SOLAR SYSTEM PHYSICS 2

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§1 The Terrestrial Planets

§1.1 Introduction to the Terrestrial Planets

Mercury

Venus

Earth (and the Moon)

Mars

All of these are:

- *Solid rocky/metallic bodies*, although Earth and Mercury have liquid metal in or around their cores.

- Have *thin atmospheres*. In the case of the Moon this is because its small size means it has a relatively low escape velocity, meaning the atmospheric gases can escape. For Mercury, the same reason applies, coupled with its proximity to the Sun which results in it being very hot. The hotter something is, the easier it is for gases in its vicinity to be expelled.

Sizes: of the order of 10^4 km in radius $\left(\sim 10^{-2} \text{ the radius of the sun, } R_S\right)$.

Masses: of the order of 10^{24} kg $\left(\sim 10^{-6} \text{ the mass of the Sun, } M_S\right)$.

§1.2 How do we measure the radius of Earth, R_E ?

Consider two points on the Earth, P_1 and P_2, which have the same longitude but different latitudes[1]. Measure the zenith angles, Z_1 and Z_2, of a star at P_1 and P_2. (An observer's "zenith" is the point in the sky directly overhead.

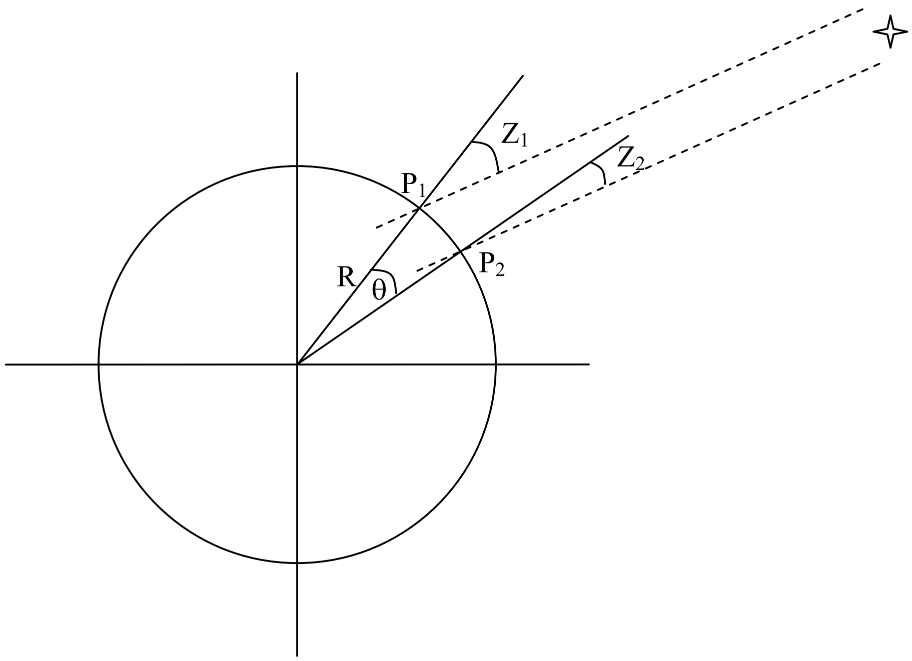

Figure 1.1 Measuring R_E.

$$\theta = Z_1 - Z_2 = \Delta Z \qquad [1.1]$$

Now, the distance along the arc from P_1 to P_2, ΔP, is $R\Delta Z$ assuming you are measuring the angle θ in radians. Therefore,

$$R = \frac{\Delta P}{\Delta Z} \qquad [1.2]$$

For the Earth, $R_E \sim 6000\,\mathrm{km}$.

[1] Lines of longitude connect the poles; lines of latitude form circles about the Earth, parallel to the equator.

§1.3 How do we measure the mass of the Earth, M_E?

Historically, this was done by Maskelyne by comparing the mass of the Earth to the mass of a mountain – Schiehallion in Perthshire. He made the comparison by looking at the pull Schiehallion exerted on a pendulum as seen from its angle against the stellar background.

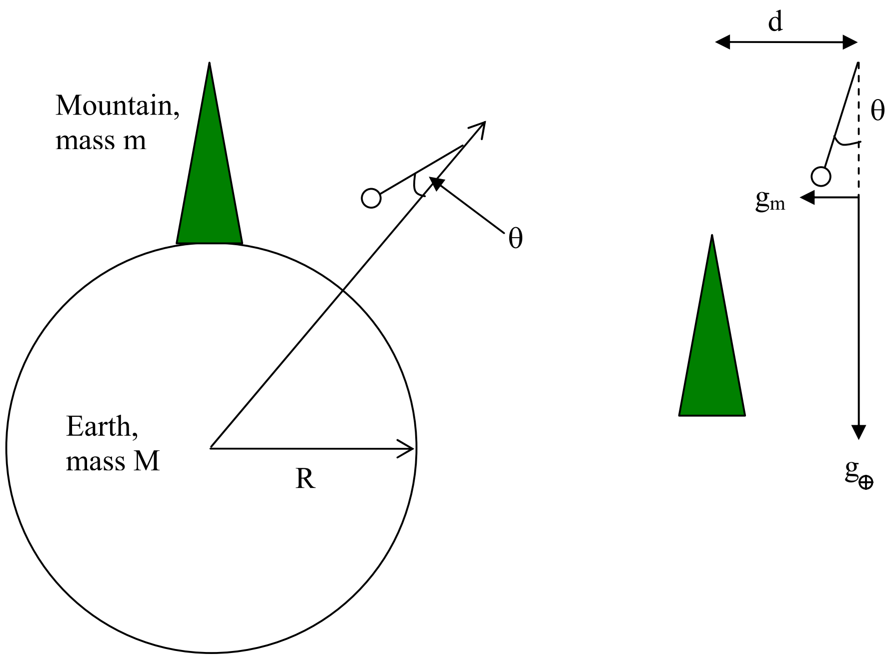

Figure 1.2 Measuring M_E.

Assume the mass of this pendulum bob is 1 kg.

Force on the 1 kg bob due to the Earth is:

$$F_{Earth} = 1 \times g_{Earth} = \frac{GM_E}{R^2} \qquad [1.3]$$

Force on the 1 kg bob due to the mountain (of mass *m*) is:

$$F_{mountain} = 1 \times g_{mountain} = \frac{Gm}{d^2} \qquad [1.4]$$

From figure 1.2,

$$\theta \sim \frac{g_{mountain}}{g_{Earth}} = \frac{Gm}{d^2} \times \frac{R^2}{GM_E}$$

$$\Rightarrow \theta \sim \frac{mR^2}{d^2 M_E} \qquad [1.5]$$

This is independent of G.

Now, $m << M_E$, but $d << R$. In fact, what we do is approximate the density of the mountain to be the same as that of the Earth. Then we can say:

$$\theta \sim \frac{d}{R} \sim \frac{1\,\text{km}}{10^4\ \text{km}}\ \text{radians} \sim 20''$$

More accurately, we measure d, and θ and then estimate the value of m based on the shape of the mountain and what rock type it is. This allows us to then calculate $M_E = 5.9 \times 10^{24}$ kg.

Note: 1" is one arcsecond.

1" = 1/3600 of a degree.

1' = 1 arcminute = 1/60 of a degree

§1.4 Probing the Earth's interior – Seismology

Following earthquakes or man-made explosions etc, waves propagate through the Earth's interior. These waves take one of two forms:

- p-waves: longitudinal *pressure waves* (compression waves – push material along direction of motion)
- s-waves: transverse *shear waves* (push material at right angles to their direction of motion).

(a) (b)

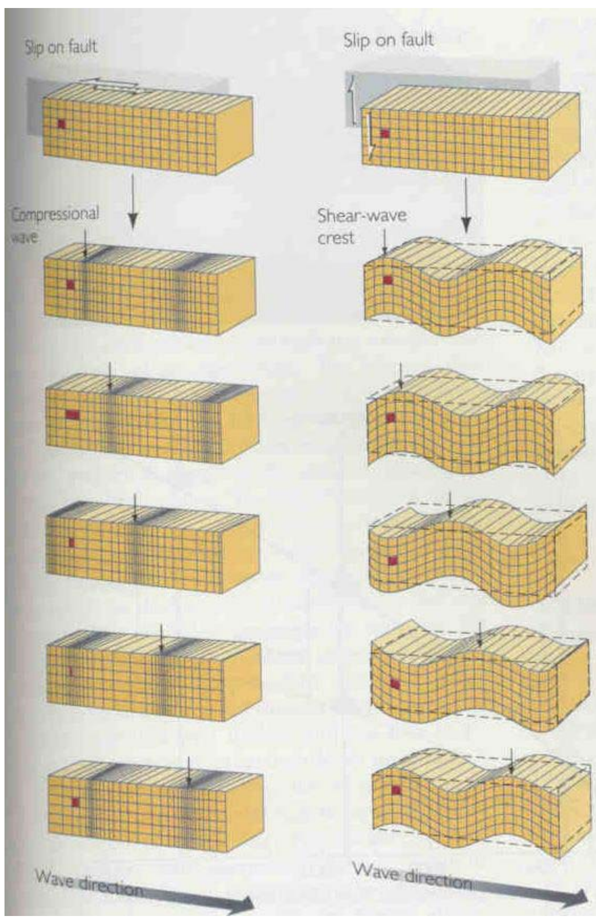

Figure 1.3 Illustration of (a) p-waves and (b) s-waves.

P-waves in rock are analogous to sound waves in air, except that p-waves travel through rock at $\sim 5\,\mathrm{kms}^{-1}$. S-waves travel at about half the speed of p-waves.

These waves are refracted according to their speed in the different materials they pass through. Their arrival pattern at places far from the original source sites tells us about the material inside the Earth.

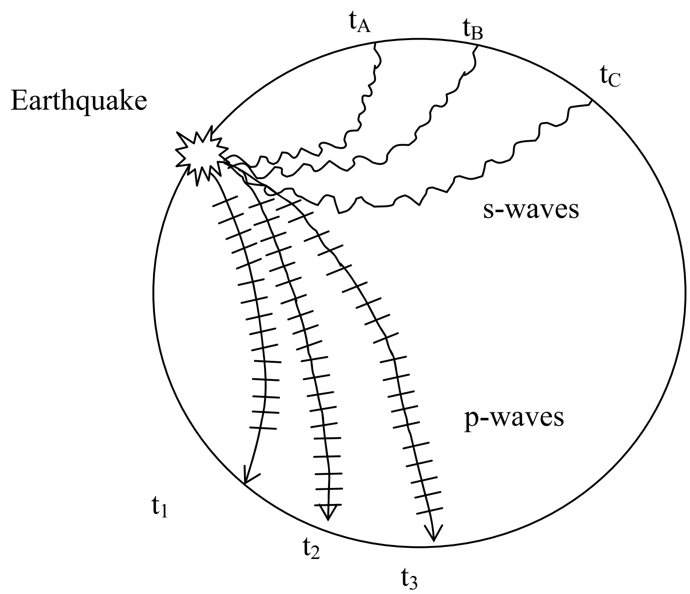

Figure 1.4 Waves generated by an earthquake.

e.g. s-waves cannot pass through liquids, therefore a liquid planetary core casts a *seismic shadow*.

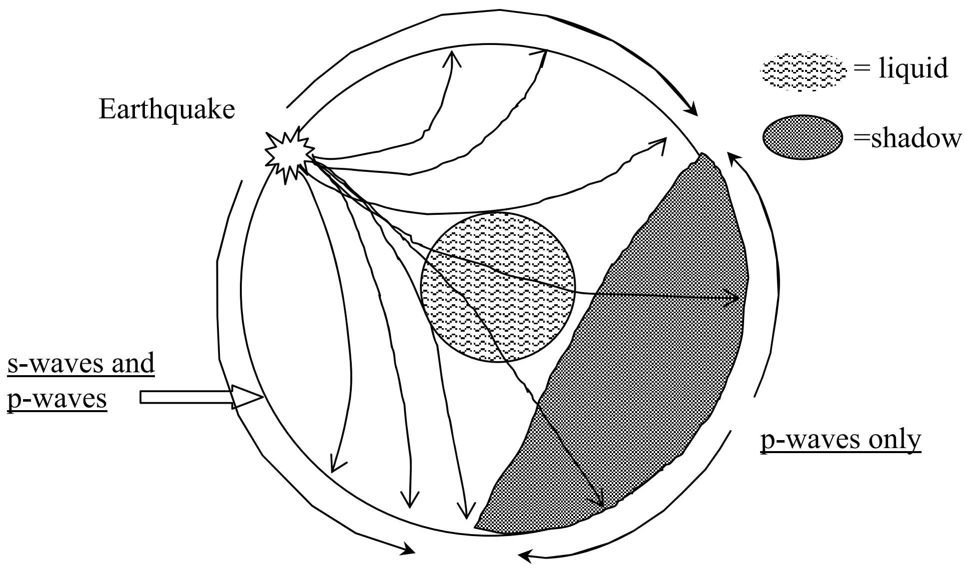

Figure 1.5 A liquid core blocks s-waves.

We can work out the size of this liquid core if we make a few approximations.

Consider the case where the core and overlying layers are separately uniform and the waves are travelling in perfectly straight directions. Figure 1.6 illustrates this situation.

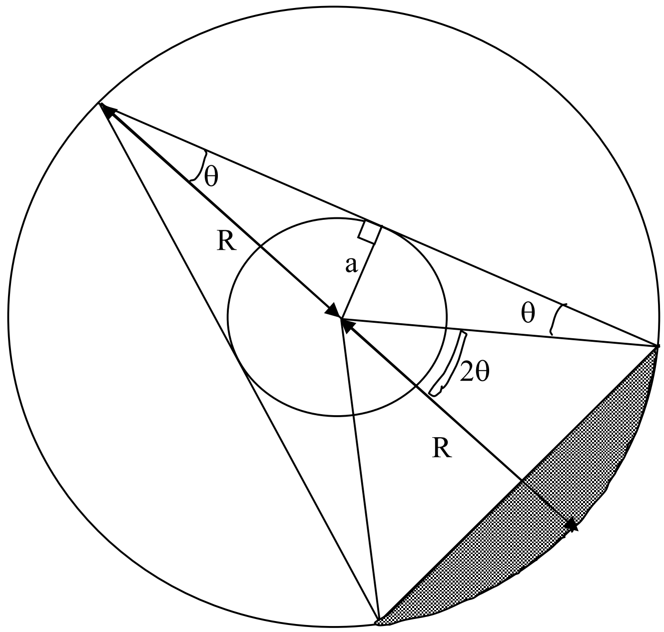

Figure 1.6 Calculating the size of the core.

We can now apply geometry to work out the size of the liquid core, of radius a.

The angular radius of the shadow zone $= 2\theta$ where,

$$\sin\theta = \frac{a}{R} \Rightarrow a = R\sin\theta \qquad [1.6]$$

Hence, if we know the radius of the Earth, then from measurements of the size of the shadow zone we can estimate the size of the molten core of the Earth. (~ half the diameter of the planet itself – see §1.5.1.)

§1.5 Overall structure of the Terrestrials

Let's take Earth as an example. In general terms, the Earth's interior is layered – it has a central core (which consists of a liquid centre surrounded by a solid outer area). Outside of this is the mantle, on top of which sits the crust. And on top of the crust sits us.

Figure 1.7 shows the general structure:

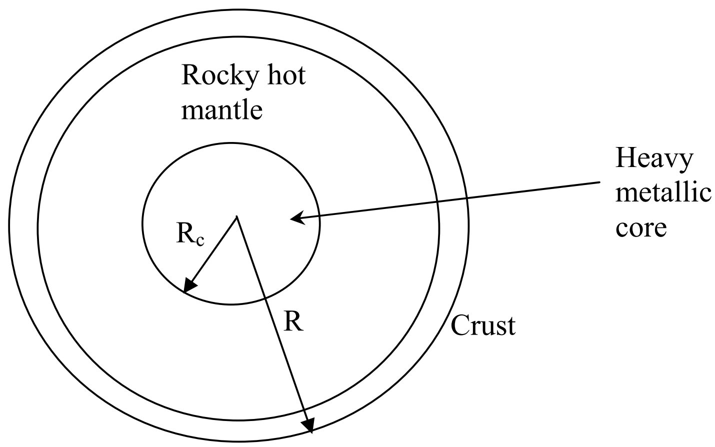

Figure 1.7: Structure of the Terrestrial planets

Mercury, Venus, Earth and Mars all have approximately the same density, ρ, given by

$$\rho = \frac{M}{\frac{4}{3}\pi R^3} \qquad [1.7]$$

where $M = \text{planetary mass}$ and $R = \text{planetary diameter}$. The specific values are given below in Table 1.1.

	Earth	Moon	Mercury	Venus	Mars
mass	1	0.012	0.06	0.82	0.11
radius	1	0.273	0.382	0.951	0.531
mean density (10^3 kgm^{-3})	5.52	3.34	5.43	5.25	3.95

Table 1.1 The comparative structure of the Earth and Terrestrial planets

The typical density of rocks on the Earth's surface is less than the mean density of Earth. This implies that *the Earth has a dense metallic core with a less dense rocky surface.*

The density of the moon is less than the mean density of the Earth. The moon is believed to have been formed as a chunk of the Earth's outer layers was knocked out by an impact (known as "*collision ejection theory*").

§1.5.1 Cores

These form as a result of *gravitational differentiation*: heavy metals sink through the semi-molten core, creating a denser central region.

Earth and Mercury have a liquid portion of their cores, as proven by their magnetic fields:

- Because the molten material (which is mostly iron) conducts electricity, the motions of the liquid portion of the planetary interiors give rise to electric currents, which in turn produce magnetic fields. (c.f. a dynamo)

For Venus, Earth and Mars: $\frac{R_{core}}{R} \sim 50\%$.

For Mercury: $\frac{R_{core}}{R} \sim 75\%$.

§2. Surface features of the Terrestrial planets

§2.1 The four basic geological processes

§2.1.1 Impact cratering

Impact craters formed by asteroids or comets striking a planet's surface.

All terrestrials show evidence of this. Around 4×10^9 years ago there was a period of "heavy bombardment" – debris rained down on planetary surfaces in the period after the formation of the planets.

§2.1.2 Volcanism

The eruption of molten rock (lava) from below the planet's crust on to the surface.

An eruption usually follows movement of magma upwards into the Earth's crust beneath a volcano and occupying a magma chamber.

Eventually, magma in the chamber is forced upwards and flows out across the planet's surface as lava, or the rising magma can heat water in the surrounding rocks and cause explosive discharges of steam.

Alternatively escaping gases from the magma can produce forceful ejections of rocks, cinders, volcanic glass, and/or volcanic ash. While always displaying powerful forces, eruptions can vary from effusive to extremely explosive.

Venus	The planet Venus is believed to be volcanically active, and its surface is 90% basalt, indicating that volcanism plays a major role in shaping its surface. Lava flows are widespread and many of its surface features are attributed to exotic forms of volcanism not present on Earth. Other Venusian phenomena, such as changes in the planet's atmosphere and observations of lightning, have been attributed to ongoing volcanic eruptions.
Mars	Olympus mons – a vast volcano 5 times the size of Everest. The evidence of whether there is current volcanic activity is uncertain.
Earth	Lots of water, volcanic activity, plate tectonics – the surface is very different from the other planets.
Moon	The density of the surface impact cratering varies – very dense in the "lunar highland". Some areas have only a few impact craters on top of a smooth surface – deep craters filled with lunar lava flow – "Lunar Mare" (from the latin for sea). Lunar impact cratering was mainly around $\sim 4\times10^9$ years ago, whilst the mare lava flows were around $\sim 3.5\times10^9$ years ago.
Mercury	Looks like the lunar highlands – lots of cratering and small lava plains.

Table 2.1 Volcanism on the Terrestrial planets

§2.1.2.1 Outgassing

Outgassing is a very important effect of volcanism. Hot early atmospheres escaped from the newly formed planets, so the only gas which existed on the planets arrived through comets, meteors etc impacting on the surface. As the early, hot, molten surfaces cooled, the gases became trapped in the planetary interiors.

The gases remained trapped until released by the effects of volcanism – as the rocks are melted again, the gases are released; or the gases are thrown back into the atmosphere during eruptions. Virtually all the gas that makes/made up the atmospheres of Venus, Earth and Mars was originally released by outgassing.

§2.1.3 Tectonics

The disruption of the planet's surface by internal stresses.

Plate tectonics was the "theory of relativity" for geologists. It allowed, for the first time, geological processes like earthquakes, mountain building and volcanoes to be viewed in terms of one unifying theory.

The outermost layers of the Earth – the crust and mantle – can be divided into two other groups: lithosphere and asthenosphere. The theory of plate tectonics says that the asthenosphere is not a constant shell, but rather it id divided up into ~ 12 large rigid plates that are in motion on the Earth's surface, riding on the asthenosphere. Figure 2.1 shows these plates.

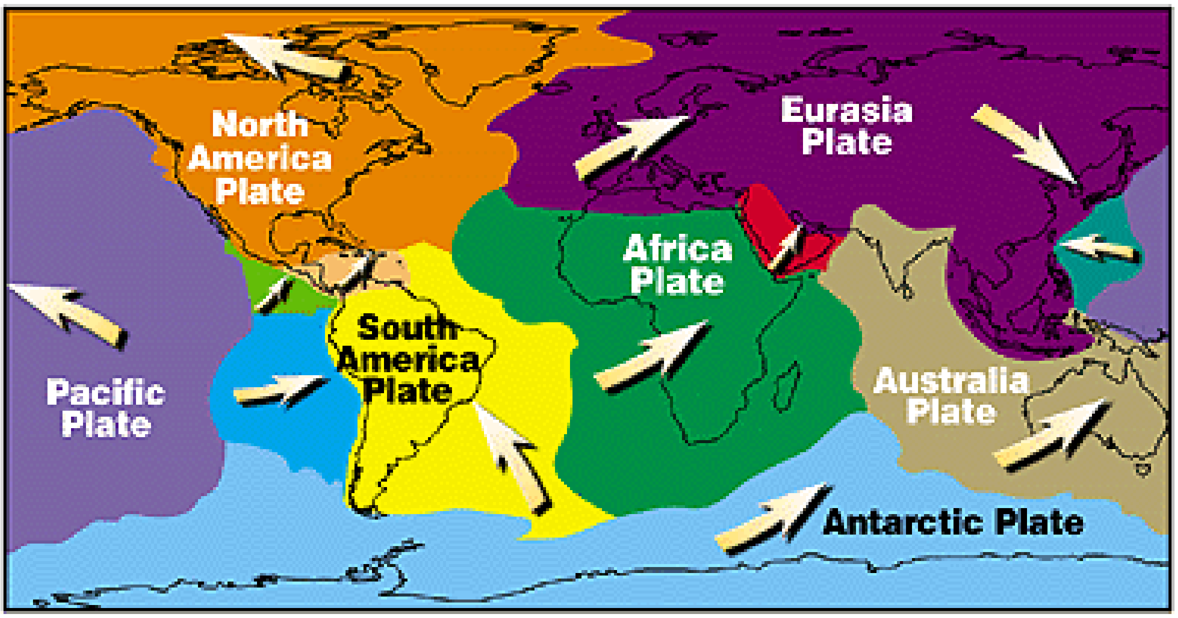

Figure 2.1 The plates of the Earth

These plates move because of convection.

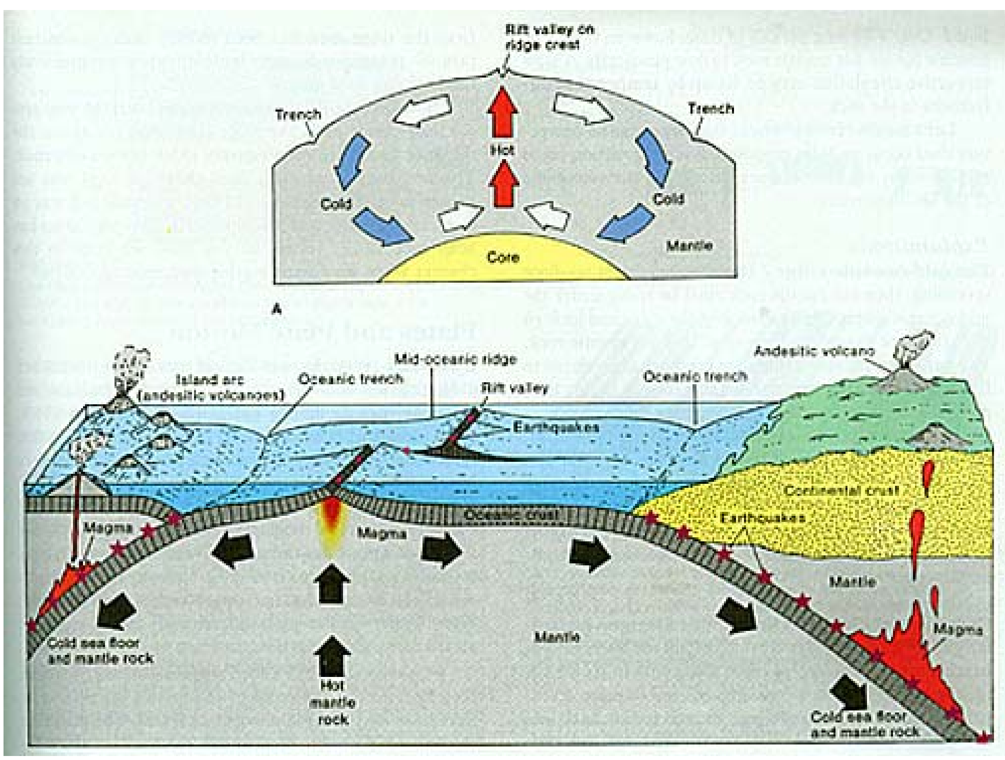

Figure 2.2 Convection current moving the plates

Large scale geologic features – earthquakes, volcanoes, mountain building – occur at the boundaries of these plates.

- At convergent boundaries, you get mountains.
- At divergent boundaries, you get rifts.

Whilst plate tectonics are primarily a feature on Earth, there is no evidence for it on the other planets, save for Mercury. Here there are "cliffs" on the surface caused by the contraction of the surface as the planet cooled.

§2.1.4 Erosion

The wearing down/building up of surface features by the action of wind/water/ice etc – i.e. by weather.

Earth	Heavy erosion – weather!
Venus	Little evidence for erosion, despite it having an atmosphere.
Mars	Substantial erosion – evidence for the existence of water flow in the past.
Mercury	No erosion.
Moon	No erosion.

Table 2.2 Erosion on the Terrestrial planets

§2.2 Atmospheres

§2.2.1. Retention of Atmospheres – Revisited

In Solar System Physics 1 you were told that a gas is lost from the atmosphere of a planet if its particles have a root-mean-square velocity, v_{rms}, greater than one sixth of that planet's escape velocity, v_{esc}. i.e.

$$v_{rms} > \frac{v_{esc}}{6} \qquad [2.1]$$

Example 2.1

Why is a gas is lost from the atmosphere of a planet if its particles have a root-mean-square velocity, v_{rms}, greater than one sixth of that planet's escape velocity, v_{esc}?

It can be shown that the distribution of molecular speeds in a gas follows a Maxwell-Boltzmann distribution. You can express the fraction of the total number of gas particles, f, which have a speed (v_{rms}) greater than a particular value (say v_1) as

$$f \approx e^{-v_1^2 / v_{rms}^2} \qquad [2.2]$$

You can therefore say that the fraction, f_{esc}, with speed greater than the escape velocity, v_{esc}, of a planet, is:

$$f \approx e^{-v_{esc}^2 / v_{rms}^2} \qquad [2.3]$$

Now, if you define the "height" of Earth's atmosphere to be R, then you can say that this fraction escapes in a time, t:

$$t \approx \frac{R}{v_{esc}} \qquad [2.4]$$

For the Earth, $R \sim 1000$ km and $v_{esc} \sim 10$ kms^{-1}, making $t \sim 10^3$ s.

If a fraction f_{esc} escapes in time t, then the time for all of the particles to escape, t_{esc}, is:

$$t_{esc} \approx \frac{t}{f_{esc}} = \frac{t}{e^{-v_{esc}^2 / v_{rms}^2}}$$

$$\Rightarrow t_{esc} = 10^3 \times e^{v_{esc}^2 / v_{rms}^2} \qquad [2.5]$$

The age of the Earth is known to be $\sim 10^{17}$ seconds. So almost all the atoms will escape from the Earth in its lifetime unless $t_{esc} > 10^{17}$. i.e.

$$10^3 \times e^{v_{esc}^2 / v_{rms}^2} > 10^{17}$$

$$\Rightarrow e^{v_{esc}^2 / v_{rms}^2} > 10^{14}$$

$$\Rightarrow \frac{v_{esc}}{v_{rms}} \geq \sqrt{\ln\left(10^{14}\right)} \approx 6 \qquad [2.6]$$

§ 2.2.2 Composition of Atmospheres

	Atmospheric pressure	Atmospheric constituents	Other comments
Earth	1 bar $\left(10^5\ \mathrm{Nm^{-2}}\right)$	$N_2 - 77\%$ $O_2 - 21\%$ $H_2 - 0.1 \rightarrow 2.8\%$ $Ar - 0.9\%$ $CO_2 - 0.03\%$	O_2 would be lost by reactions in rock, etc. It only exists because of the planet life.
Moon	10^{-14} bar (very thin)		
Mercury	10^{-14} bar (very thin)		
Venus	90 bar	$CO_2 - 96\%$ $N_2 - 3.5\%$	
Mars	0.006 bar	$CO_2 - 95\%$ $N_2 - 3\%$ $Ar - 1.5\%$ $O_2 - 0.1\%$	CO_2 is frozen in the polar caps; the atmosphere is dusty; wind blown patterns.

Table 2.3 Atmospheric constituents and pressures of the Terrestrial planets.

Consider oxygen (O_2) – suppose $f_{esc:O_2} = 10^{-13}$, then, again assuming $t = 10^3$ s,

$$t_{esc} = \frac{t}{f_{esc}} = \frac{10^3}{10^{-13}} = 10^{16}\text{ s} \qquad [2.7]$$

i.e. the fraction escapes in a time $\sim 10\%$ of the age of the Earth.

Now, for a molecule, $v_{rms} = \sqrt{\frac{3RT}{M}}$, therefore the heavier the molecule, the lower the v_{rms} (assuming the temperature is constant). f_{esc} will therefore be smaller.

Now consider carbon dioxide (CO_2) - $m_{CO_2} = \frac{44}{32} \times m_{O_2}$.

Here $f \sim 10^{-18}$ [PROVE THIS FOR YOURSELF] and it takes 10000 times the age of the Earth for all the molecules to escape. So the retention of a gas is very sensitive to the ratio ${}^{v_{esc}}\!/\!{}_{v_{rms}}$.

For example, Earth loses hydrogen and helium very quickly, but retains oxygen, nitrogen, carbon dioxide and water. The hydrogen and helium on Earth exist only when locked up in rocks, water, etc. This is in contrast to the Jovian planets, which are mostly H and He.

§2.3 Magnetosphere

This shields the atmosphere from solar winds, which you will recall were low density streams of charged particles. Some of these penetrate the Earth's magnetosphere at the poles – charged particles trapped in the magnetosphere create the Aurora.

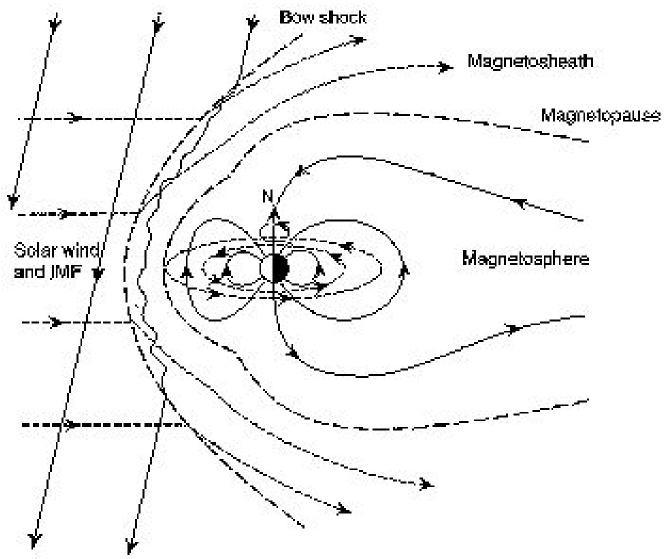

Figure 2.3 The Earth's magnetosphere

(Ref: A Beginner's Guide to the Earth's Magnetosphere by Stanley W. H. Cowley, Department of Physics and Astronomy, University of Leicester, Leicester, United Kingdom http://www.agu.org/sci_soc/cowley.html)

§3 Interior features of the Terrestrial planets – Earth

In this section we will deal exclusively with the Earth for the simple reason that the Earth is the only Terrestrial planet we have – so far – been able to probe beneath the surface.

§3.1 The layers of the Earth

Figure 3.1 summarises the various layers of the Earth's crust:

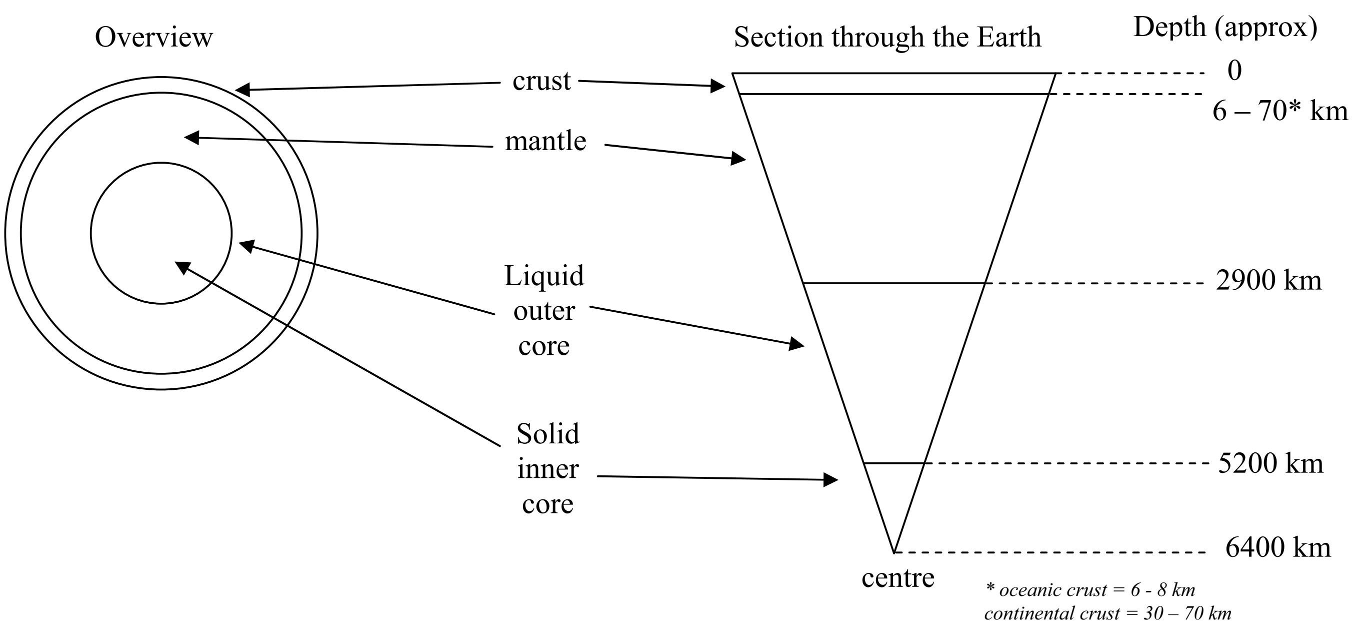

Figure 3.1 Schematic picture of the structure of the Earth

The crust:

- Consists of silicates and oxides.
- Has a temperature ranging 300 – 700 K.
- The rock types are mainly igneous, but there are also sedimentary and metamorphic rocks.

The mantle:

- Consists of silicates and oxides.
- Has a temperature ranging 700 - 4500 K.
- The rock types are mainly “plastic”, basalt-like in composition.

The outer core:

- Liquid in state.
- Consists of molten iron and nickel.

The inner core:

- Solid in state.
- Consists of iron and nickel.
- Central temperatures are ~ 6000 K.
- Solidification caused by high pressure.

§3.2 The rocks of the Earth

Igneous rocks:

- Get their name from the Latin word for fire: “ignis”.
- They form from the crystallisation of a magma – a mass of melted rock that originates deep in the crust or upper mantle, where temperatures reach the 700 $^{\circ}C$ needed to melt most types of rock.

- e.g. Granite (intrusive igneous) or basalt (extrusive) The former results from slow cooling within the crust, the latter from fast cooling on the surface (e.g. after a volcanic eruption).

Sedimentary rocks:

- Derive from sediments – layers of loose particles, such as sand, silt and shells of organisms.
- Weather erodes other rock types, creating the finer particles which can be washed away in rivers to the sea, where they are laid down on the sea bed. Sediments then undergo lithification – they harden into solid rock through compaction (as the grains are squeezed tighter and tighter together), and by cementation (minerals precipitate from the grains after deposition and bind the deposits together).
- Characterised by bedding – the formation of parallel layers by the settling of the particles on the sea bed.
- e.g. calcite, dolomite

Metamorphic rocks:

- From the Greek “meta” meaning change and “morphe” meaning form.
- Produced by mineralogical and textural transformations of all kinds of rocks – igneous, sedimentary, metamorphic – under the influence of high temperatures ($250\ ^{o}\mathrm{C}$ to $700\ ^{o}\mathrm{C}$) and pressures deep in the Earth. Can occur in regions of tectonic plate collision, or around the edges of magma pockets.

- e.g. mica, garnet

§3.3 The rock cycle

The various types of rock listed above are not independent of each other. As figure 3.2 shows, each type of rock can, at some stage in its life, become another type.

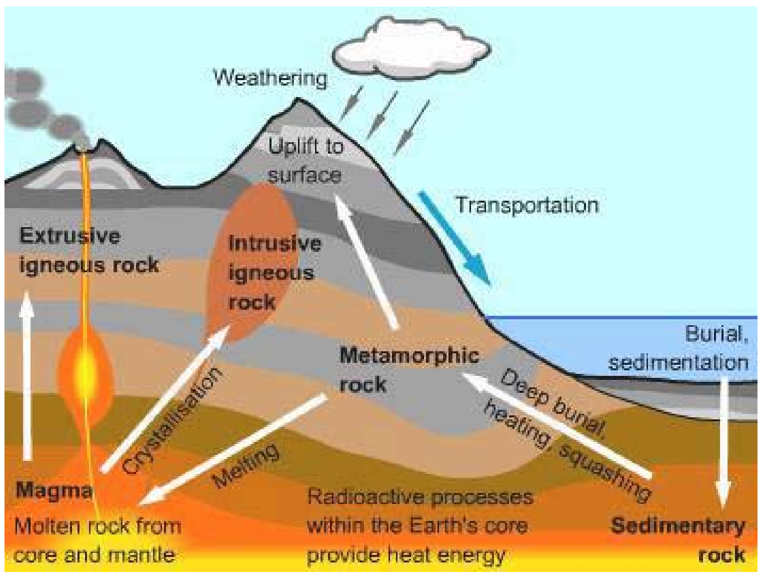

Figure 3.2 The rock cycle

§3.4 Other rocks in our solar system

There is another type of rock to be found in the solar system – *Primitive rock*. This is rock which has undergone no reforming. It is formed from the primitive matter in our solar system. It has not been melted or compacted nor has anything else affect it, as would have happened if it were on the Earth. It's typically found in asteroids and meteorites, which we will discuss in more detail later.

§4 Why are planets hot?

There are two reasons:

1. *Gravitational energy* released when the planet is formed.
2. *Geothermal energy* due to tidal stressing of rocks and radioactive decay of heavy elements (of which more in §4)

§4.1 Gravitational energy

Consider the case of a 1 kg mass at a distance r from a proto-planet of mass M and radius R.

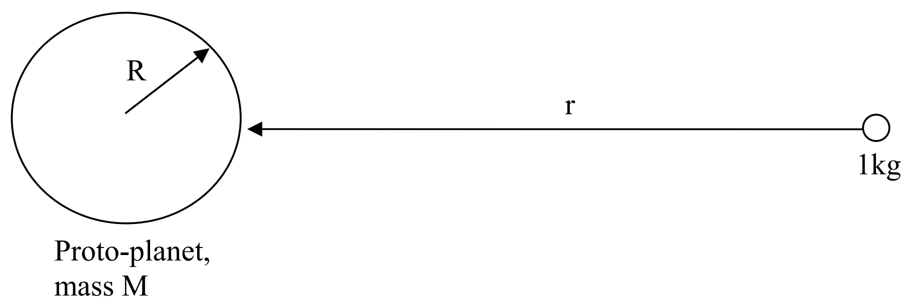

Figure 4.1 A mass near a proto-planet.

The gravitational energy of the 1 kg mass, m, due to the presence of the proto-planet of mass M is:

$$U = -\frac{GMm}{r} = -\frac{GM}{r} \qquad [4.1]$$

The gravitational energy the 1 kg mass loses as it falls from a distance r to R is:

$$U_{\text{lost}} = U_{\text{at r}} - U_{\text{at R}} \qquad [4.2a]$$

$$\Rightarrow U_{\text{lost}} = -\frac{GM}{r} - \left(-\frac{GM}{R}\right) \approx \frac{GM}{R} \qquad [4.2b]$$

assuming $r >> R$, i.e. that the 1 kg mass is initially at a very great distance from the proto-planet.

So, as the planet forms, each kg of mass loses energy GM/R as *heat*.

§4.2 Temperature of formation

Example 4.1

Calculate the temperature of formation of the Earth.

Assume that in 1 kg of material there are N atoms, each of mass $\overline{m}$. You can then say

$$N = \frac{\text{total mass}}{\text{mass per atom}} = \frac{1}{\overline{m}} \qquad [4.3]$$

By the equipartition theorem, each atom has energy $\frac{3}{2}k_B T$, where $k_B = 1.38 \times 10^{-23}$ JK^{-1} = the Boltzmann contant and T is temperature. The amount of energy, E, in N atoms is therefore

$$E = \frac{3}{2} N k_B T$$

$$\Rightarrow E = \frac{3}{2} \frac{k_B T}{\overline{m}} \qquad [4.4]$$

since $N = \frac{1}{\overline{m}}$.

It is then possible to evaluate the temperature rise, T, by equating this equation with the energy loss per kg, i.e. combining equations [4.2] and [4.4]:

$$\frac{3}{2} \frac{k_B T}{\overline{m}} \approx \frac{GM}{R}$$

$$\Rightarrow T \approx \frac{2}{3}\frac{GM\overline{m}}{k_B R} \qquad [4.5]$$

Let's assume that the 1 kg mass is carbon and that it is falling in towards a planet with an Earth-like radius and mass.

Here, $\overline{m} = 12m_H$ where m_H = mass of hydrogen atom $= 1.673 \times 10^{-27}$ kg .

Putting this into equation [4.5], together with the known values for constants G, M, k_B and R, you find that $T \sim 60000\,\mathrm{K}$.

This means that the Earth was obviously a lot hotter when it was created than it is now.

§4.3 How long does the cooling take?

Example 4.2

How long, approximately, did the Earth take to cool to today's temperature?

To estimate the cooling time, we need to consider the luminosity of the cooling object. As mentioned in §1, the luminosity of a body is the energy emitted per second by that body. If the body is a hot sphere of radius R, and is at temperature T, then its luminosity, L, is given by equation [4.6]:

$$L = 4\pi R^2 \sigma T^4 \qquad [4.6]$$

$\sigma = 5.7 \times 10^{-8}$ Wm^{-2}K^{-4} is the Stefan-Boltzman Constant. Again recall that luminosity is a power, and hence is measured in Watts. Luminosity is sometimes referred to as a radiating power.

Check: does equation [4.6] agree with the idea that luminosity has the units of power?

$$[L] = [4][\pi]\left[R^2\right][\sigma]\left[T^4\right]$$

$$\begin{aligned}
[4] &= 1 \\
[\pi] &= 1 \\
\left[R^2\right] &= \mathrm{m}^2 \\
[\sigma] &= \mathrm{Wm^{-2}K^{-4}} \\
\left[T^4\right] &= \mathrm{K}^4
\end{aligned}$$

Therefore, $[L] = 1 \times 1 \times \mathrm{m}^2 \times \mathrm{Wm}^{-2}\mathrm{K}^{-4} \times \mathrm{K}^4$.

i.e. $[L] = \dfrac{\mathrm{m}^2 \times \mathrm{W} \times \mathrm{K}^4}{\mathrm{m}^2 \times \mathrm{K}^4} = \mathrm{W} = [\text{power}]$

Since luminosity is a rate of energy flow with respect to time, it can also be represented by the following expression:

$$L = \frac{dE}{dt} \qquad [4.7]$$

i.e. luminosity is the first derivative of energy with respect to time. Equation [4.4] expressed E in terms of T for a 1 kg mass. Generalised for the case of a mass M, equation [4.4] becomes

$$E = \frac{3}{2}\frac{Mk_B T}{\overline{m}} \qquad [4.4^*]$$

If we want to use this expression to work out L, we need to apply the chain rule as follows:

$$\frac{dE}{dt} = \frac{dE}{dT}\frac{dT}{dt} \qquad [4.8]$$

Therefore,

$$L = \frac{dE}{dt} = \frac{dE}{dT}\frac{dT}{dt} = \frac{d}{dT}\left(\frac{3}{2}\frac{Mk_BT}{\overline{m}}\right)\frac{dT}{dt}$$

$$\Rightarrow L = \frac{3}{2}\frac{Mk_BT}{\overline{m}}\frac{dT}{dt}$$

$$\Rightarrow \frac{3}{2}\frac{Mk_BT}{\overline{m}}\frac{dT}{dt} = 4\pi R^2\sigma T^4$$

$$\Rightarrow dt = \frac{3}{2}\frac{Mk_B}{\overline{m}4\pi R^2\sigma T^4}dT \qquad [4.9]$$

Equation [4.9] can, theoretically, then be used to work out the age of the Earth by integrating its right hand side from the initial formation temperature T_i to the temperature today, T_f. i.e.

$$\text{Age of Earth} = \tau_{\text{cool}} = \int dt = \int_{T_i}^{T_f} \frac{3}{2}\frac{Mk_B}{\overline{m}4\pi R^2\sigma T^4}dT$$

$$\Rightarrow \tau_{\text{cool}} = \frac{3}{2}\frac{Mk_B}{\overline{m}4\pi R^2\sigma}\int_{T_i}^{T_f}\frac{1}{T^4}dT$$

$$\left|\tau_{\text{cool}}\right| = \frac{1}{2}\frac{Mk_B}{\overline{m}4\pi R^2\sigma}\left[\frac{1}{T_f^3} - \frac{1}{T_i^3}\right] \qquad [4.10]$$

If we assume, again, that $m_a = 12 m_H$, and take a final temperature of $300\,\mathrm{K}$, then [4.10] tells us that the cooling time of the Earth, and hence its age, is $\sim 8 \times 10^4$ years, which is considerably shorter than the commonly accepted age. So what's wrong?

The problem is that our model is incomplete:

- heat is also supplied by radioactive decay in Earth's crust – *radiogenic heating*
- *radiative heating* loss is shielded by the Earth's crust – solid rock is a very poor thermal conductor.

§5. How old are the Earth and the planets?

§5.1 Natural radioactivity and radioactive dating

Some heavy elements decay by nuclear disintegration – *radioactive decay* (see, e.g. University Physics by Freedman and Young). The number of radioactive nuclei present in any sample of a radioactive material decreases continuously as some of those nuclei decay. The rate of decay varies for different types of nuclei.

Consider a radioactive sample. Let N be the number of radioactive nuclei in the sample at some time t and let dN be the number of those nuclei that decay in a short time interval, dt.

Then, the decrease in the number of radioactive nuclei is $-dN$, and the rate of change of N is $-\frac{dN}{dt}$.

The larger N at the start, the larger dN is for a given dt.

i.e.

$$\frac{dN}{dt} = -\lambda N \qquad [5.1]$$

where λ is a constant.

Now let N_0 be the initial number of nuclei, i.e. the number at $t = 0$. Then a solution of the above equation is:

$$N(t) = N_0 e^{-\lambda t} \qquad [5.2]$$

§5.2 Radioactive half-life

The half-life for a given type of radioactive material is the time taken for half of the original number to decay. i.e. the time for $N \to N_0/2$. This is given the symbol $t_{1/2}$.

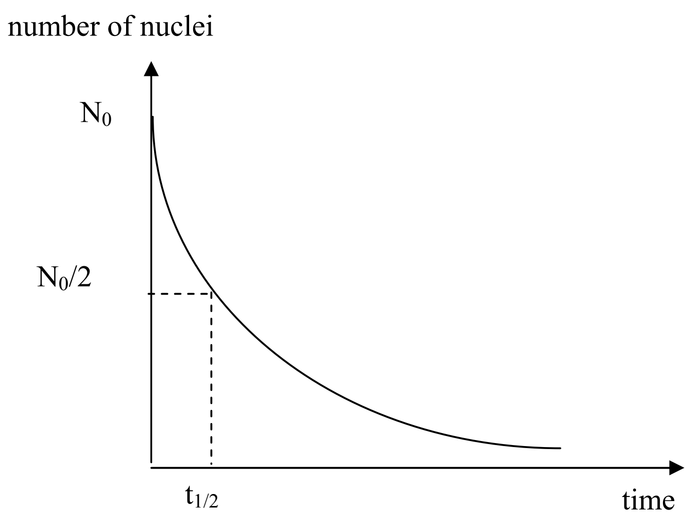

Figure 5.1 The decay of nuclei.

Example 5.1

Derive an expression for the half-life of a radioactive decay.

From equation [5.2], we can say that

$$\frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}} \Rightarrow e^{-\lambda t_{1/2}} = \frac{1}{2}$$

$$\Rightarrow -\lambda t_{1/2} = \ln\left(\frac{1}{2}\right) = -0.693$$

$$\Rightarrow t_{1/2} = \frac{0.693}{\lambda} \qquad [5.3]$$

§5.3 Parents and daughters

Consider the case of an element P (parent) decaying to element D (daughter) in a time t_1. If N_P and N_D are the numbers of atoms of each element, respectively, then we find that

$$N_P(t_0) = N_P(t_1) + N_D(t_1) \qquad [5.4]$$

i.e. the total number of atoms stays the same – N is conserved.

Equation [5.2] can be rewritten as

$$N_P(t_1) = N_P(t_0)e^{-\lambda(t_1 - t_0)} \qquad [5.2^*]$$

This can be rearranged as follows:

$$e^{-\lambda(t_1 - t_0)} = \frac{N_P(t_1)}{N_P(t_0)} = \frac{N_P(t_1)}{N_P(t_1) + N_D(t_1)} \qquad [5.5]$$

In other words, by comparing the ratio of parent and daughter isotopes in a rock sample, we can date it. i.e. find $t_1 - t_0$, where we take t_1 as now and t_0 as the time of the rock formation.

§5.4 Actual radioactive dating

The most important example of this is Potassium to Argon decay: ${}^{40}_{19}K \rightarrow {}^{40}_{18}Ar$ for which $t_{1/2} = 1.3 \times 10^9$ years.

Note:

(1) The technique above assumes:

(a) no argon is initially present in the rock at the time of its formation;

(b) no argon has escaped from the rock once since it formed.

(2) From chemistry, we know:

(a) argon does not react with rock;

(b) argon is a gas and escapes from liquids – but is trapped in rock.

Thus, any ${}^{40}_{18}Ar$ in a rock got there by the decay of ${}^{40}_{19}K$ after the rock solidified.

Other cases can be more complex due to multiple products and/or contamination.

(Can also use ${}^{238}U \rightarrow {}^{206}Pb$, which has a half-life of 4.5×10^9 years for rock dating.)

§5.5 The Result

By radioactive dating meteorites the age of our solar system has been set at $\sim 4.55 \times 10^9$ years. The oldest surface rocks on the Moon and Earth, though, are younger. Especially for the Earth, whose surface is constantly being changed by *erosion, volcanism and plate tectonics*.

§6 Radiation theory

§6.1 Introduction

A hot, dense body radiates a spectrum that depends on its temperature. The radiated spectrum, or flux, for a body is denoted by F_λ, and is measured in Wm^{-2} per unit wavelength).

For a "black body", we use the symbol B_λ, which represents the "Planck Spectrum".

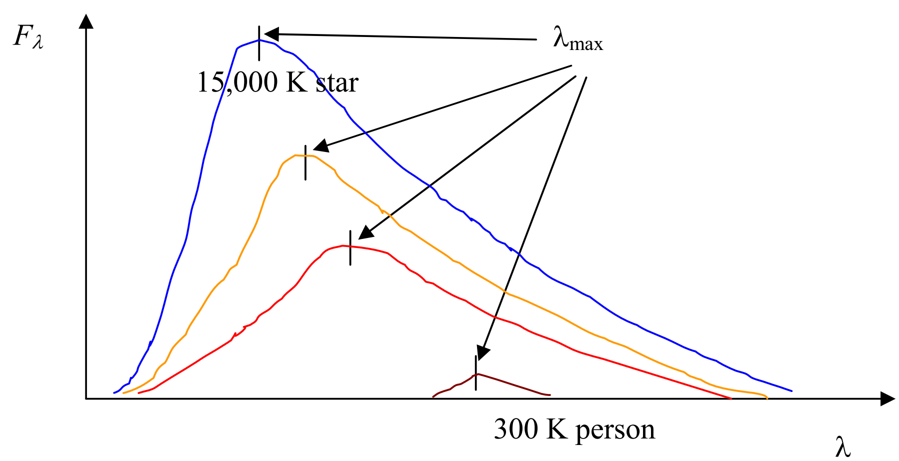

Figure 6.1 Radiated intensity distribution

Figure 6.1 shows that the hotter an object is:

- the greater the total amount of radiation it emits (area under the curve);
- the shorter the wavelength at which its emission peaks.

§6.1.1 Important features about the curves on figure 6.1

These curves have the following important properties:

(i) Total intensity (denoted by F, or B, or I)

The total intensity in all wavelengths λ is given by the Stefan-Boltzmann Law:

$$B = F = \int_0^\infty B_\lambda d\lambda = \sigma T^4$$

$$\Rightarrow \boxed{F = \sigma T^4} \qquad [6.1]$$

Stefan-Boltzmann Law

where σ = the Stefan-Boltzmann Constant = 5.6705×10^{-8} $\mathrm{Wm^{-2}K^{-4}}$.

(Not to be confused with the Boltzmann constant!)

The units of F = units of intensity = Power per unit area; i.e. $\mathrm{Wm^{-2}}$.

(ii) Luminosity of a hot sphere (as we've seen before)

This is defined as the area of the body, the sphere, multiplied by the intensity. i.e.

$$L = \text{area} \times \sigma T^4 = 4\pi R^2 \sigma T^4 \qquad [6.2]$$

(iii) Peak intensity

This occurs when $\lambda = \lambda_{\text{max}}$, such that

$$\boxed{\lambda_{\text{max}} T = \text{CONSTANT} = 2.9 \times 10^{-3} \text{ Km}} \quad [6.3]$$

Wien's Displacement Law

§6.2 Applications of Radiation Laws

§6.2.1 Determining the temperature of the solar surface and the flux.

We'll do this as an example.

Example 6.1

If the peak emission frequency of the Sun's surface is $\sim 6\times10^{14}$ Hz, show that its effective temperature is ~ 5800 K. Hence determine the flux at the solar surface.

First we need to find the peak wavelength:

$$c = f\lambda \Rightarrow \lambda = \frac{c}{f} \sim \frac{3\times10^{8}}{6\times10^{14}} = 5\times10^{-7}\text{ m}$$

Now we use Wien's displacement law to find $T_{effective}$:

$$\lambda_{\max}T = 2.9\times10^{-3} \Rightarrow T = \frac{2.9\times10^{-3}}{5\times10^{-7}} = 5800\text{ K}$$

Now we use the Stefan-Boltzmann law:

$$F = \sigma T^{4} \sim 5.671\times10^{-8}\times5800^{4}$$

$$\Rightarrow F \sim 6.42\times10^{7}\text{ Wm}^{-2}$$

$$\Rightarrow F \sim 64.2\text{ MWm}^{-2}$$

§6.2.2 Determining the flux of sunlight at the Earth, or a general distance D

Again, we'll do this as an example.

Example 6.2

Determine the flux of sunlight at the Earth, or a general distance D

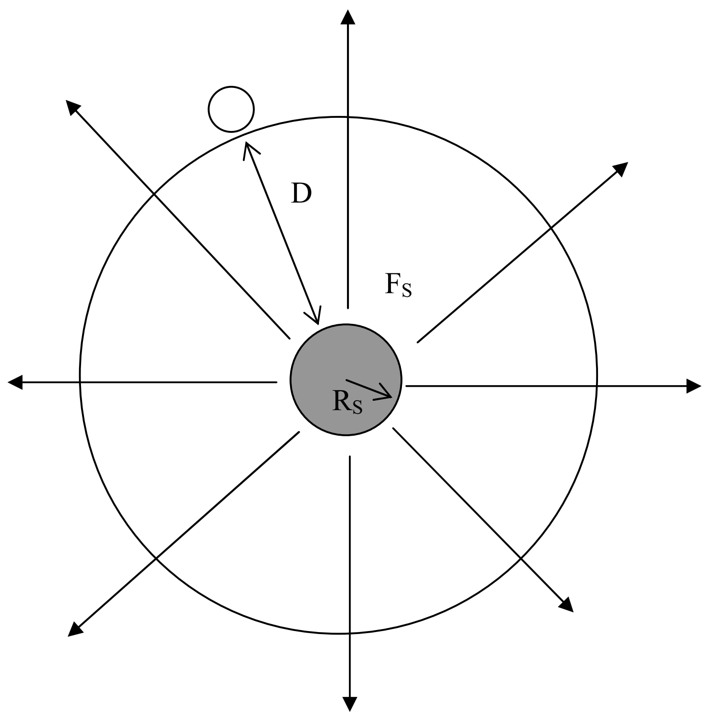

Figure 6.2 Calculating flux at a distance D from the sun.

For steady radiation flow:

$$L = 4\pi R^2 \sigma T^4 = 4\pi D^2 F(D) \qquad [6.4]$$

since the Stefan-Boltzmann law says $F = \sigma T^4$ and we're interested in a distance D from the sun. Therefore,

$$F(D) = \frac{L}{4\pi D^2} \qquad [6.5]$$

$$\Rightarrow F(D) \propto \frac{1}{D^2} \qquad [6.6]$$

Now, since the luminosity of the sun, $L_S = 4\pi R_S{}^2 F_S$, equation [6.5] can be expressed as:

$$F(D) = \frac{4\pi R_S{}^2 F_S}{4\pi D^2} = \left(\frac{R_S}{D}\right)^3 F_S \qquad [6.7]$$

$$\Rightarrow F(D) = \theta^2 F_S \qquad [6.8]$$

where $\theta = \dfrac{R_S}{D}$ is the angular size of the sun, in radians.

At the Earth, $\theta = 4.7 \times 10^{-3}$ radians, so

$$\boxed{\text{F at Earth} = 1.36 \times 10^3 \text{ Wm}^{-2}}$$

The Solar Constant

i.e. the sun is equivalent to a 1.36 kW heat at a distance of 1 m.

§6.3 Theoretical temperature of planets

And again, we'll calculate this via an example.

Example 6.3

Calculate the theoretical temperature of a planet.

Once the heat of a planet's formation is lost, the temperature of that planet, T_P, is fixed by the balance of the amount of solar radiation the planet receives against the amount that planet emits.

Consider a planet, or radius a, bathed in solar power, as illustrated in figure 6.3:

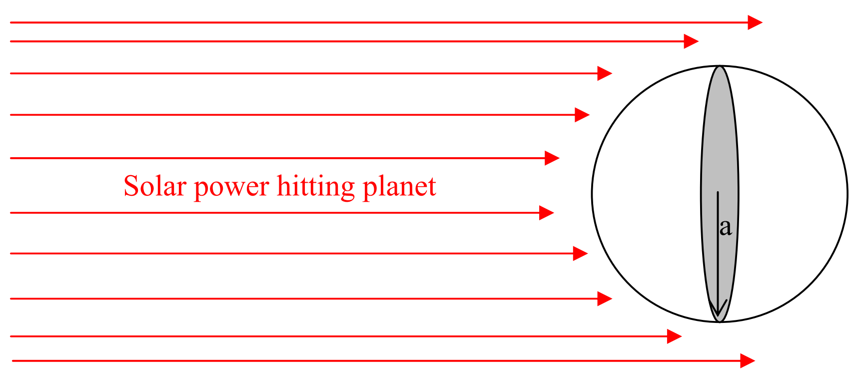

Figure 6.3 Solar power hitting a planet

The solar power, P, arriving at the planet at any time is the area times the power per unit area. i.e.

$$P = \pi a^2 \times F(D) \qquad [6.9]$$

The ratio of the amount of sunlight which a planet reflects (from its surface and from its atmosphere), to the amount of light incident upon it, is known as the planet's albedo, denoted by A .i.e.

$$\frac{\text{sunlight reflected}}{\text{sunlight received}} = A$$

The power absorbed by the planet, P_{in}, is therefore given by:

$$P_{in} = (1 - A)P = (1 - A)\pi a^2 F(D) \qquad [6.10]$$

The power the planet emits when at temperature T_P (i.e. its luminosity) is:

$$P_{out} = 4\pi a^2 \sigma T_P{}^4 \qquad [6.11]$$

The temperature of the planet will increase until there is a balance between the power in and out, i.e. until equilibrium is reached. At equilibrium $P_{in} = P_{out}$, i.e.

$$(1 - A)\pi a^2 F(D) = 4\pi a^2 \sigma T_P{}^4$$

$$\Rightarrow (1 - A)F(D) = 4\sigma T_P{}^4$$

$$T_P{}^4 = \frac{(1 - A)F(D)}{4\sigma} \qquad [6.12]$$

But, $F(D) = \theta^2 F_S = \theta^2 \sigma T_S{}^4$. Therefore,

$$T_P{}^4 = \frac{(1-A)\theta^2 \sigma T_S{}^4}{4\sigma} = \frac{\theta^2}{4}(1-A)T_S{}^4$$

$$\Rightarrow T_P = \left(\frac{\theta}{2}\right)^{1/2}(1-A)^{1/4} T_S \qquad [6.13]$$

This assumes that:

- T_P is uniform over the planet;
- the planet behaves like a perfect "black body";
- all latitudes received an equal amount of incoming solar radiation.

Table 6.1 gives the temperatures – both calculated and observed – for the planets in our Solar system. Also included are the albedoes.

	Albedo	Mean distance from Sun (AU)	Theoretical temperature (K)	Observed temperature (K)
Mercury	0.106	0.387	437	100 – 700 (ave = 440)
Venus	0.65	0.723	253	730
Earth	0.37	1	249	200 – 300 (ave = 288)
Mars	0.15	1.52	218	130 – 290 (ave = 218)
Jupiter	0.52	5.2	102	165
Saturn	0.47	9.54	77	134
Uranus	0.50	19.2	54	76
Neptune	0.50	30.1	43	74
Pluto	0.50	39.5	37	40

($1\ \text{AU} = 1.5\times10^{11}$ m, radius of Sun $= 6.97\times10^{8}$ m, effective temp of Sun = 5800 K)

Table 6.1 The temperatures (calculated and observed) of the planets

Note:

- The theoretical temperature, T_P, agrees reasonably well with observations of Mars, Jupiter, Saturn, Uranus, Nepture and Pluto.

- For Earth, $T > T_P$.

- For Venus, at the surface $T >> T_P$! This is because of …

§6.4 The Greenhouse Effect

Certain gases in the atmosphere are transparent to visible light but not to infra-red radiation.

(CO_2, H_2O and CH_4 – these absorb photons and re-radiate – they slow down the escape of IR).

The result of this is that the temperature at the surface becomes greater than that at the top of the dense clouds.

Earth has a modest greenhouse effect though it's increasing due to CO_2 emissions. The average temperature is greater than the calculated T_P by a few 10s of degrees.

Venus has a runaway greenhouse effect:

- Early water evaporated – H_2O in cloud form can be ionised, allowing H to escape.

- This reduced the available water to dissolve CO_2 from the atmosphere.

- The CO_2 therefore built up and trapped IR, which further increased the temperature.

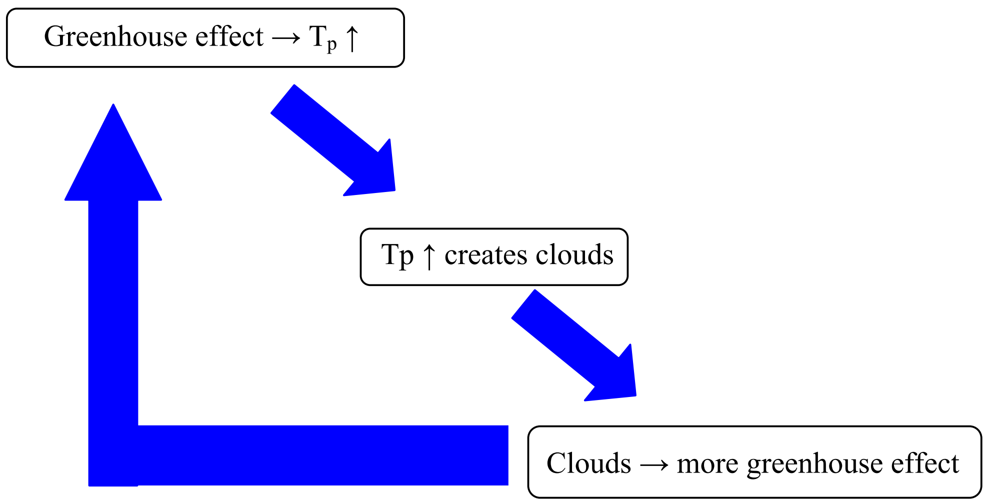

Figure 6.4 Runaway greenhouse effect.

§7. The Minor Bodies of the Solar System

There are three main types – asteroids, meteors and comets.

§7.1 Asteroids

The term asteroid literally means "star-like bodies", though the name is misleading as they are not, in anyway, stars. Rather they are small, rocky bodies. The vast majority are small, with diameters less than 100 m. A handful are larger – over 300 km across.

The largest asteroid is called Ceres and was detected in 1801 by the Italian astronomer Guiseppe Piazza. It is still not large in solar system terms, though - it measures 940 km across and has a mass of only 1/10000 that of the Earth. In fact, the total mass of every asteroid in the solar system is less than the mass of the Moon.

By the start of the 20th century. Several hundred asteroids had been discovered. A century later this number is now something like 200 000. (Although only ~75 000 have well understood orbits.)

§7.1.1 Orbital properties

The first asteroids discovered – Ceres, Pallas, Juno and Vesta all have orbital semi-major axes of ~2.8 AU. They reside in the asteroid belt – a band stretching from 2.1 to 3.3 AU, roughly halfway between Mars and Jupiter.

There are two possible explanations for the existence of the asteroid belt:

- The asteroids are the debris left over from the break up of a planet.

- They are primal rocks that never managed to accumulate into a genuine planet.

Current theories favour the latter option. This is for two reasons – firstly, as already mentioned, there isn't enough matter in the belt to make up a sizeable moon, never mind a planet. Secondly, there are marked chemical differences amongst the individual asteroids, which implies that they did not all originate from the same source. It is believed that the strong gravitational pull of Jupiter has prevented the asteroids from clumping together as they are constantly being shoved around.

§7.1.2 Physical properties

Not a lot is known about the make up of asteroids as they are too small to be clearly observed by Earth-based telescopes. What information we do have, though, suggests that they vary wildly from the other bodies in the solar system, as well as from each other.

Composition

There are three main classifications of asteroids: C-type, S-type and M-type

- *C-type:* these are the darkest – i.e. least reflective – of the asteroids and contain a lot of carbon.
- *S-type:* these are shinier thanks to being composed of silicates.
- *M-type:* contain a lot of nickel and iron.

C-type asteroids make up ~75 % of the asteroid belt, S-type make up 15 %, with M-types and others making up the rest. S-type dominate the inner regions of the asteroid belt, with C-types becoming more common as you move outwards.

C-type asteroids are believed to be primal, dating back to the creation of the solar system 4.6 billion years ago.

Size

The sizes of asteroids are usually estimated by the amount of light they reflect or the heat they give off. (~1 000 asteroids have been sized this way.) A few, though, have been measured when they pass in front of stars. This method can produce very accurate measurements.

The largest asteroids are roughly spherical, but smaller ones are highly irregular. Of all those know, only ~25 exceed 250 km across.

Vesta is unique amongst the asteroids in that it appears to have undergone volcanism in its distant past. This suggests that this asteroid used to be part of a larger, planetary body.

§7.1.3 Asteroid observations from space

The first close up shot of an asteroid was by the Galileo probe as it headed for Jupiter. In 1991 it passed Gaspra, and then two years later it studied Ida. Both Gaspra and Ida are S-type asteroids. They are irregularly shaped with diameters of ~20 and ~60 km.

Both are covered in craters, though Ida has far more of them than Gaspra. This is explained because Ida is thought to be over 1 billion years older than Gaspra. (Which, itself, is thought to be 200 000 years old.) They are both thought to be fragments of larger bodies.

Galileo's investigation of Ida revealed that it has a moon. Named Dactyl, it is 1.5 km across and orbits at a distance of 90 km. The presence of Dactyl allowed the mass of Ida to be estimated from Newton's law of gravity at $5-10\times10^{16}$ kg. This, in turn, allowed its density to be calculated as 2200-2900 kgm^{-3}, in line with S-type classification.

In the late 1990s, the Near Earth Asteroid Rendezvous spacecraft was sent to observe Eros, an S-type asteroid. It spent a year in orbit about it, determining its mass to be 7×10^{15} kg and its density as 2700 kgm^{-3}. The craft then landed on the asteroid, taking images all the way down. It survived the landing and continued sending radio signals, though no more photographs, for 16 days before shutting down.

§7.1.4 Earth crossing asteroids

An Earth crossing asteroid is one whose orbital path crosses that of the Earth and could, therefore, result in it hitting our world. These probably originated in the asteroid belt but were knocked out of their original orbits by the action of Mars or Jupiter.

There are two types:

- Apollo asteroid – these have semimajor axes greater than 1AU.
- Aten asteroid – these have semimajor axes less than 1 AU.

There are also Amor asteroids, whose orbits only cross that of Mars.

There are more than 2500 known Earth-crossing asteroids, most of which were found in the early 1990s after Jupiter was hit by asteroids.

600 are deemed "potentially hazardous" because they are over 150 m in diameters and move in orbits that bring them within 0.05 AU of us. Most Earth-crossers are small though – about 1 km across – but even an impact with one of these would be bad. Such an asteroid would have the destructive force of 1 million megaton nuclear bombs and would devastate and area 100 km in diameter.

A major impact is thought to have extinguished the dinosaurs ~ 65 million years ago. What are the chances of it happening again? Here's what JPL's Near Earth Asteroid Tracking team has to say[2]:

> *"The most dangerous asteroids, capable of a global disaster, are extremely rare. The threshold size is believed to be 1/2 to 1 km. These bodies impact the Earth only once every 1,000 centuries on*

[2] Taken from http://imagine.gsfc.nasa.gov/docs/ask_astro/answers/danger.html

average. Comets in this size range are thought to impact even less frequently, perhaps once every 5,000 centuries or so."

And whilst we haven't had any major impacts recently, there have been a series of near misses …

- In 1968, asteroid Icarus missed earth by "only" 6 million km.
- In 1991, asteroid 1991 BA (no name) passed at 170000 km.
- In 1994, asteroid 1994 XMI missed by 105000 km

In total, over the last decade more than 800 asteroids have come within 15 million kms of Earth. (That we know of!)

§7.1.5 Orbital resonances

Additional to the asteroid belt, there is another group of asteroid known as the Trojans. Several hundred of these have been found – they orbit at the distance of Jupiter. They are locked in a 1:1 orbital resonance with Jupiter thanks to its gravitational affect. (i.e. they remain in the same position relative to Jupiter throughout their orbit.)

In 1772 the French mathematician Joseph Lagrange demonstrated that there are exactly 5 places in the solar system where a small body can orbit the sun in synchrony with Jupiter, subject to the combined gravitational influence of both large bodies. These are the Lagrange points of the planet's orbit. See figure 7.1.

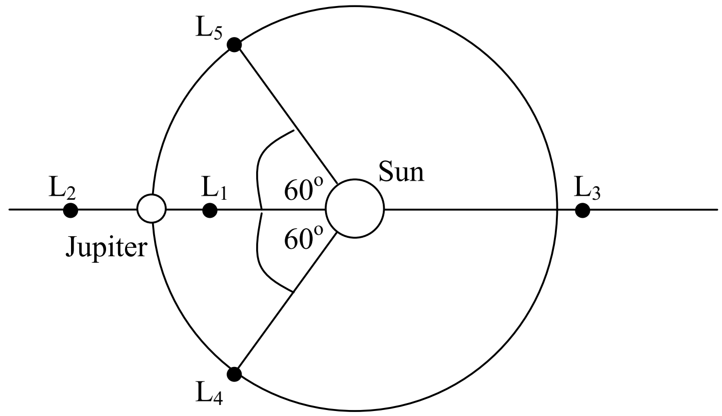

Figure 7.1: The Lagrange points of the orbit of Jupiter.

Three points are on the line joining Jupiter and the sun, or its extension in either direction. The other two are located on Jupiter's orbit, exactly 60 degrees ahead or behind the planet. All 5 points revolve around the sun at the same rate as Jupiter.

In theory, any asteroid placed at one of the points will retain the same relative position about Jupiter at all times. The three points along the linking line, however, are known as unstable Lagrange points – if an object is disturbed from those points it will continue to drift away. And since there is so much clutter in the solar system, no asteroids are found there. The two points on Jupiter's orbital line, though, are stable, therefore asteroids tend to accumulate there. For an unknown reason, the Trojans tend to be near the leading Lagrange point, rather than the trailing one. Similar, small asteroids have been found in the Lagrange points of Venus, Earth and Mars.

The main asteroid belt also has structure. Figure 7.2 shows a graph of the number of asteroids as a function of orbital semimajor axes. It illustrates that there are several prominently under-populated regions – known as Kirkwood gaps. (Named after Daniel Kirkwood who discovered them in the 19th century.)

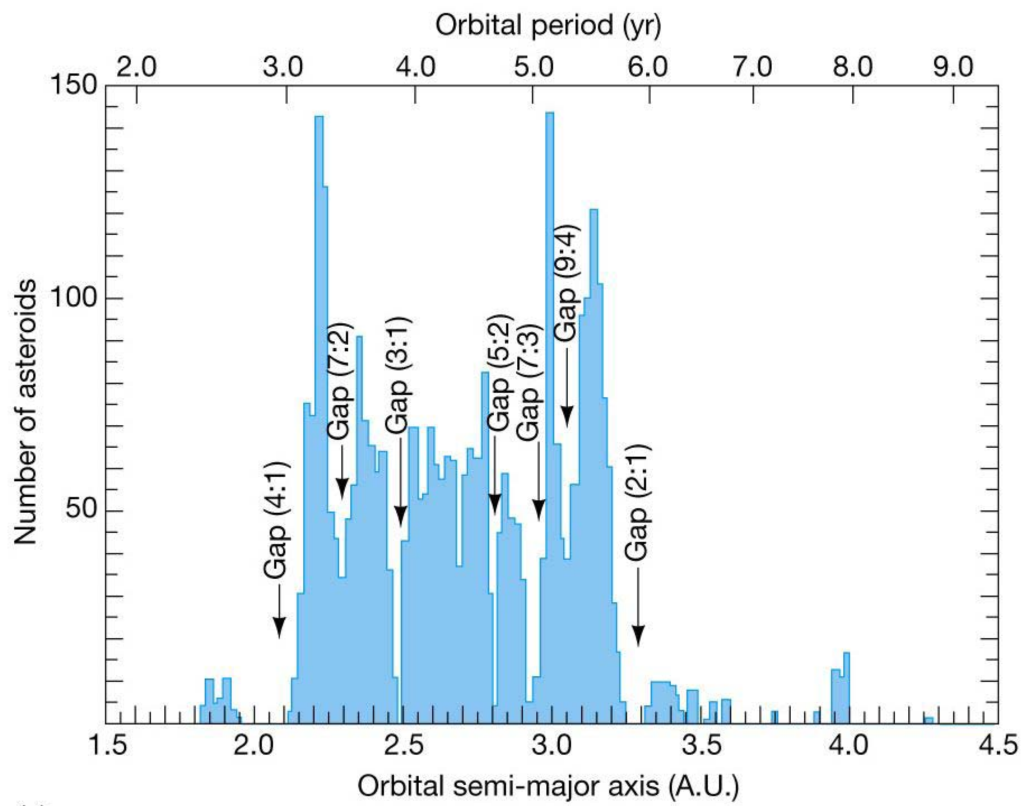

Figure 7.2: The Kirkwood gaps in the asteroid belt.

Trojans share an orbit with Jupiter orbiting in 1:1 resonance with the planet. Kirkwood gaps result from more complex orbital resonances with Jupiter. E.g. an asteroid with semimajor axis of 3.3 AU would, according to Kepler's 3rd law) orbit the sun in exactly half the time taken by Jupiter. The gap at 3.3 AU then corresponds to a 2:1 resonance. An asteroid at that particular resonance receives a regular periodic tug from Jupiter at the same point in every orbit. The cumulative effect of all the tugs is to deflect the asteroid into an elongated orbit, one that crosses the orbit of Mars or Earth. Eventually the asteroid hits one of those planets or gets close enough to be pushed off on a whole new trajectory. In this way Jupiter's gravity creates the gaps and some of the cleared asteroids become Apollos or Amors.

§7.2 Meteors (-oids and -ites)

The following table provides some definitions:

Meteoroid	The particle itself
Meteor	Visible phenomenon
Meteorite	Meteoroid (large) and not burnt up before reaching the Earth
Micrometeorite	Meteoroid (small) dust decelerated before it burns up – falls slowly to the Earth.

Table 7.1 Meteor related definitions

- Meteorites are – as mentioned above – the physical part of the meteor, the bit that you would find on Earth if it makes planetfall. They're thought to be debris from asteroids or comets that has been thrown into the path of the Earth's orbit.

- Meteors are the light displays produced when the meteor enters our atmosphere.

Where do they come from?

- ~20/hour from any direction in the sky – small pieces of dust vaporised in a brief flash.

- Follow the orbit of a comet – meteor showers. Get anything from 50 – 100000 per hour.

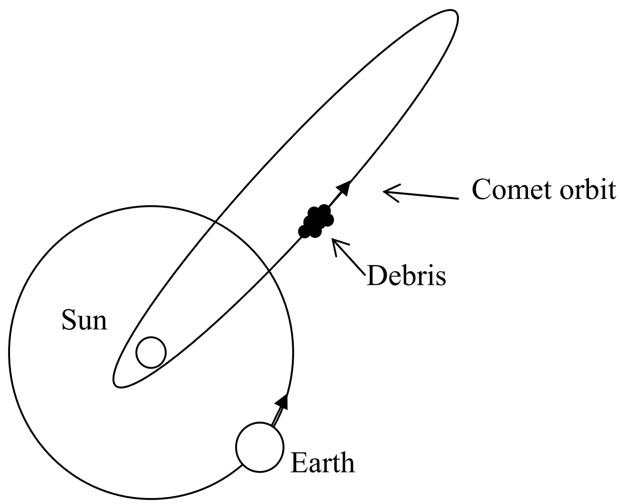

Figure 7.3 Meteors following a comet.

More meteors are visible after midnight than before it because we are on the side of the Earth facing forward along the direction of the Earth's motion around the Sun ...

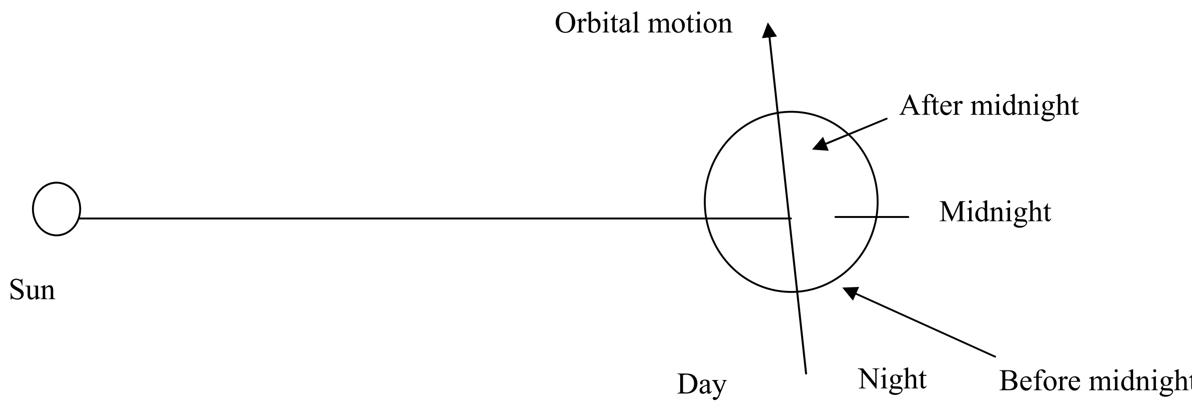

Figure 7.4 Times for seeing meteors.

§7.2.1 Classification of meteorites by composition

Types:

- *Stony* ~ 93 % – "primitive" – remnants from the birth of the solar system; ages $\sim 4.6 \times 10^9$ years (established from radioactive dating).

- *Iron* ~ 6 % – probably "processed" – have been part of some asteroids that had enough mass to undergo differentiation. A few may come from Mars or the Moon as ejecta – these are younger.

- *Mix* ~ 1 %

§7.2.2 Classification of meteorites by parent body

(i) Carbonaceous chondrites (aka primitive meteorites):

- These are rich in organic compounds and contain chondrules – spherical objects made predominantly of SiO_2, M_gO and FeO, that have cooled rapidly from a molten state. Some also contain water. These are always stony meteorites.

(ii) Chemically differentiated meteorites:

- These have no inclusions – i.e. no chondrules – and were formed entirely out of molten rock. All the iron and mixed meteorites fall into this category.

Since these tend to be igneous in nature they may have originated in the crusts of planetary bodies.

§7.2.3 Meteor physics

The kinetic energy and momentum of a meteor are reduced by air friction. This creates a hot meteorite which loses mass and a hot ionised trail in the atmosphere. We see this trail glowing but can also bounce radar off of it (daytime radar meteors).

- Very small meteoroids – slowed down before they vaporise because radiation cools them. (They have a large surface to mass ratio ~ $d^2/d^3 = 1/d$.) = MICROMETEORITES.

- Large meteoroids (d ~ 10 cm to 1 m) – these reach the ground while still travelling fast with only some loss = METEORITES.

- Medium meteoroids (d ~ mm to 10 cm) – vaporise in the air

§7.2.4 Source of meteor luminosity

Meteors have a huge kinetic energy per kg. A typical velocity for a meteor is $30\,\text{kms}^{-1}$, hence

$$K.E. = \frac{1}{2}mv^2$$

$$\Rightarrow \frac{K.E.}{m} = \frac{v^2}{2} = \frac{9\times10^8}{2} = 4.5\times10^8 \text{ Jkg}^{-1}$$

Compare this with the explosive energy of TNT:

$$\sim 10^7 \text{ Jkg}^{-1} \sim \frac{1}{50} \text{ the above kinetic energy.}$$

Consider a cubical meteor of side d and a typical density of $4000\,\text{kgm}^{-3}$:

$$\text{Mass} = \rho \times \text{volume} \sim 4\times10^3 \times d^3$$

The product of the mass of the meteoroid and its kinetic energy per kg, divided by the energy per kg of TNT gives you the mass of TNT. Therefore the mass of the TNT is equivalent to the energy released.

$$m_{\text{TNT}} \sim 2\times10^5 \times d^3$$

d	m_{TNT}
10 mm	0.2 kg
100 mm	200 kg
1 m	200 tonnes
10 m	0.2 Megatonnes
100 m	200 Megatonnes

Table 7.2 Meteoroid/TNT equivalence

Fortunately, high mass (i.e. large d) meteoroids are very rare.

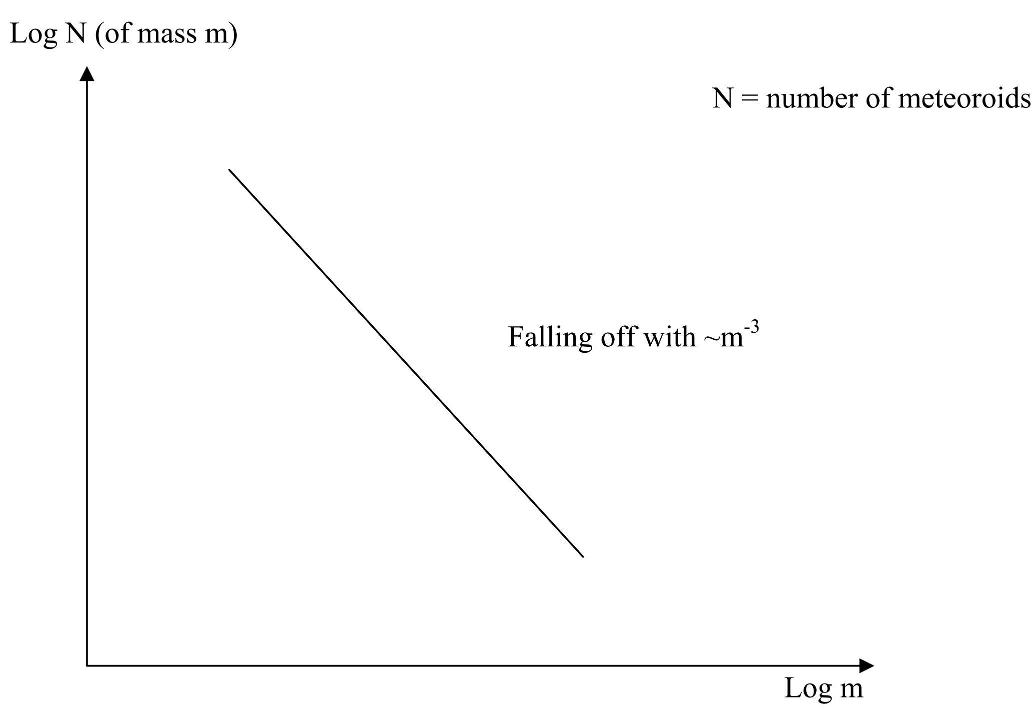

Figure 7.4 Numbers of meteoroids with respect to their mass.

§7.2.5 Meteoroid deceleration

Example 7.1

Calculate the height at which an meteoroid, descending through the atmosphere of the Earth, stops due to the effect of air resistance.

As the meteoroid descends through the atmosphere of the Earth it encounters a varying air density, $\rho_{\text{air}}(z)$, where z is the height above the ground. This air density can be expressed as follows:

$$\rho_{\text{air}}(z) = \rho_0 \exp\left\{-\frac{z}{H}\right\} \qquad [7.1]$$

where $H = \text{scale height} \sim 8\,\text{km}$ and ρ_0 is the density at ground level.

When the meteoroid reaches a height z, the air mass it has encountered per unit area is:

$$m_{\text{air}} = \int_{z}^{\infty} \rho_{\text{air}}\, dz$$

$$\Rightarrow m_{\text{air}} = \int_{z}^{\infty} \rho_0 \exp\left\{-\frac{z}{H}\right\}$$

$$\Rightarrow m_{\text{air}} = \rho_0 H \exp\left\{-\frac{z}{H}\right\} \qquad [7.2]$$

The air mass encountered by a meteoroid of dimension $d = \frac{\text{mass}}{\text{unit area}} \times \text{area}$ is therefore,

$$m_{\text{air}} \sim \rho_0 H \exp\left\{-\frac{z}{H}\right\} d^2 \qquad [7.3]$$

The meteoroid "stops" when it has encountered its own mass. If the mass of the meteoroid is $m = \rho_m d^3$ then it stops when:

$$\rho_m d^3 = \rho_0 H \exp\left\{-\frac{z}{H}\right\} d^2$$

$$\Rightarrow \rho_m d = \rho_0 H \exp\left\{-\frac{z}{H}\right\}$$

$$\Rightarrow z = H \ln\left[\frac{\rho_0}{\rho_m}\frac{H}{d}\right] \qquad [7.4]$$

This is the stopping height, though the meteoroid may vaporise earlier.

Numerically,

$$z = H \ln\left[\frac{\rho_0}{\rho_m}\frac{H}{d}\right] = H \ln\left[\frac{1}{4\times 10^3}\frac{8\times 10^3}{d}\right]$$

$$\Rightarrow z \sim H \ln\left[\frac{2}{d}\right] \qquad [7.5]$$

Note – for $d > 2$ m, $z < 0$. This implies that it reaches the ground.

Most of the deceleration occurs in the final Δz = one scale height, where ρ is highest. The deceleration time, therefore, is: $\Delta t = \dfrac{H}{v}$.

Example 7.2

How much power is dissipated during the deceleration?

The power dissipated in that time is:

$$P_{diss} = \frac{\text{energy}}{\text{time}} = \frac{\frac{1}{2}mv^2}{\Delta t} \sim \frac{\frac{1}{2}\rho_m d^3 v^3}{H}$$

$$\Rightarrow P_{diss} \sim 10^{13} \times d^3 \text{ Watts}$$

(again assuming $v \sim 30\,\text{kms}^{-1}$.)

So, for $d = 10\,\text{mm}, P \sim 10^7\ \text{W}$.

If we see this from a distance D of 30 km and 10 % of the power dissipated goes into light, then the radiation flux seen is:

$$F = \frac{0.1P_{diss}}{4\pi D^2} \sim 10^{-4}\ \text{Wm}^{-2}$$

This is $\sim 10^{-7}$ of F_S and ~ 0.1 of F_moon.

§7.3 Comets

- Basically dusty ice balls < 10 km across (nucleus) – "dirty snow balls".

- Melt and vaporise near the sun (in eccentric orbit) producing a coma and tail (or tails) which can be huge. (The coma is a cloud of gas and dust released when the nucleus gets close to the sun. Illuminating this with sunlight produces the tail.)

- Comets are visible due to reflected sunlight and luminescence – they become much more luminous as they change size.

- Gas in the coma can absorb UV and fluoresce.

- Since it melts on approach to the sun and releases dust, meteor showers are sometimes associated with comets.

- All of these have eccentric orbits. The majority are highly eccentric, in fact, with perihelion[3] of ~1 AU and aphelion[4] of $\sim 10^3 - 10^5$ AU.

- They can have periods of $10^4 - 10^7$ years. Though you also get some short period comets, e.g. Halley's, which has a period of 76 years.

[3] Perihelion = point where orbiting body is closest to the Sun
[4] Aphelion = point where orbiting body is furthest from the Sun

§7.3.1. Origins

Most comets are found in the Oort cloud and the Kuiper belt.

Oort cloud comets:

- Believed to be in the spherical Oort cloud of frozen debris left over from the formation of the solar system. Found $\sim 10^4 - 10^5$ AU from the Sun.

- Oort cloud objects can be perturbed from orbit by passing stars (they're only loosely bound to the Sun) – they fall in near the Sun and make a comet (these typically have a long period and can come from any direction/angle).

<u>The Oort cloud:</u>

Postulated in 1950 by Jan Oort, based on very careful study of the orbits of long-period comets. It's never been observed, but its existence seems certain. It's thought to contain $10^{12} - 10^{13}$ objects with a total mass of the order of ~ 100 the mass of the Earth.

Kuiper belt comets:

- The Kuiper belt objects have orbits ~ 30 – 100 AU.

- Short period comets result when these objects are perturbed by gravitational effects of the outer planets (or when the orbit of a long period comet is perturbed by planetary influences).

§7.3.2 Comet tails

- Nucleus of ice and dust/rock starts to evaporate on approach to the Sun

- Coma of gas and dust forms, surrounding the nucleus

- Sun ionises the gas released from the comet. Charged particles are swept straight out by solar wind – tail points straight away from the Sun.

- Dust particles feel solar radiation pressure – weaker effect – get curved dust tail.

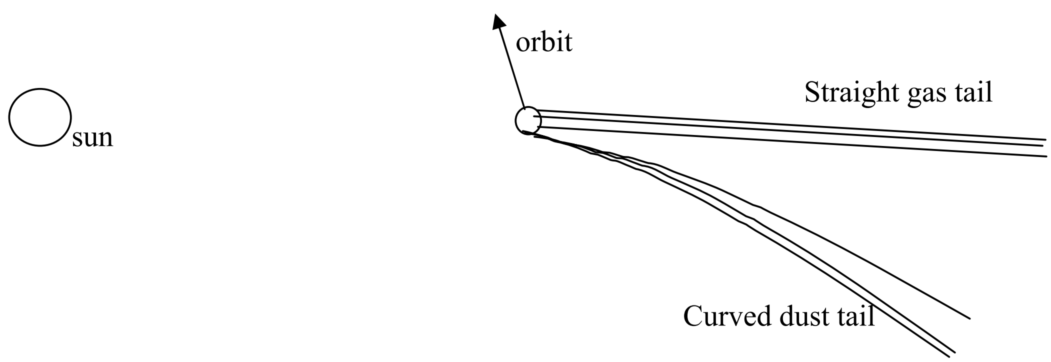

Figure 7.6 The tail of a comet.

§8 The Sun

§8.1 General statistics

§8.1.1 Size

The Sun is huge – it dwarfs the size of the planets in both radius and mass.

Radius, $R_S \sim 7\times10^5$ km

Mass, $M_S \sim 2\times10^{30}$ kg

The mass of the Sun can be estimated using Kepler's 3rd Law.

Johannes Kepler (1571 – 1630) was a German mathematician who worked at the Uraniborg observatory on an island near Denmark. He was working to prove the theory put forward by Copernicus that the Sun was at the centre of our solar system, and not the Earth. Analysing data from observations made of stars as they wandered about the sky, he came up with three laws:

1. (1609) A planet's orbit about its parent sun is elliptical, with the sun as one focus of that ellipse.
2. (1609) A line connecting a planet to the Sun sweeps out equal areas in equal time intervals. i.e. a planet's speed is not constant as it orbits its parent.
3. (1619) There is a direct mathematical relationship between the period of a planet's orbit, P and the semimajor axis of the elliptical orbit, a:

$$P^2 \propto a^3 \qquad [8.1]$$

Much later, Newton would adapt this relationship to create a more precise formula:

$$P^2 = \frac{4\pi^2}{G(M_S + m)} a^3 \qquad [8.2]$$

where m is the mass of the orbiting planet and G is the gravitational constant of the universe. ($G = 6.673 \times 10^{-11}\ \mathrm{Nm^2kg^{-2}}$)

Now, since $M_S >> m$, [8.2] can be simplified and rearranged to allow a value for M_S to be calculated.

$$P^2 = \frac{4\pi^2}{G(M_S + m)} a^3 \approx \frac{4\pi^2}{GM_S} a^3$$

$$\Rightarrow M_S \approx \frac{4\pi^2 a^3}{GP^2} \qquad [8.3]$$

Newton's version of Kepler's 3rd law is exceedingly useful, as it can be applied to any two bodies – e.g. a moon orbiting a planet, a galaxy orbiting another galaxy and so on.

Example 8.1

The orbital period of Io, one of the moons of Jupiter, is 1.77 days. The semi-major axis of its orbit is $4.22\times10^{8}\ \mathrm{m}$. Using Kepler's third law, estimate the mass of Jupiter. You can assume that the mass of Io is negligible in comparison.

First convert the orbital period into SI units:

$$1.77\ \mathrm{days} = 1.77\times24\times60\times60\ \mathrm{s} = 1.53\times10^{5}\ \mathrm{s}$$

Now use equation [8.3]:

$$M_{Jupiter} \approx \frac{4\pi^{2}}{G}\frac{a^{3}}{P^{2}} = \frac{4\pi^{2}}{G}\frac{\left(4.22\times10^{8}\right)^{3}}{\left(1.53\times10^{5}\right)^{2}}$$

$$\Rightarrow M_{Jupiter} \approx \frac{4\pi^{2}}{6.67\times10^{-11}}\times3.21\times10^{15}$$

$$\Rightarrow M_{Jupiter} \approx 5.92\times10^{11}\times3.21\times10^{15} = 1.90\times10^{27}\ \mathrm{kg}$$

§8.1.2 Temperature

The different regions of the sun vary in temperature. The apparent, optical surface, though, known as the Photosphere has a temperature of $\sim 5800\,\mathrm{K}$. This is determined by studying the Sun's emitted radiation spectrum. We'll deal with this in detail later.

§8.1.3 Luminosity

The luminosity of a body is the energy emitted per second by that body. In other words it is a power, and is therefore measured in Watts. (We'll deal with luminosity In more detail later.) The luminosity, L, is related to the temperature of the body, T:

$$L \propto T^4 \qquad [8.4]$$

The luminosity of the Sun is $4 \times 10^{26}\ \mathrm{W}$. Which is a lot of lightbulbs.

§8.1.4 Period of rotation

The Sun revolved once every ~25 days. This is very slow in comparison to some stars and is an important clue to the origin of the solar system.

§8.2 The outer layers of the Sun

§8.2.1 The photosphere

The Sun doesn't have a real, physical surface. When the internal energy of the Sun reaches the outer region it is released again as radiation. This area of release is called the Photosphere. It is this "sphere of light" that we see on a sunny day – this is the "surface" that we see in visible radiation. It has a temperature of $\sim 5800\,\mathrm{K}$. Solar telescopes reveal that this surface has a churning, granulated structure with a few dark spots called sunspots which also change in size and shape. The churning, granulated structure is our view of the top of the solar convection - much like looking down on a pan of boiling water. See §1.3.1 for more.

§8.2.2 The corona

At a total solar eclipse the photosphere is hidden by the moon. We can then see a very faint solar corona, which extends out to a distance of several times the radii of the Sun. The corona can also be seen, directly, using x-ray telescopes since it is very hot – $T_{\text{corona}} \sim 2\times10^{6}\ \text{K}$.

§8.2.3 The chromosphere

This is a very thin layer that separates the corona and photosphere. It is more visually transparent than the photosphere. At the eclipse this is seen as a pink/red ring around the moon. This is due to the presence of Hydrogen which is formed in the chromosphere. The temperature of the chromosphere is somewhere in the region of $6000-10000\ \text{K}$.

The temperatures of the various exterior regions of the Sun are summarised in figure 1.1 below.

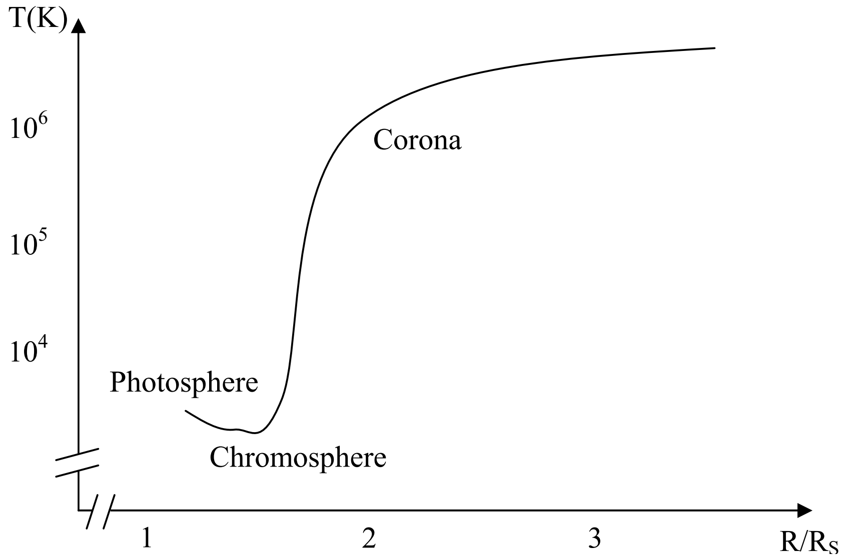

Figure 8.1 Temperature of the outer layers of the Sun

The relative sizes of these regions are summarised in figure 8.2.

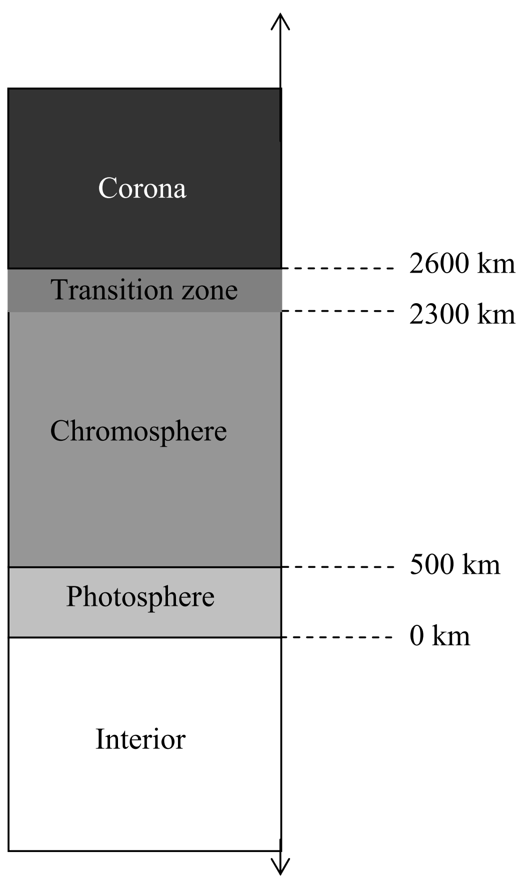

Figure 8.2 The thicknesses of the components of the Sun's atmosphere

§8.3 The coronal heating problem

Consider figure 1.1 – why is $T_{\text{corona}} >> T_{\text{photosphere}}$?

The second law of thermodynamics says that you "cannot transfer heat from a cool body to a hot body without doing mechanical work". So we need a "mechanism" to pump energy from the $5800\,\text{K}$ photosphere to the $2000000\,\text{K}$ corona.

What are the possibilities?

§8.3.1 Shockwaves developed from sound waves driven by interior convection cells.

When the base of the photosphere is observed it appears as a patchwork of bright and dark regions that are constantly changing, with individual regions appearing and then disappearing. With a spatial extent of roughly 700 km, the characteristic lifetime for one of these regions is 5 – 10 minutes. This patchwork structure is known as granulation and is the top of the convection zone protruding into the base of the photosphere. This zone is made up of a series of individual convection cells.

The motion of particles in the convection cells generates sound waves, which in turn result in shock waves analogous to the sonic boom when an object exceeds the speed of sound. These shock waves excite the atmosphere they enter into, causing them to heat up. However, the energy for heating comes from the mechanical energy of the shockwave, therefore the waves soon dissipate. There is not enough energy available for the shockwave to account for the extra heat so far out from the photosphere.

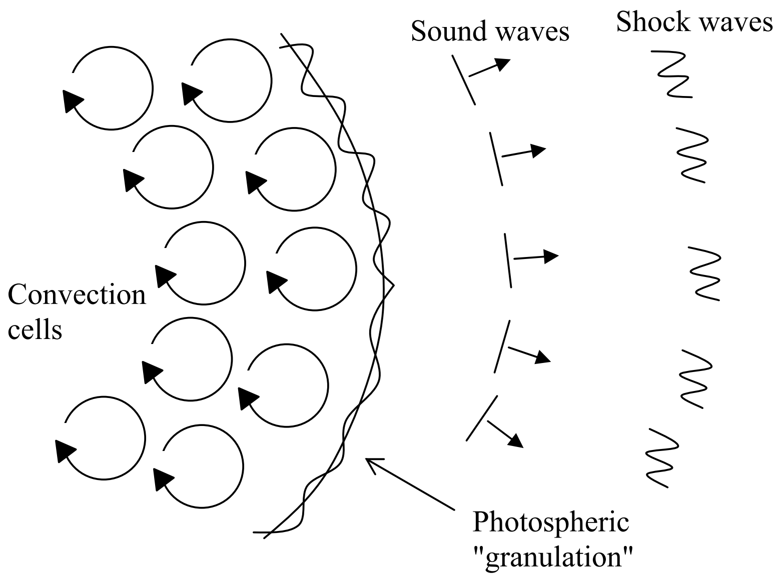

Figure 8.3 Development of shock waves.

§8.3.2 Currents and magnetic energy

When looked at in detail, the corona is full of fine structure with many magnetic loops.

The sun is composed of plasma – i.e. hot, ionised gas. Ions are charged particles, and when a charged particle moves, it creates a changing magnetic field. And a changing magnetic field induces the flow of currents. This current flow is thought to be the most likely source of the heating "mechanism" that allows $T_{\text{corona}} >> T_{\text{photosphere}}$.

§8.4 The solar wind

The solar wind was first noticed by its effect on the tails of comets. It can now be measured directly in space. It comprises a fast flow of plasma (hot, ionised gas particles), emitted from the Sun with a speed $\sim 300 - 600\,\text{kms}^{-1}$. This escape of particles is removing mass from the sun at the following rate:

$$\frac{dM_{\text{wind}}}{dt} \sim 10^{-14}\, M_S \text{ per year}$$

Example 8.2

How much mass does the Sun lose due to the effects of solar wind during this lecture?

How does this compare to the mass loss rate by radiation?

Calculate mass loss per second:

$$\frac{dM_{\text{wind}}}{dt} \sim \frac{10^{-14} \times 2 \times 10^{30}}{365 \times 24 \times 60 \times 60} = 6 \times 10^{8} \text{ kgs}^{-1}$$

Length of lecture = 50 mins = 3000 seconds

Therefore, mass loss in 1 lecture = $3000 \times 6 \times 10^{8} = 1.8 \times 10^{12}$ kg .

(Equivalent to, roughly, 24 billion students.)

Mass loss per second by radiation is:

$$\frac{dM_{\text{rad}}}{\text{dt}} = \frac{1}{c^2}\frac{dE}{dt} \sim \frac{L}{c^2} = \frac{4 \times 10^{26}}{9 \times 10^{16}} \sim 4 \times 10^{9} \text{ kgs}^{-1}$$

Total mass = 1.2×10^{13} kg .

§8.4.1 Why does the wind exist?

Consider how wind occurs on Earth? It is the result of pressure differences in the atmosphere – wind blows from areas of high pressure to areas of low pressure as the atmosphere tries to re-establish equilibrium. It is this imbalance in pressure which results in the solar wind.

We have already seen that the corona is at a very high temperature. This high temperature implies that the pressure of the corona, P_{corona}, is also very high. In fact, $P_{\text{corona}} \gg$ pressure of gas in interstellar regions. Therefore, the corona expands outwards as the solar wind. (The same process results in a hot planet losing its atmosphere.)

§8.5 The Solar Interior

We have already seen, in §1.3.1, that the outermost section of the Sun's interior consists of a region of convection cells. This region extends to a depth of $\sim 0.3R_S$. Beneath this is a region of radiation which in turn surrounds a central core, $\sim 0.2R_S$ in radius. Figure 1.4 summarises this.

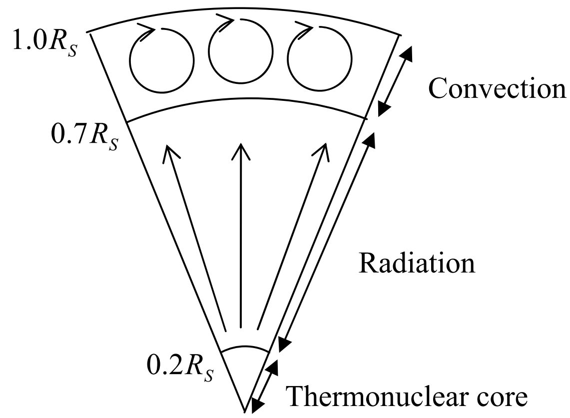

Figure 8.4 The solar interior

Unsurprisingly, the interior regions of the Sun are very hot. We shall now derive an expression that allows this temperature to be estimated. Like all good astronomy, it requires some approximations to be made to keep the maths within the realms of Mortal Man.

We can consider the Sun to be a sphere of gas in hydrostatic equilibrium – i.e. the interior pressure force balances the weight of the overlying material.

Consider a slice through the centre of the Sun:

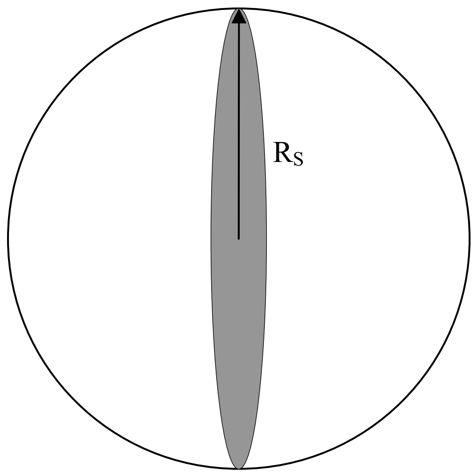

Figure 8.5 A slice through the Sun

If the force from the weight of the overlying material on this slice is F_W and the pressure force from the interior gas is F_P, then for equilibrium, $F_W = F_P$. So, if

$$F_W = gM_S \sim \frac{G{M_S}^2}{{R_S}^2} \qquad [8.5]$$

and

$$F_P = P \times \pi {R_S}^2 \qquad [8.6]$$

then the equilibrium condition requires that

$$\frac{G{M_S}^2}{{R_S}^2} = P\pi {R_S}^2$$

$$\Rightarrow P = \frac{GM_S^{\,2}}{\pi R_S^{\,4}} \qquad [8.7]$$

Now, for an ideal gas of hydrogen, $P = 2nk_BT$ where:

n = number of particles per unit volume $\sim \frac{\rho}{m_p}$; ρ = density and m_p = proton mass.

Substituting this into equation [8.4], we get:

$$2nk_BT = \frac{GM_S^{\,2}}{\pi R_S^{\,4}} \Rightarrow 2\frac{\rho}{m_p}k_BT = \frac{GM_S^{\,2}}{\pi R_S^{\,4}}$$

$$\Rightarrow T = \frac{GM_S^{\,2} m_p}{2\pi R_S^{\,4} k_B \rho} \qquad [8.8]$$

But ...

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{M_S}{\frac{4}{3}\pi R_S^{\,3}}$$

so

$$T \sim \frac{2}{3}\frac{Gm_p M_S}{k_B R_S} \qquad [8.9]$$

$G = 6.673\times10^{-11}$ Nm^{2}kg^{-2}; $m_p = 1.673\times10^{-27}$ kg; $M_S \sim 2\times10^{30}$ kg;

$k_B = 1.381\times10^{-23}$ JK^{-1}; $R_S \sim 7\times10^{8}$ m.

$$\Rightarrow T \sim \frac{2}{3}\times\frac{6.673\times10^{-11}\times1.673\times10^{-27}\times2\times10^{30}}{1.381\times10^{-23}\times7\times10^{8}}$$

$$\Rightarrow T \sim \frac{2}{3}\times\frac{22.328\times10^{-8}}{9.667\times10^{-15}}$$

$$\Rightarrow T_{\text{centre}} \sim 1.5\times10^{7}\text{ K}$$

This is so high that, together with high density, the solar core is undergoing continuous nuclear fusion:

$$4 \times {}^{1}_{1}H \rightarrow {}^{4}_{2}He$$

The core's composition is:

~ 73 % H

~ 25 % He

+ traces of other elements.

The energy released from the fusion reactions supports the sun against gravity and supplies its luminosity, L_S.

There is sufficient solar mass to supply L_S for $> 10^{10}$ years.

Note:

Equation [1.9] can be rearranged into the following form:

$$\frac{3}{2} k_B T \sim \frac{G m_p M_S}{R_S} \qquad [8.9^*]$$

i.e. In this form, we see that the thermal energy per proton is equal to the gravitational energy per proton.

§8.5.1 Interior puzzle

The density of the sun is greater than the density of water, and the density of the core is a lot greater than the density of water. So how can the sun be treated as a gas?

The answer is that because the temperature of the sun is so high, the Hydrogen is ionised into protons and electrons, whose sizes are a lot less than that of the hydrogen atoms. The particles can, therefore, move freely.

§8.6 Summary of the structure of the sun

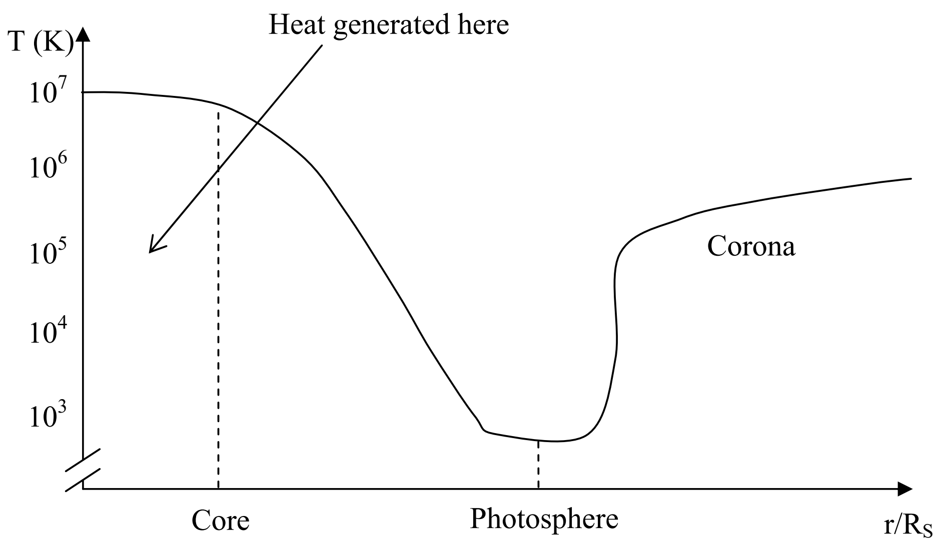

Figure 8.6 Temperature throughout the Sun

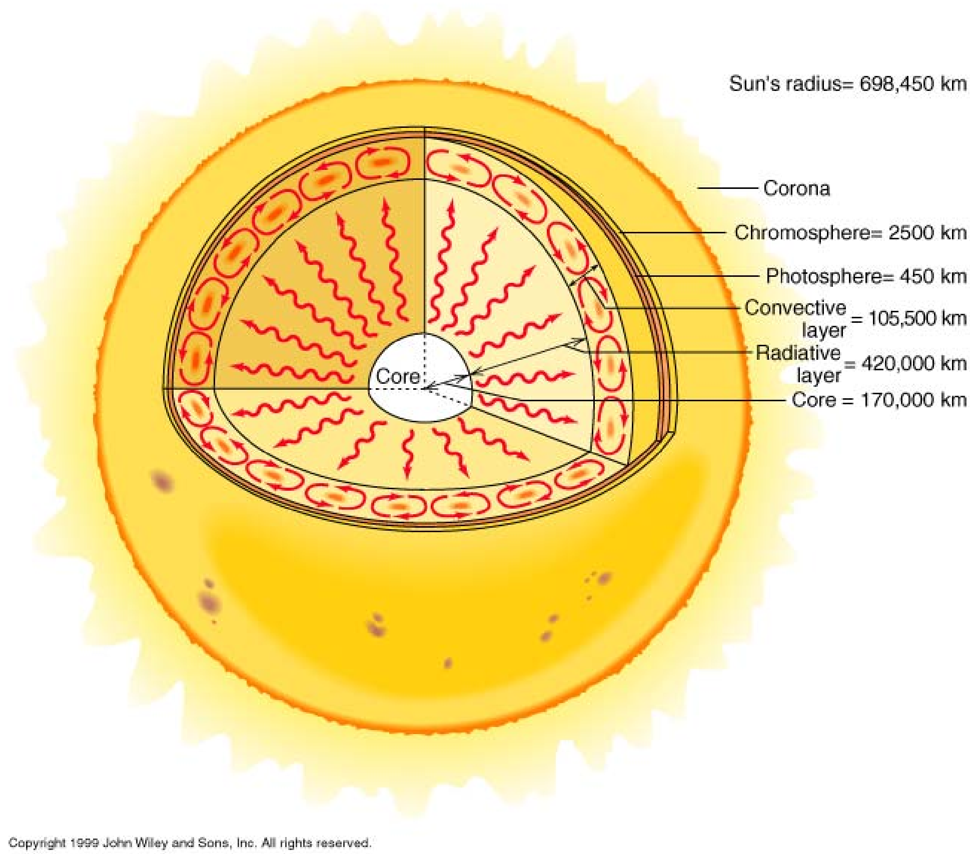

Figure 8.7 Overall structure of the sun.

§8.7 Solar activity

The Sun is magnetically active. This activity has a terrestrial impact.

§8.7.1 Main features of solar activity – Sunspots

Sunspots are dark spots in the photosphere.

Solar gas consists of charged particles, whose movement results in electric currents and magnetic fields. These fields in turn control the direction of movement of the particles – they move along the magnetic field lines. (Recall that magnetic field lines are a means of denoting the direction of force a positive charge would feel if placed near another charged particle. The more densely packed the field lines, the stronger the field.)

Sunspots tend to occur in pairs where closely spaced field lines come out of and loop back into the surface.

Charges can move long (spiral around) lines, but not perpendicular to them. This reduces the flow of hot plasma into strong magnetic field regions, and hence reduces the temperature from $5800\,\mathrm{K}$ to $\sim 3500\,\mathrm{K}$. This temperature drop results in a reduction in the Sun's brightness by a factor of $\left(35/58\right)^4$, since $L \propto T^4$.)

The magnetic field loops often loop high above the photosphere, right out into the corona – they form "solar prominances" in which the field traps the gas.

§8.7.2 Sunspot cycle

The total number of sunspots varies over ~ 11 years with a switch in N-S polarity in alternate cycles (so really the cycle is ~22 years). In fact, the interval between maximum and minimum can vary from 15 to 17 years.

Associated changes are:

(i) Shape of the corona is affected by the magnetic field, B.

(ii) X-ray, UV and other emissions are enhanced when B is large.

(iii) Magnetic energy released in solar storms:

- Solar flares: (Fast – typically a few minutes)

  These occur in the vicinity of sunspots

  - thought to occur when the magnetic field lines get so twisted that they cannot be sustained and suddenly "snap", releasing huge amounts of energy.
  - emit hard X and γ rays and high energy ions (GeV) which are a hazard in space and at high altitude.

- Coronal mass ejections (CME) (Slow – typically a few hours)

  - Huge expanding magnetic "bubbles" in the corona.
  - Mass released during a CME $\sim 10^9$ tonnes – high temperature coronal gas is sent into space at speeds of hundreds of kms^{-1}.

If the material from the solar storms is directed towards the Earth, a stream of high energy particles reaches us a few days later. These can hit the Earth's magnetosphere, generating high voltages (and so damage power lines and telecommunications).

There are some videos of solar activity at sohowww.nascom.nasa.gov (from the Solar and Heliospheric Observatory).

§9. The Origin of the Solar System

Any theory of the origin of the solar system must be able to explain the existing structure.

§9.1 The main features of the existing structure

(1) (Almost) all of the planetary orbits are in the same plane.

(2) Most of the planets rotate in the same direction as they orbit. i.e. they share the same rotation axis and the angular momentum of the bodies and orbits are ~ parallel. The exceptions to this are Venus, Uranus and Pluto.

(3) Most of the solar system's angular momentum is in orbits (Jupiter, Saturn) and not in the rotation of the Sun.

(4) The age of the Solar system is $\sim 4.55 \times 10^9$ years – the oldest surfaces are debris (e.g. comets and meteors).

(5) The inner planets are small, rocky/metallic. The outer planets are large, gaseous.

(6) There is an asteroid belt exists between Mars and Jupiter.

(7) Comets are icy bodies originating in a disk – the Kuiper belt – and a spherical distribution – the Oort cloud.

§9.1.1 "Location" of angular momentum

You know from Physics that a body moving in a linear direction with velocity $\underline{v}$ and mass m has an associated momentum, $\underline{p}_{lin}$, given by:

$$\underline{p}_{lin} = m\underline{v} \qquad [9.1]$$

There is an equivalent term associated with the angular movement of the body: angular momentum, $\underline{p}_{ang}$.

$$\underline{p}_{ang} = \underline{r} \times \underline{p}_{lin} = \underline{r} \times m\underline{v} \qquad [9.2]$$

where $\underline{r}$ is the position vector of the body with respect to some origin.

Nb. In Physics 1X angular momentum is given the symbol $\underline{L}$. We use $\underline{p}_{ang}$ here to avoid confusion with luminosity.

The magnitude of this, $\left|\underline{p}_{ang}\right|$ is:

$$\left|\underline{p}_{ang}\right| = rmv \qquad [9.3]$$

Example 9.1

How does the (rotational) angular momentum of the Sun compare to the (orbital) angular momentum of Jupiter?

(Rotational) Angular momentum of the Sun

The angular velocity of the Sun is:

$$v_S = \omega R_S = \frac{2\pi}{\tau} R_S$$

$$\Rightarrow v_S = \frac{2\pi}{25 \times 24 \times 60 \times 60} \times 7 \times 10^8 \sim 2 \times 10^3 \text{ ms}^{-1}$$

where $\omega = 2\pi f$ is the angular frequency of rotation.

(Recall that it takes 25 days for the Sun to revolve once.)

Therefore,

$$\left| \underline{p}_{ang} \right|_S = R_S M_S v_S = 7 \times 10^8 \times 2 \times 10^{30}$$

$$\Rightarrow \left| \underline{p}_{ang} \right|_S \sim 3 \times 10^{42} \text{ kgm}^2\text{s}^{-1}$$

<u>(Orbital) Angular momentum of Jupiter</u>

Here, $m_J = 1.8 \times 10^{27}$ kg ; $v_J = 1.3 \times 10^4$ ms^{-1}; $r_J = 7.8 \times 10^{11}$ m (= 5.2 AU).

(Nb. r_J is the radius of Jupiter's orbit, not the planet itself. Here we take the origin to be the centre of the Sun.)

Therefore,

$$\left| \underline{p}_{ang} \right|_J = r_J m_J v_J = 7 \times 10^{11} \times 1.8 \times 10^{27} \times 1.3 \times 10^4$$

$$\Rightarrow \left| \underline{p}_{ang} \right|_J \sim 1.8 \times 10^{43} \text{ kgm}^2\text{s}^{-1}$$

i.e. $\left| \underline{p}_{ang} \right|_J = 6 \times \left| \underline{p}_{ang} \right|_S$, even though the mass of the Sun is 1000 times that of Jupiter.

§9.2 Scenario for the creation of the Solar System

Many theories have been postulated as to the origins of our solar system. In the 1770s, it was proposed that a giant comet collided with the Sun, causing the ejection of a disk of material that ultimately condensed to form the planets. Alternatively, tidal theories argued that a close encounter with a passing star ripped material from the Sun. It was also suggested that the Sun accreted planetary material from interstellar space. But the basis of today's models argues that Sun and planets and asteroids and comets etc all formed at the same time, from the same nebula. The general idea is as follows.

- The solar system formed from a pre-solar nebula – a rotating gas cloud shrinking under its own gravity.
- This heats up and then radiates due to gravitational energy loss.
- It spins up due to angular momentum conservation.
- Centrifugal force spins off the outer layers in a gas disk.
- The gas disk cools and fragments into gas clouds and dust. Clumps accrete by gravity into condensing "planetessimals".
- Angular momentum of the Sun is reduced by magnetic coupling – angular momentum is transferred to charged particles in early strong solar magnetic fields. Strong solar wind blew particles out of inner solar system.

- Planets form – debris forms asteroids, meteoroids, moons and comets.

§9.3 Testing the scenario

We first test the scenario by considering the Sun on its own. We will assume that as it was created, its mass remained constant, only its density changing as it condensed from a protosun.

During solar formation from protosun, angular momentum is conserved. i.e.

$$\left|\underline{p}_{ang}\right| = rmv = rm\omega r = r^2 m\omega = \text{constant}$$

So, the angular momentum at time = 0 is the same as the angular momentum at some time t, later. i.e.

$$r_0{}^2 m\omega_0 = [r(t)]^2 m\omega(t)$$

$$\Rightarrow r_0{}^2 \omega_0 = [r(t)]^2 \omega(t)$$

$$\Rightarrow \frac{\omega(t)}{\omega_0} = \left[\frac{r_0}{r(t)}\right]^2 \qquad [9.4]$$

assuming that the mass, m, is also constant. r_0 is the initial radius of the body we're considering and ω_0 is it's initial angular frequency.

Suppose the Sun started out as a gas cloud of interstellar size. i.e.

$$R \sim 100000 \text{ AU} \sim 10^7 \; R_S \text{ (today)}$$

but with the same mass it has now.

Also, suppose it was rotating with the galaxy with a period of

$$\tau_0 \sim 2.3 \times 10^8 \text{ years} \sim 3 \times 10^{15} \text{ s}$$

Now, today $R = R_S$ which means that the period today, τ_S, would be ... (recalling that $\omega_S = \dfrac{2\pi}{\tau_S}$)

$$\frac{\tau_S}{\tau_0} = \frac{\omega_0}{\omega_S} = \left(\frac{R_S}{R_0}\right)^2 = 10^{-14}$$

$$\Rightarrow \tau_S = 30 \text{ s}$$

i.e. the Sun, today, would be complete one rotation in 30 seconds, which is patently absurd. If it was doing this the Sun would tear itself apart by centrifugal force. Therefore the Sun must have been created out of something more massive than itself, with the surplus matter going elsewhere. This is explained mathematically by the existence of a centrifugal limit, though the maths is a bit beyond this course.