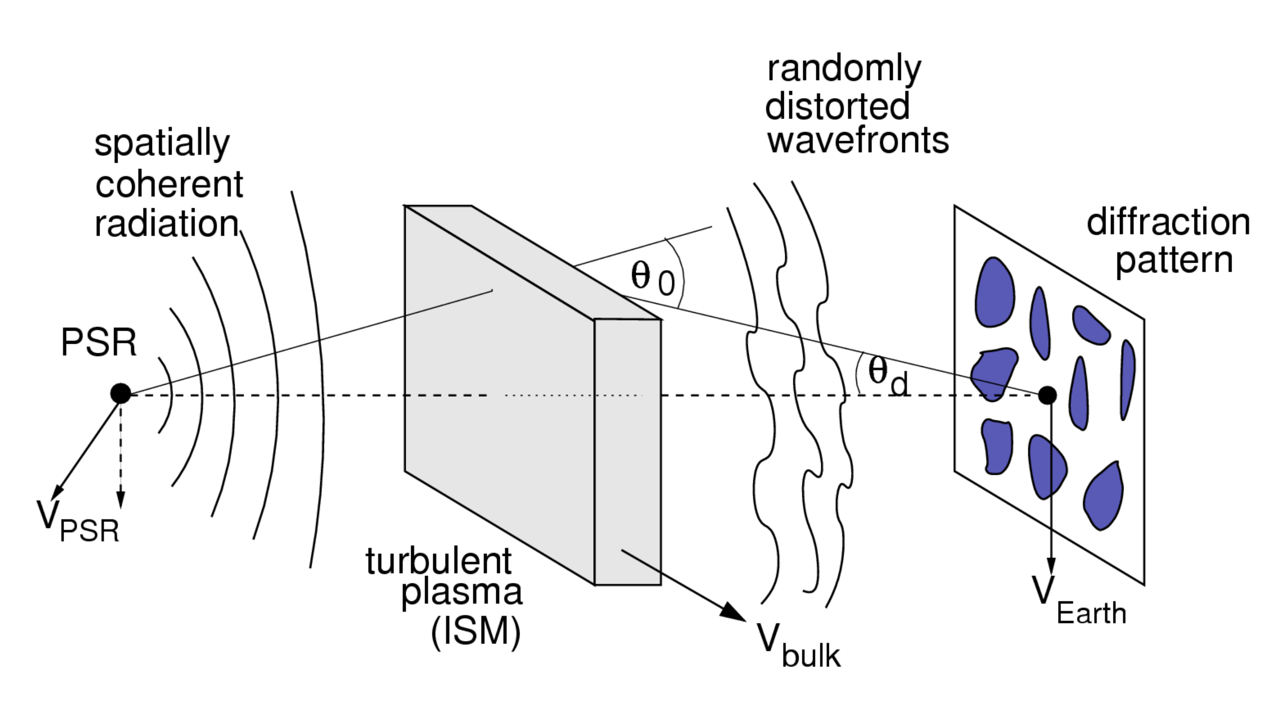
Residuals:
Subtract expected time of arrival from actual time.
<- 100ns white noise

Actual data:
J0437-4715
(one of the better pulsars)



Data challenges

Noise mostly due to the interstellar medium
Frequency dependent (goes as 1/f^2)



$$t_g(\nu) = K\ DM/(\nu^2)$$

$$K \equiv 4.15 \times 10^{15}\ \mathrm{Hz}^2\ \mathrm{cm}^3\ \mathrm{pc}^{-1}\ \mathrm{s}$$

$$\mathrm{DM} = \int_0^L n_e \mathrm{d}l.$$

Data challenges

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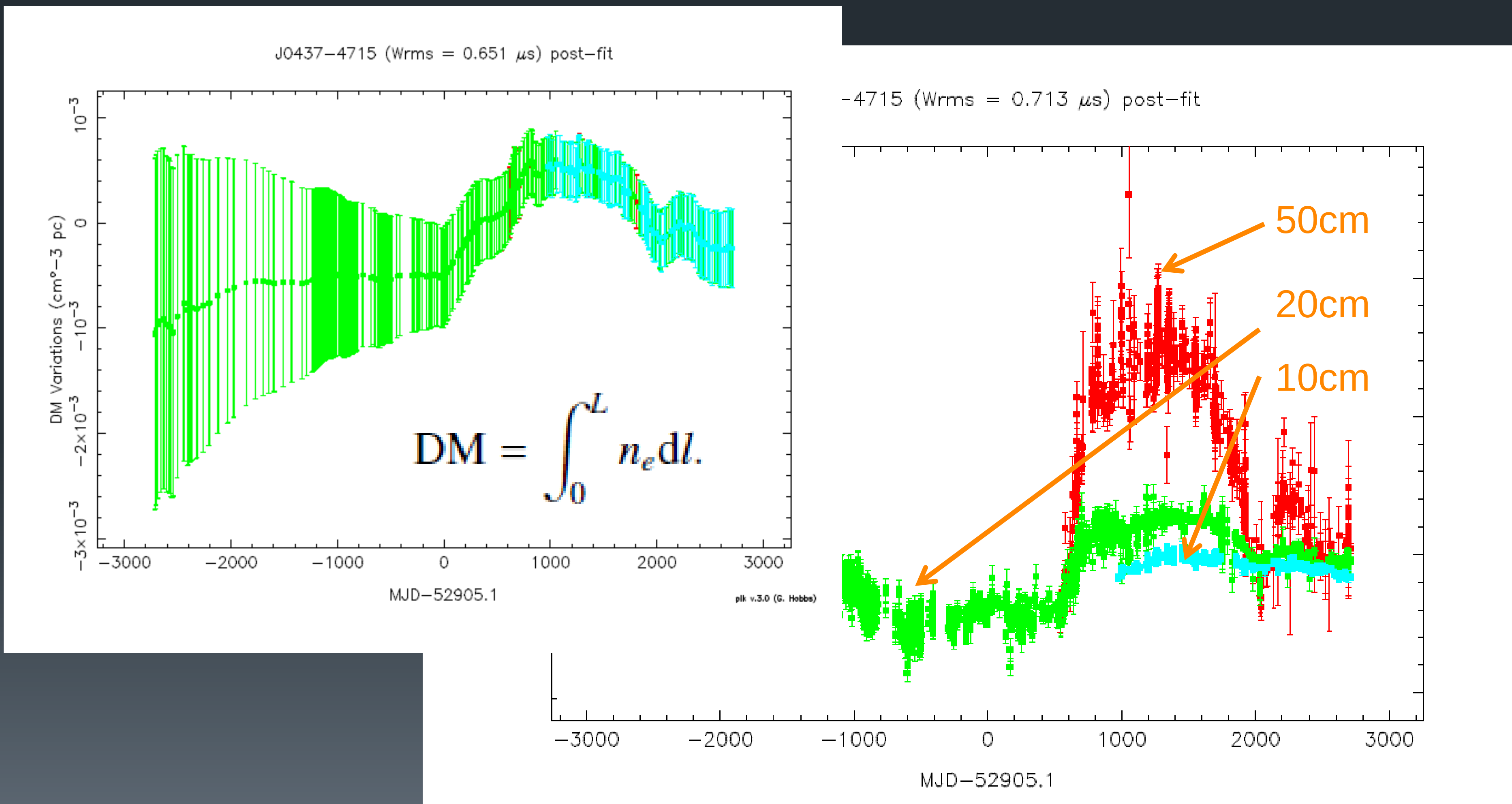
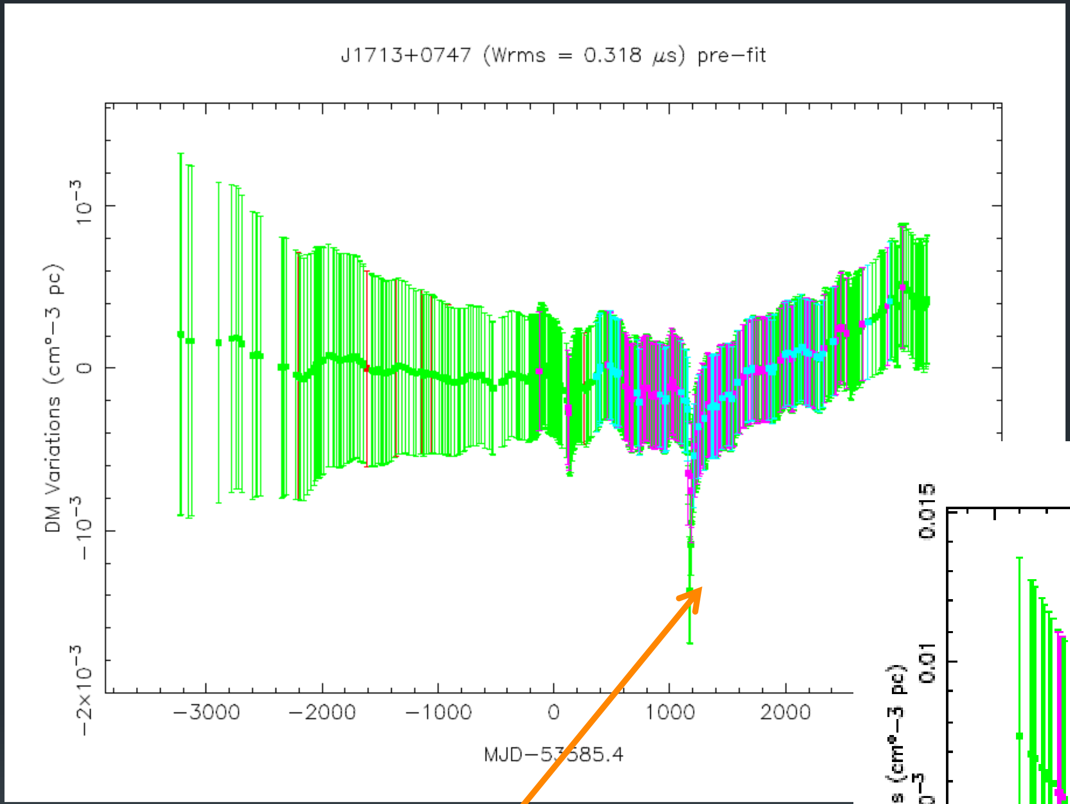
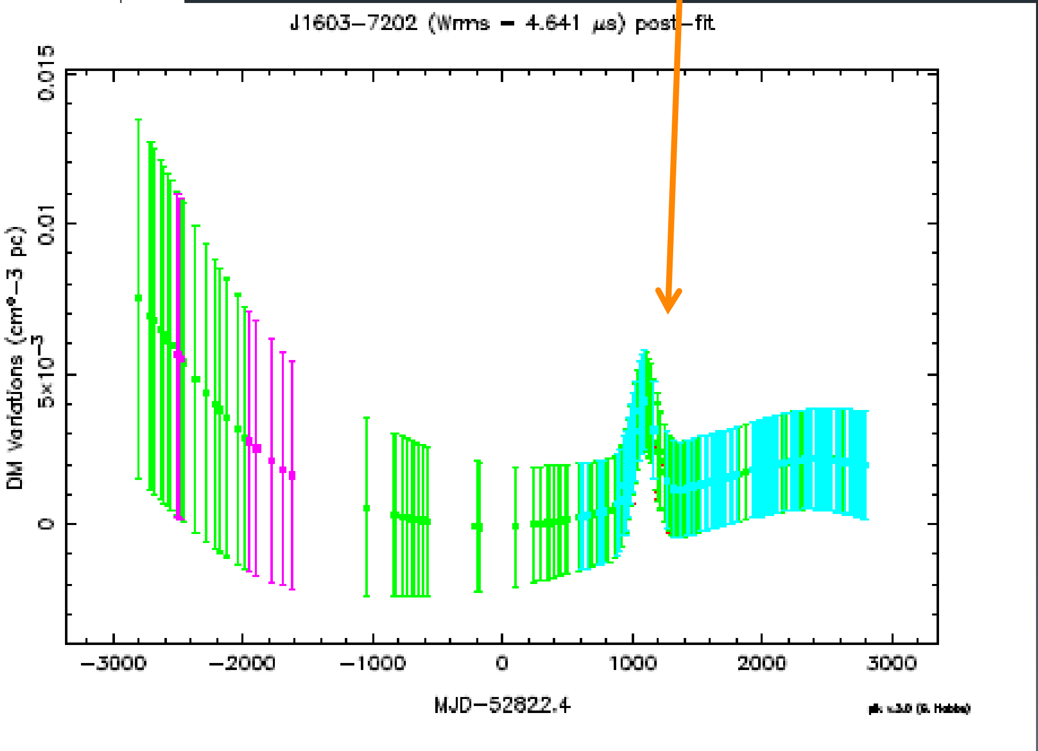


Data challenges

Not Time stationary

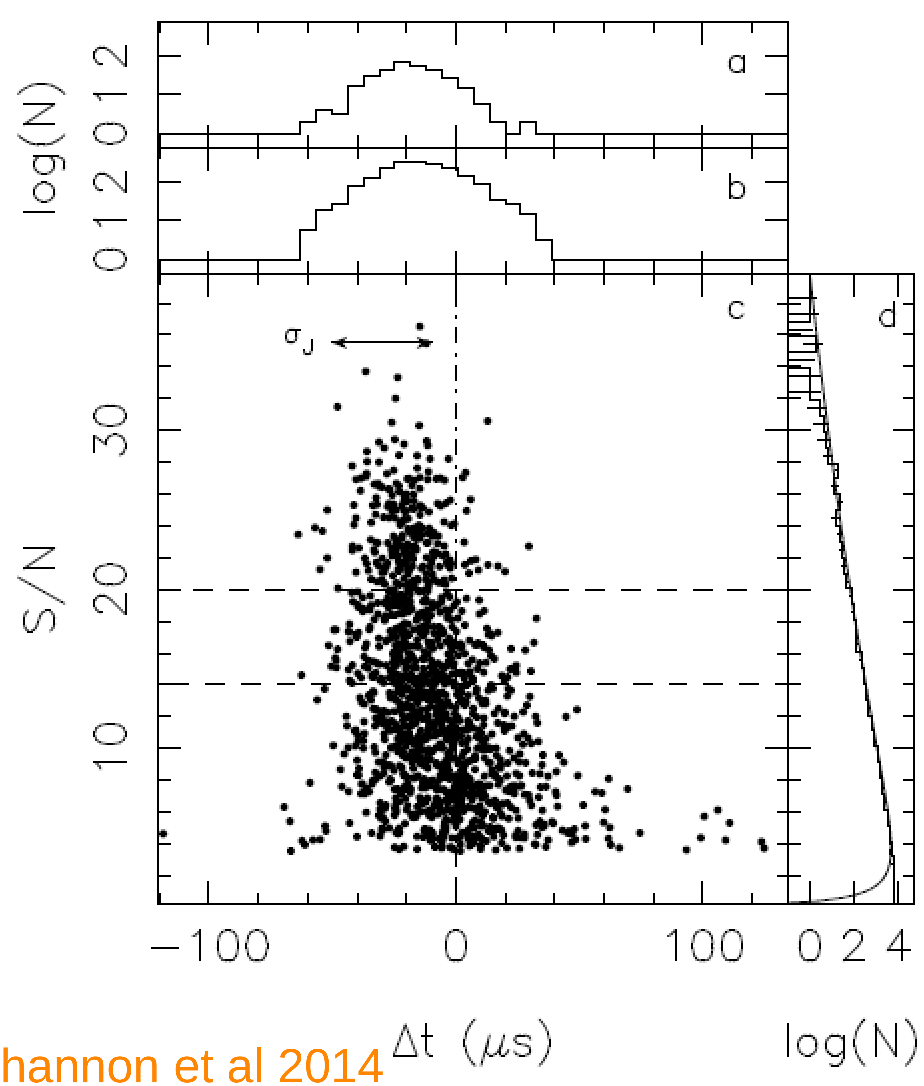
Over density in the ISM





Void in the ISM

Data challenges



Shannon et al 2014

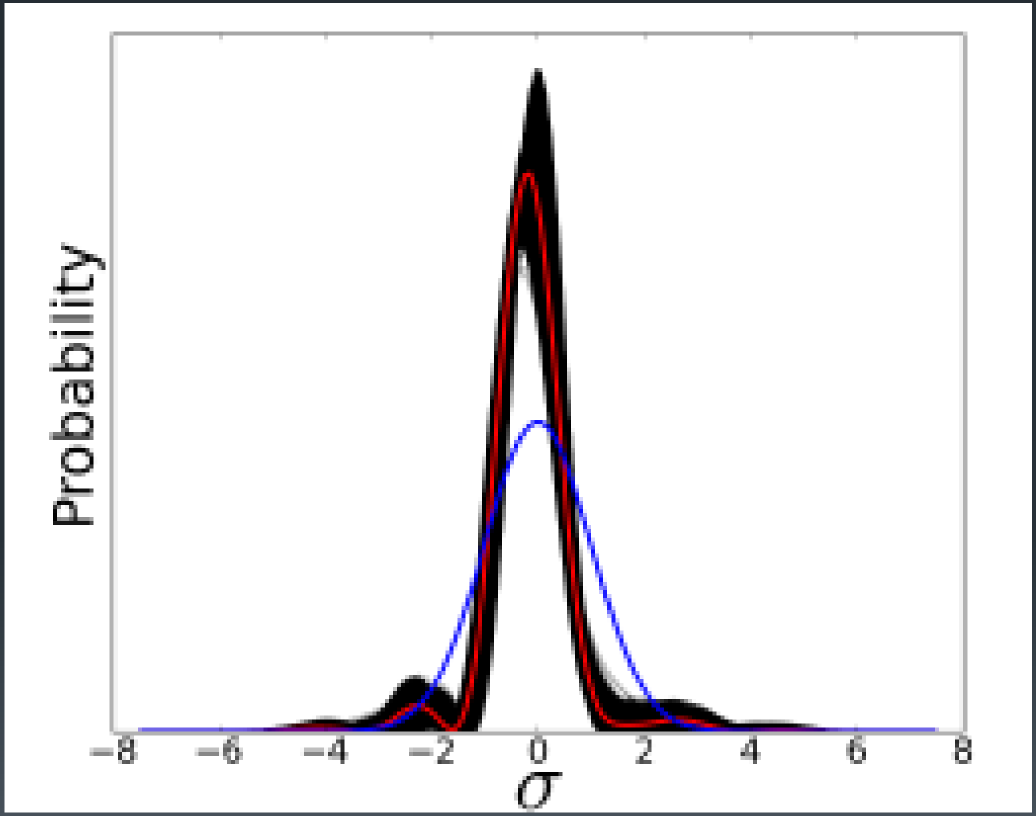
Intrinsic High Frequency in arrival times

Known as ‘Jitter’

Better telescopes wont help

Some pulsars already at limit

Data challenges



Lentati et al 2014

Intrinsic High Frequency variation in arrival times

Known as ‘Jitter’

Better telescopes wont help

Some pulsars already at limit

Not necessarily Gaussian

Data challenges

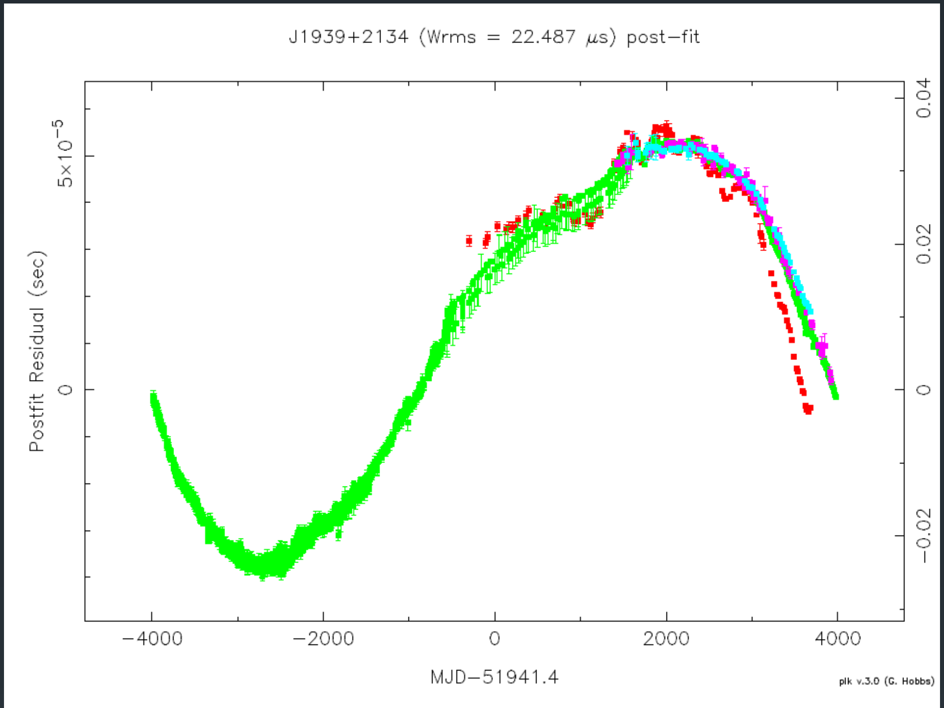
Finally, Intrinsic Low Frequency variation in arrival times

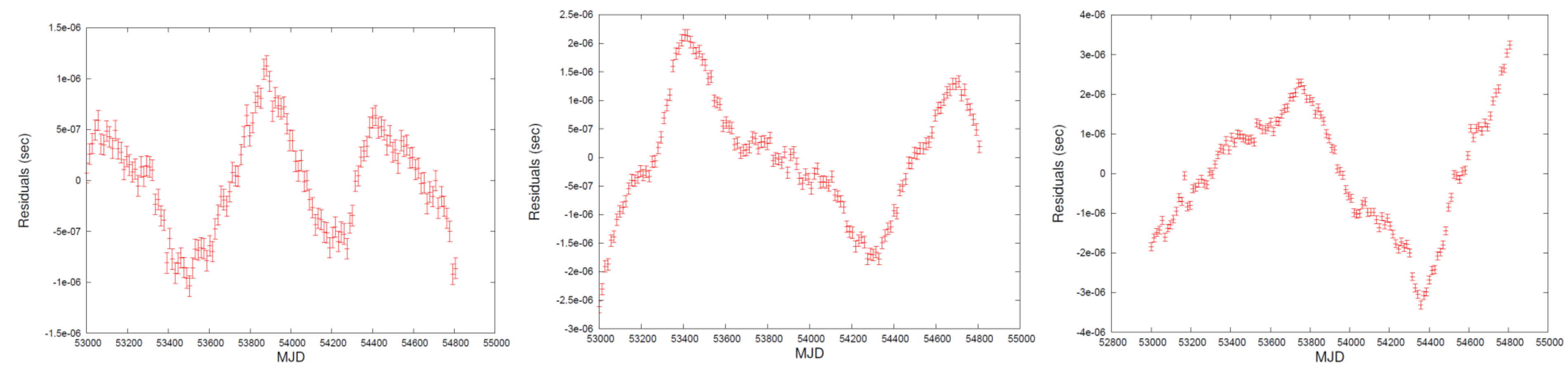
Known as ‘Timing Noise’

Either from magnetosphere, or core.. Origins mostly unknown

Stochastic Process as with DM

Individually can look just like Gravitational Waves





Computational challenges

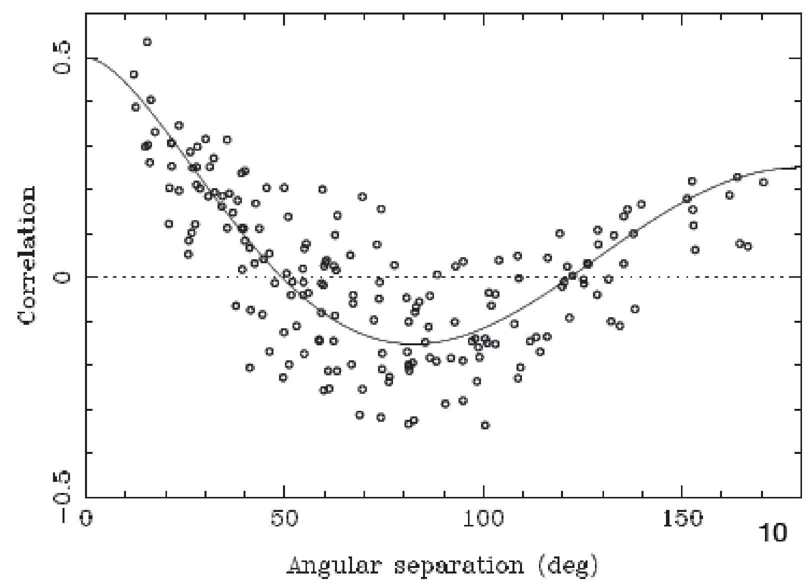
$$p(\mathbf{r}|\vec{\theta}) = \frac{1}{\sqrt{\det 2\pi \boldsymbol{\Sigma}(\vec{\theta})}} \exp\left(-\frac{1}{2}\mathbf{r}^T \boldsymbol{\Sigma}^{-1}(\vec{\theta})\mathbf{r}\right)$$

Residuals $\mathbf{r} = \begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \vdots \\ \mathbf{r}_l \end{bmatrix}$

The signal in this case is in the covariance matrix!

Covariance matrix for residuals:

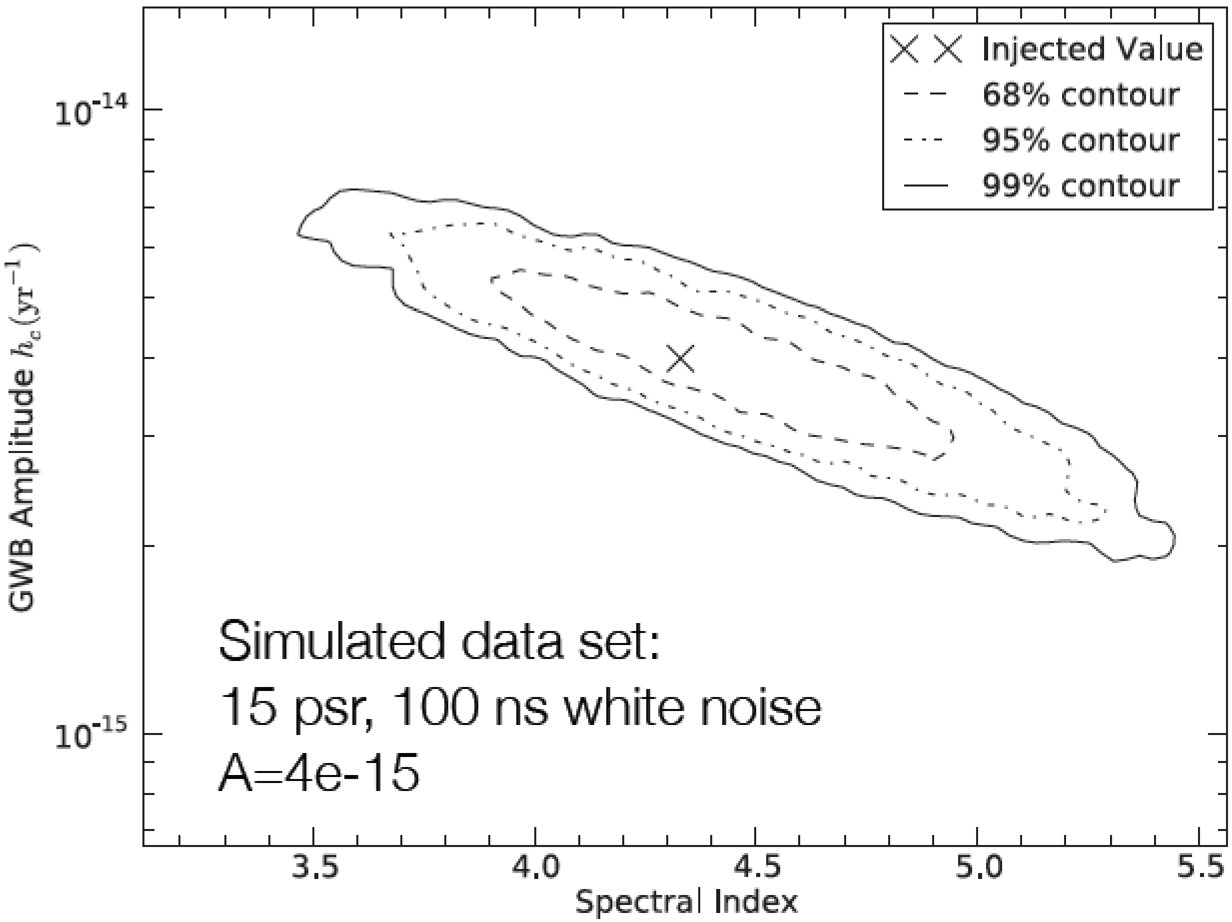
$$\boldsymbol{\Sigma}_r = \langle \mathbf{r}\mathbf{r}^{\mathrm{T}} \rangle = \begin{bmatrix} \mathbf{P}_1 & \mathbf{S}_{12} & \cdots & \mathbf{S}_{1l} \\ \mathbf{S}_{21} & \mathbf{P}_2 & \cdots & \mathbf{S}_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{S}_{l1} & \mathbf{S}_{l2} & \cdots & \mathbf{P}_l \end{bmatrix}$$



Computational challenges

$$p(\mathbf{r}|\vec{\theta}) = \frac{1}{\sqrt{\det 2\pi\mathbf{\Sigma}(\vec{\theta})}} \exp\left(-\frac{1}{2}\mathbf{r}^T\mathbf{\Sigma}^{-1}(\vec{\theta})\mathbf{r}\right)$$

$$\vec{\theta} = (\text{GWB Amplitude, Spectral Index})$$



First Bayesian analysis
In time domain.
(van Haasteren 2011)

Big Matrices
(30k x 30k inversions)

Current Bayesian
analysis
In Fourier domain.
(Lentati 2013)

Much smaller matrices
Much faster

Computational challenges

Dimensionality becoming an issue:

Up to 100 parameters for a single pulsar
Total can reach many hundreds or thousands

Most parameters are white noise related (scaling and quadrature terms):

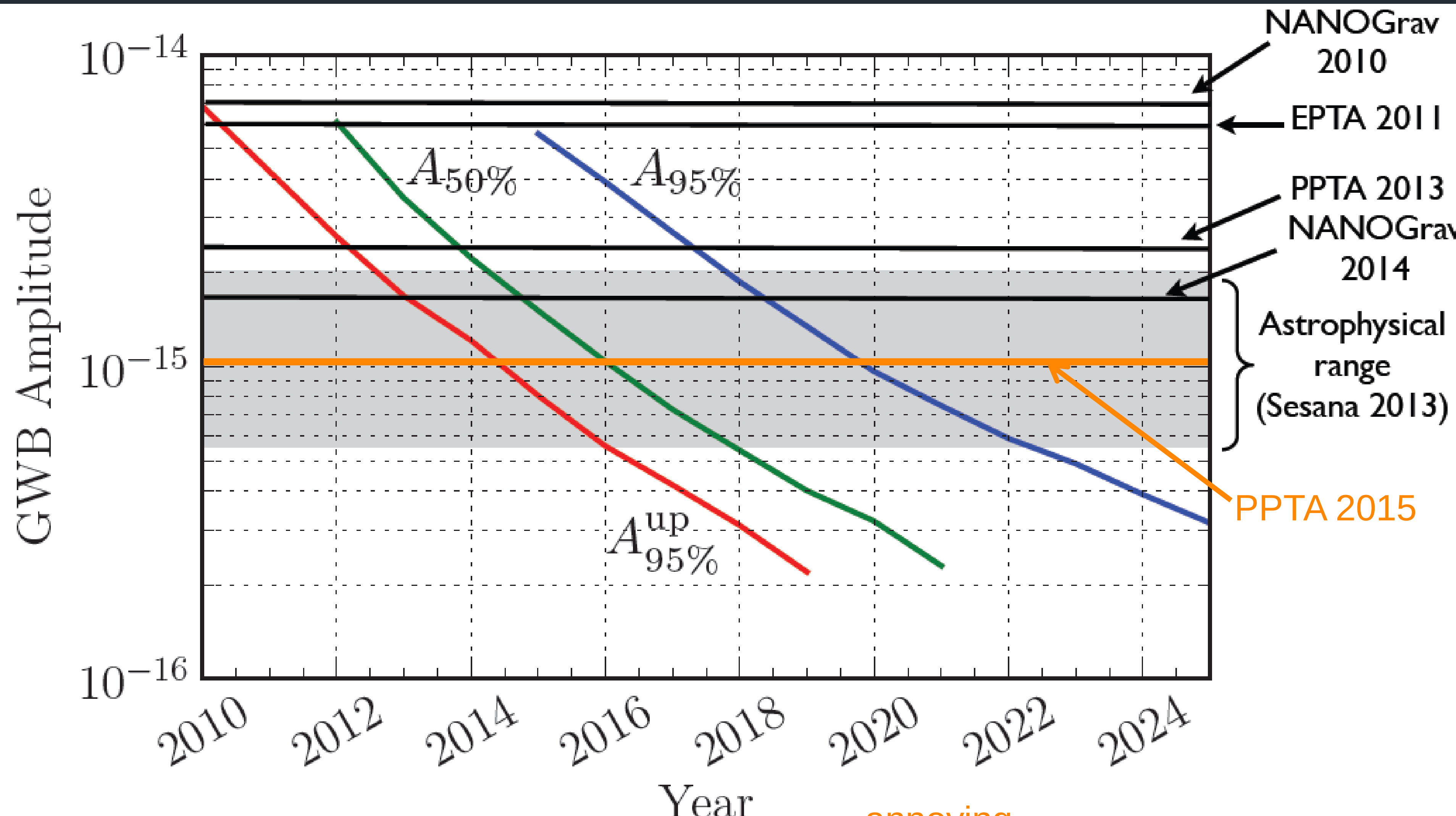
Not very covariant with low frequency noise
Fix based on single pulsar analysis
Can reduce parameter space to 50-100
Use standard MCMC/MultiNest
But not ideal

Options – Different Samplers for large dimensional problems

Gibbs Sampling (van Haasteren et al 2014)
Hamiltonian Sampling (Lentati et al 2013)

Still in general a problem

Astrophysical problems



We are already in astrophysically ~~interesting~~ annoying territory!!

Astrophysical problems

Ruling out large fractions of published models:

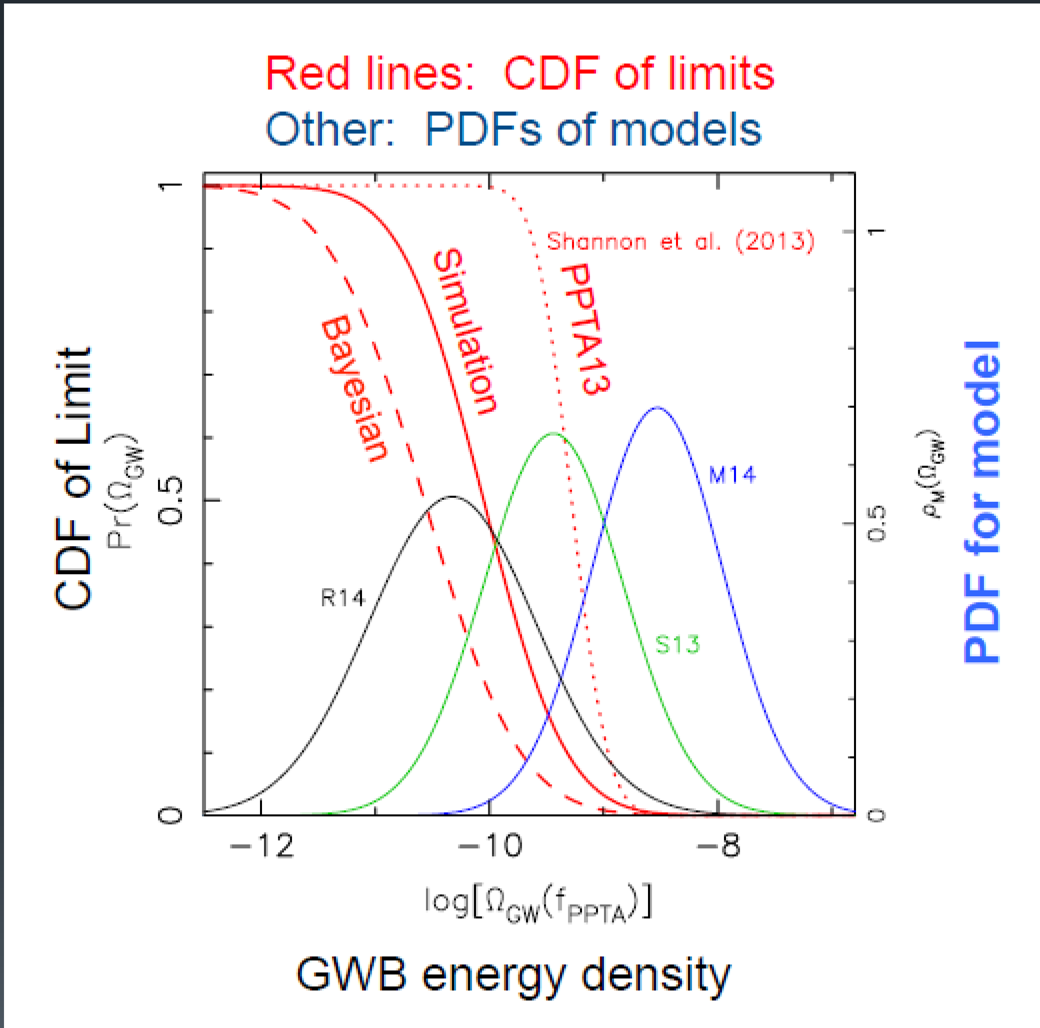
(M14) McWilliams 2014
(S13) Sesana 2013
(R14) Ravi 2014

Significant implications for Cosmology:

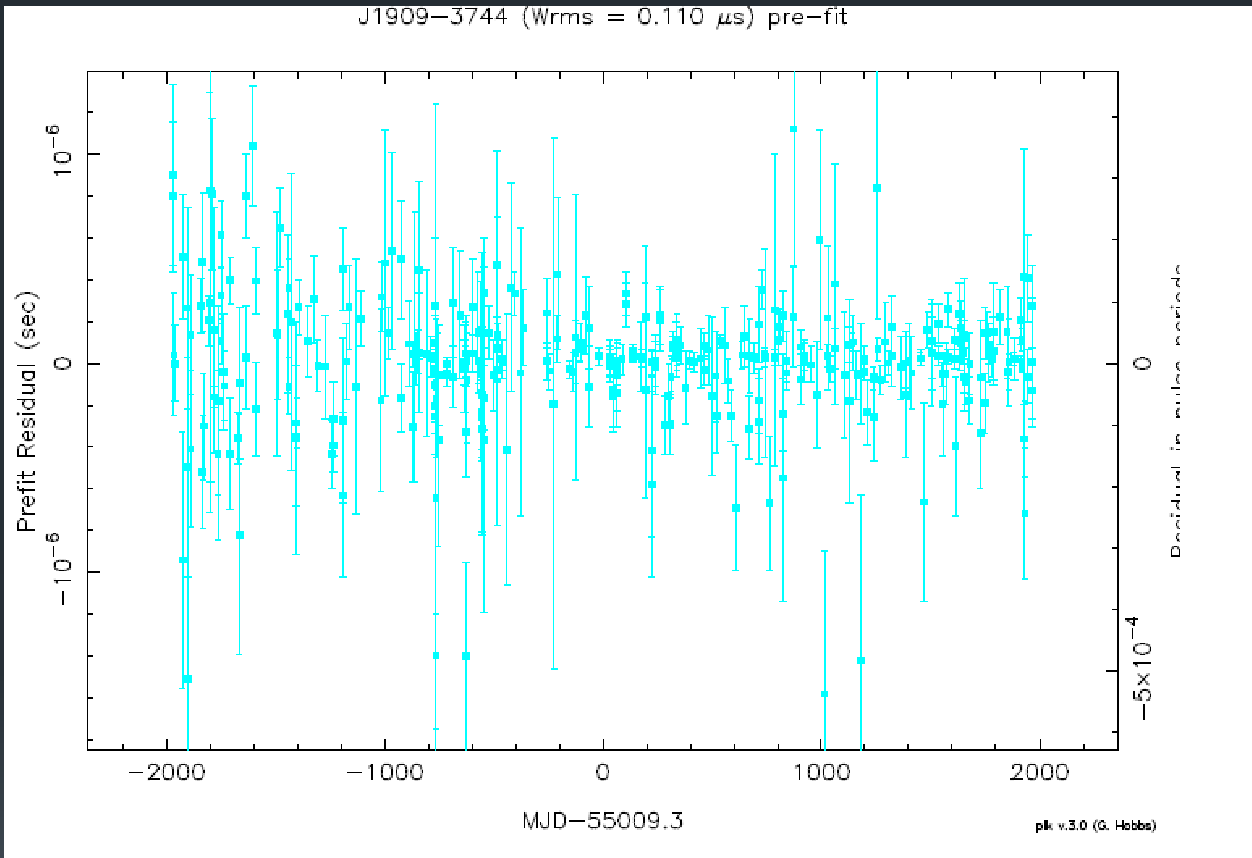
Mergers less frequent?

Energy lost through environment?

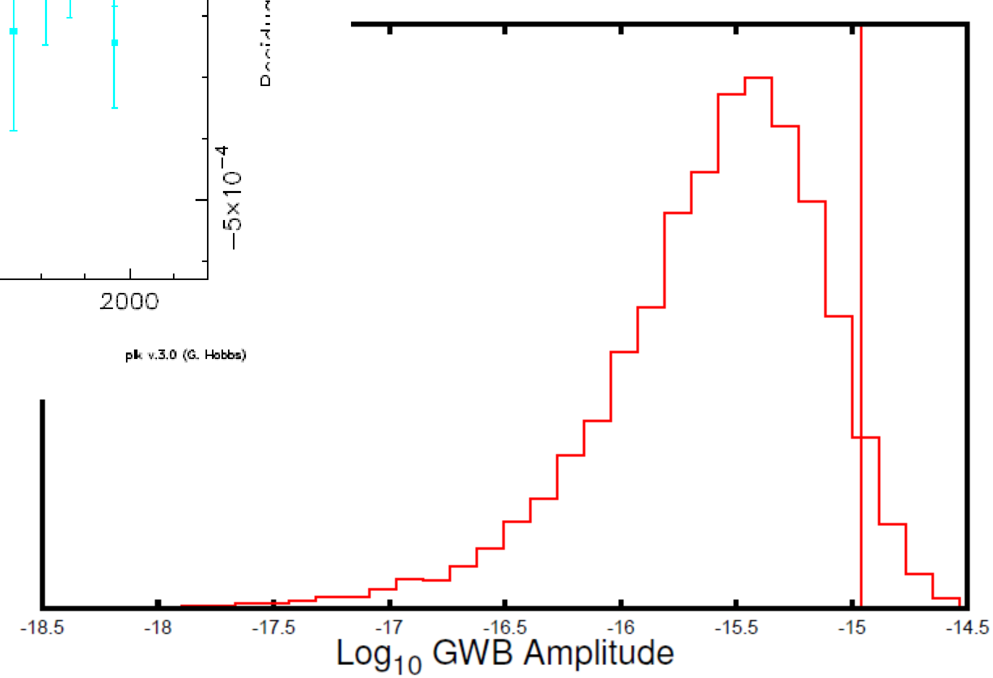
Decreases predicted amplitude compared to GW only evolution.



Astrophysical problems



11 years of data
Very stable pulsar
High frequency (avoids ISM)
100ns rms



No evidence for low frequency noise of any kind!

Summary

Current challenges:

Modelling the pulsars themselves, and the ISM in a reasonable way
Large dimensionality of total problem (hundreds/thousands)
Still fairly large matrices to deal with (few thousand x few thousand)
- Can do algebra on GPUs

Even so:

At the point where we *might* expect to see something..
..but still nothing!

Cheers