Department of Physics and Astronomy

Astronomy 1X

Session 2007-08

Solar System Physics I

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5 lectures, beginning Autumn 2007

UNIVERSITY of GLASGOW

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Course information and handouts:
access via A1X moodle site

http://moodle.gla.ac.uk/physics/moodle/

Astronomy A1X 2007-08
Solar System Physics I – Lecture Plan

2 lectures

Introductory Tour of the Solar System

- Qualitative description of the Sun, planets, moons and minor bodies, contrasting Jovian and terrestrial planets
- Some Solar System vital statistics
- Overview of Solar System formation

Gravitation and Solar System physics

- Newton's law of gravitation
- Surface gravity and escape speed
- Tidal forces

Links to A1X Dynamical Astronomy

Astronomy A1X 2007-08
Solar System Physics I – Lecture Plan

3 lectures

The physics of planetary atmospheres

- The ideal gas law and velocity of gases
- Hydrostatic equilibrium and atmospheric scale heights

The Jovian planets and their moons

- Internal and atmospheric structure and composition
- Ring systems and Roche stability
- Physical properties of the main satellites
- Case study: the Galilean moons

Section 1: A Tour of the Solar System

Some vital statistics:-

The Solar System consists of:-

- the Sun,
- its **8** planets,
- their moons,
- dwarf planets, asteroids and comets,
- the 'Solar wind'

- Astronomers have studied the motions of the Sun, Moon and planets for thousands of years (see A1X Positional Astronomy)

- Before the invention of the telescope, however, we knew almost nothing about their true nature.

The Sun: some vital statistics:

The Sun is a star: a ball of (mainly) hydrogen gas, 700,000 km in radius (about 100 Earth radii)

It generates heat and light through **nuclear fusion**:

Surface temperature = 5800K

Central temperature = ~15 million K

Balance (hydrostatic equilibrium) maintained between *pressure* and *gravity*

The Sun's outer atmosphere, or *corona*, is very hot (several million K) – heated by twisting of the Sun's magnetic field

Section 1: A Tour of the Solar System

The Planets: some vital statistics:-

Name	Diameter* (Earth=1)	Mass (Earth=1)	Mean distance from the Sun
Mercury	4880 km (0.383)	3.302×10^{23} kg (0.055)	5.79×10^{7} km (0.387 AU)
Venus	12104 km (0.949)	4.869×10^{24} kg (0.815)	1.082×10^{8} km (0.723 AU)
Earth	12756 km (1.000)	5.974×10^{24} kg (1.000)	1.496×10^{8} km (1.000 AU)
Mars	6794 km (0.533)	6.418×10^{23} kg (0.107)	2.279×10^{8} km (1.524 AU)
Jupiter	142984 km (11.209)	1.899×10^{27} kg (317.8)	7.783×10^{8} km (5.203 AU)
Saturn	120536 km (9.449)	5.685×10^{26} kg (95.16)	1.432×10^{9} km (9.572 AU)
Uranus	51118 km (4.007)	8.682×10^{25} kg (14.53)	2.871×10^{9} km (19.194 AU)
Neptune	49528 km (3.883)	1.024×10^{26} kg (17.15)	4.498×10^{9} km (30.066 AU)
Pluto	~2300 km (0.18)	1.3×10^{22} kg (0.0021)	5.915×10^{9} km (39.537 AU)

* Equatorial diameter

See also table 6.1 in Astronomy Today

Mean Earth – Sun distance = **Astronomical Unit**

$$\boxed{149{,}597{,}870 \text{ km}}$$

1 A.U. = 107 solar diameters

The orbits of the planets are ellipses and lie in, or close to, a plane – the ecliptic. (See A1X Dynamical Astronomy).

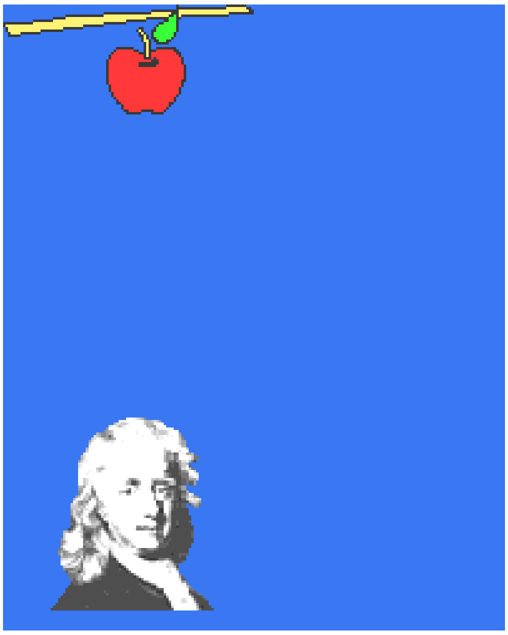
The planets divide into two groups:

Inner *Terrestrial* planets: small, rocky — **Mercury, Venus, Earth, Mars**

Outer *Jovian* planets: gas giants — **Jupiter, Saturn, Uranus, Neptune**

Pluto is a 'misfit' – Kuiper Belt object; along with asteroids and comets, 'debris' from formation of the Solar System.

Section 2: Newton's Law of Gravitation

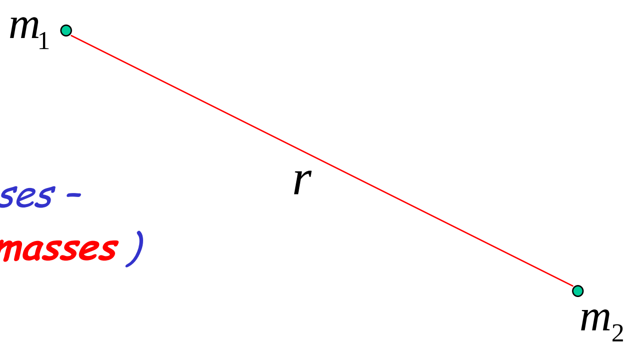


In 1686 Isaac Newton published his **Universal Law of Gravitation**.

This explained gravity as a force of attraction between all matter in the Universe, causing e.g. apples to fall from trees and the Moon to orbit the Earth.

(See also A1X Dynamical Astronomy)

Consider two masses m_1 and m_2, separated by distance r



(we ignore for the moment the physical extent of the two masses – i.e. we say that they are ***point masses*** *)*

Gravitational force on m_1 due to m_2 is

$$\boxed{\vec{F}_{12} = \frac{G\, m_1 m_2}{r^2}\,\hat{r}_{12}} \qquad (2.1)$$

Notes

1. The gravitational force is a **vector** – i.e. it has both magnitude and direction.

2. $\hat{r}_{12}$ is a unit vector from m_1 to m_2. In other words, $\vec{F}_{12}$ acts along the straight line joining the two masses.

3. The **gravitational constant** G is a fundamental constant of nature, believed to be the same everywhere in the Universe.

$$\boxed{G = 6.673\times 10^{-11}\ \mathrm{Nm}^2\,\mathrm{kg}^{-2}} \qquad (2.2)$$

4. The gravitational force on m_1 due to m_2 is of equal magnitude, but in the opposite direction, i.e.

$$\boxed{\vec{F}_{21} = -\vec{F}_{12}} \qquad (2.3)$$

5. Gravity is described as an **Inverse-Square Law**. i.e. the gravitational force between two bodies is inversely proportional to the **square** of their separation.

6. The gravitational force per unit mass is known as the **gravitational field**, or **gravitational acceleration**.

It is usually denoted by $\vec{g}$

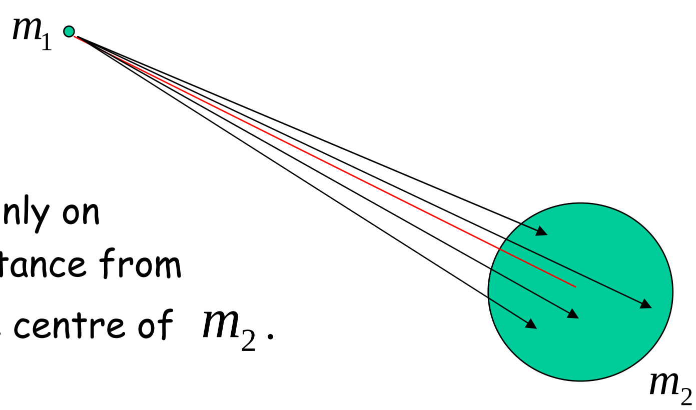
Aside

We shouldn't be too surprised that $\vec{g}$ is an acceleration: Newton's 2nd law states that "Force = mass x acceleration".

*However, Newton's 2nd law concerns **inertial mass** while Newton's law of gravitation concerns **gravitational mass**. That these two quantities are measured to be identical to each other is a very profound fact, for which Newton had no explanation, but which much later led Einstein to his theory of relativity.*

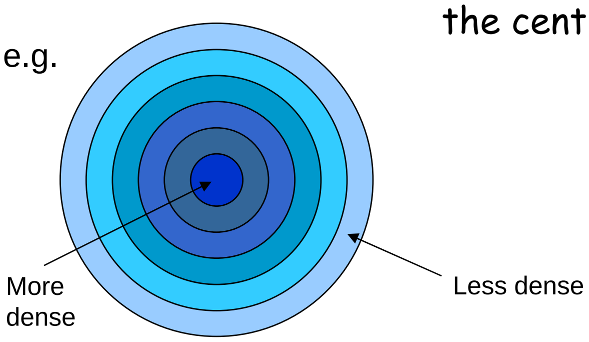
See P1 dynamics & relativity, and A2 special relativity

Planets and stars are ***not*** point mass objects. To determine the net force on m_1 due to m_2 we must add together the forces from all parts of m_2.



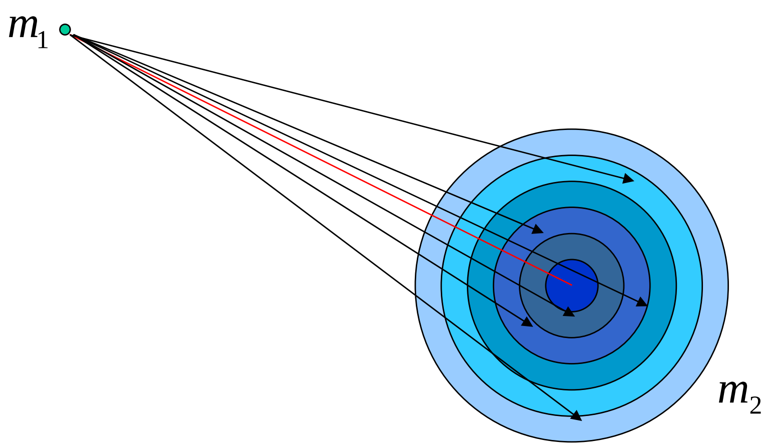
Suppose that m_2 is **spherical**, and its **density** (amount of mass per unit volume) depends only on distance from the centre of m_2.

e.g.



*We say that the density is **spherically symmetric***

In this special case, the net gravitational force on m_1 due to m_2 is exactly the same as if all of the matter in m_2 were concentrated in a point at the centre of m_2 .



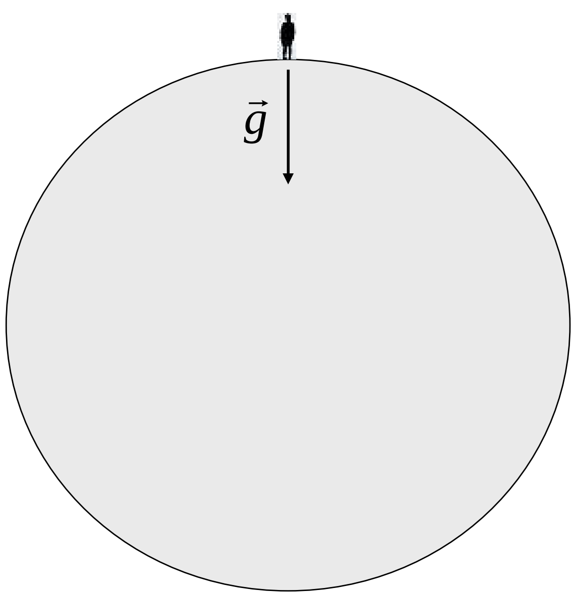
Section 3: Surface Gravity and Escape Speed

Consider, therefore, a spherical planet of radius R and total mass M which has a spherically symmetric density distribution.

The gravitational field $\vec{g}$ at the planet's surface is directed radially towards the planet's centre, and

$$\boxed{|\vec{g}| = \frac{GM}{R^2}} \qquad (3.1)$$

Magnitude of $\vec{g}$



The magnitude of $\vec{g}$ is known as the **surface gravity**, often just denoted by g (i.e. it is not a vector).

For example

$$M_{\text{Earth}} = 5.974 \times 10^{24}\ \text{kg}$$

$$R_{\text{Earth}} = 6.378 \times 10^{6}\ \text{m}$$

This means (assuming the Earth is spherical)

$$\boxed{g_{\text{Earth}} = 9.80\,\text{ms}^{-2}} \qquad (3.2)$$

g measures the rate of acceleration of falling objects (neglecting air resistance).

For any other body P (e.g. another planet or moon) it is useful to write

$$\boxed{\frac{g_P}{g_{\text{Earth}}} = \left(\frac{R_{\text{Earth}}}{R_P}\right)^2 \left(\frac{M_P}{M_{\text{Earth}}}\right)} \qquad (3.3)$$

e.g. for **Mars** :

$$R_{\text{Mars}} = 0.533\, R_{\text{Earth}}$$

$$M_{\text{Mars}} = 0.107\, M_{\text{Earth}}$$

So

$$\boxed{g_{\text{Mars}} = 0.377\, g_{\text{Earth}} = 3.69\,\text{ms}^{-2}} \qquad (3.4)$$

Exercise: Use the table of planetary data from the textbook and Section 1 to compute g for all the planets.

We can also express g in terms of average density $\overline{\rho}$

$$\overline{\rho} = \frac{\text{mass}}{\text{volume}} = \frac{M}{\frac{4}{3}\pi R^3}$$

i.e.

$$\boxed{M = \tfrac{4}{3}\pi R^3 \overline{\rho}} \qquad (3.5)$$

So from (3.1)

$$\boxed{g = \frac{GM}{R^2} = \tfrac{4}{3}\pi G R \overline{\rho}} \qquad (3.6)$$

From eq. (3.6)

1. If two planets have the same average density, the **larger** planet will have the higher surface gravity.

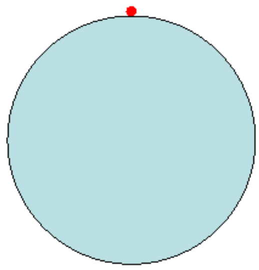
2. If two planets have the same radius, the **denser** planet will have the higher surface gravity.

Escape Speed

Consider a projectile launched vertically upwards at speed υ from the surface of a planet, with surface gravity g.

As the projectile climbs, the planet's gravity slows it down – its **kinetic energy** converted to **potential energy**.

In the animation we see the projectile slow to a stop, then accelerate back to the surface.



If the initial speed is high enough, the projectile will *never* return to the surface.

We say that the projectile **escapes** the planet's gravity.

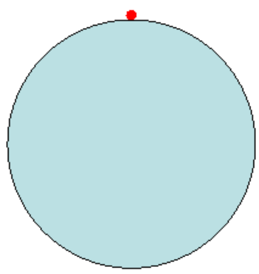
The minimum speed required to achieve this escape is known as the **escape speed**, and

$$\boxed{\upsilon_{\text{escape}} = \sqrt{\frac{2GM}{R}}} \qquad (3.7)$$

M = mass of planet

R = radius of planet

See A1X Dynamical Astronomy



For the Earth $$\upsilon_{\text{escape}} = 11.2\,\text{kms}^{-1}$$

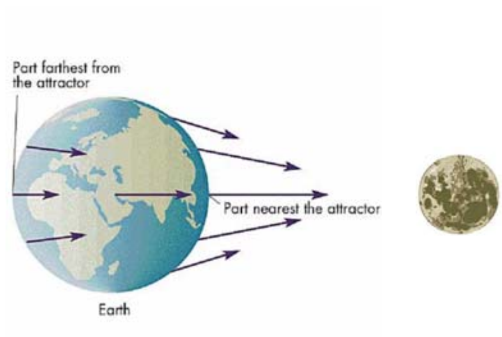
For Jupiter $$\upsilon_{\text{escape}} = 59.6\,\text{kms}^{-1}$$

Note that the escape speed does not depend on the mass of the projectile.

Section 4: Tidal Forces

In Section 2 we pointed out that planets and moons (and indeed stars) are not point mass objects. Consequently, they will be subjected to **tidal forces** since different parts of their interior and surface experience a different gravitational pull from neighbouring bodies.

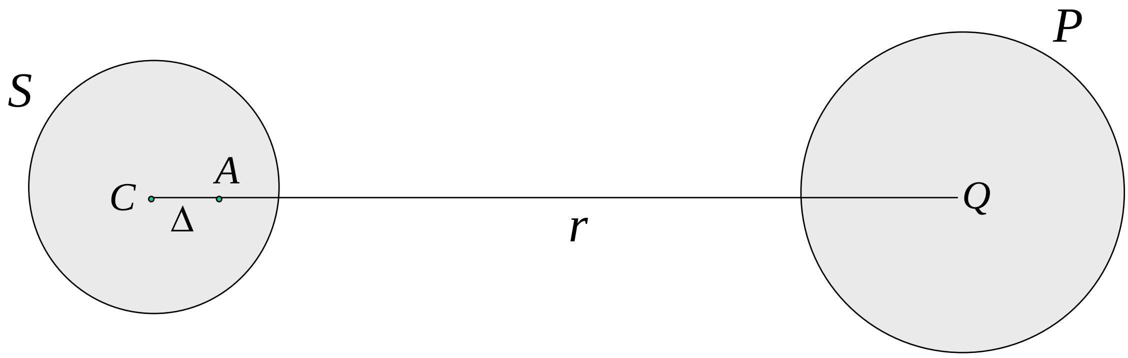
We see this differential effect with e.g. the Earth's tides, due to the Moon (and the Sun).



We now consider briefly the maths of tidal forces, before later exploring some applications to planets and moons in the Solar System.

Suppose that planet P and moon S are separated by distance r (centre to centre)

Consider a small mass m at position A and C



Tidal force between A and C due to the planet is equal to the *difference* in the gravitational force on A and C due to P

Let the distance from A to C equal Δ, and assume $\Delta << r$

Gravitational force on C due to P : $$F_C = \frac{Gm_P m}{r^2}$$

Gravitational force on A due to P : $$F_A = \frac{Gm_P m}{(r-\Delta)^2}$$

(We needn't worry about forces being vectors here, since A, C and Q lie along a straight line)

So $$F_A - F_C = \frac{Gm_P m}{(r-\Delta)^2} - \frac{Gm_P m}{r^2}$$

$$= \frac{Gm_P m}{r^2(1-\Delta/r)^2} - \frac{Gm_P m}{r^2}$$

$$= \frac{Gm_P m}{r^2}\left[\frac{1}{(1-\Delta/r)^2} - 1\right] \qquad (4.1)$$

We can write eq. (4.1) as $F_A - F_C = \frac{G m_P m}{r^2}\left[\left(1-\frac{\Delta}{r}\right)^{-2} - 1\right]$

If we now use that $\Delta << r$ then $\left(1-\frac{\Delta}{r}\right)^{-2} \approx 1 + \frac{2\Delta}{r}$ (4.2)

(Aside: Eq. (4.2) follows from the **Binomial expansion** for $(1+x)^n$ which is approximately $1+nx$ if $x << 1$)

So

$$\boxed{F_A - F_C = \frac{2 G m_P m \Delta}{r^3}} \qquad (4.3)$$

The important point here is that the magnitude of the tidal force is an **inverse-cube law**: i.e. it falls off more rapidly with distance than does the force of gravity.

So, if the planet P is far from the moon S, the tidal force experienced by the moon (and vice versa) will be small.

Conversely, however, if the moon lies very close to the planet, then the tidal forces on its interior may be considerable.

In a later section we will explore the consequences of this for the stability of moons in the Solar System.

Section 5: The Ideal Gas Law

The atmospheres of planets (and the Sun too) can be modelled as an **Ideal Gas** – i.e. consisting of point-like particles (atoms or molecules) moving in random directions and interacting through perfectly elastic collisions.

We assume that the atmosphere has an **equation of state**, which links its pressure, density and temperature:

Volume of the gas
m^3

Gas temperature, a measure of the K.E. of the particles
degrees Kelvin K

$$\boxed{PV = NkT} \qquad (5.1)$$

Gas pressure, a measure of the force of the collisions
$Pa \equiv Nm^{-2}$

Number of particles in the gas

Boltzmann constant:
$1.381\times10^{-23}\ J\,K^{-1}$

We can also write eq. (5.1) in the form
$$\boxed{P = nkT} \qquad (5.2)$$

Here $n = N/V$ is the **number density** of gas particles.

Also we can write
$$\boxed{n = \frac{\rho}{\overline{m}}} \qquad (5.3)$$

Mass density of the gas

Average mass of a gas particle

So if we introduce
$$\boxed{\mu = \frac{\overline{m}}{m_H}} \qquad (5.4)$$

1.674×10^{-27} kg

Average mass of a gas particle in units of mass of hydrogen atom

Then, from eqs. (5.2) – (5.4)
$$\boxed{P = \frac{\rho kT}{\mu m_H}} \qquad (5.5)$$

The temperature of the gas is a measure of the **average kinetic energy** of the particles.

Suppose all particles have mass m. Then we define

$$\boxed{\tfrac{1}{2} m \overline{\upsilon^2} = \tfrac{3}{2} kT} \qquad (5.6)$$

Note

- $\overline{\upsilon^2}$ is the "mean square speed" of the gas particles
- Factor of 3 on the RHS comes from the 3 dimensions ("degrees of freedom") in which the particles can move.

  The gas has energy of $\tfrac{1}{2} kT$ per degree of freedom

From eq. (5.5)

$$\boxed{\tfrac{1}{3} m \overline{\upsilon^2} = kT} \qquad (5.7)$$

Substituting from eqs. (5.2) and (5.3)

$$\boxed{P = \tfrac{1}{3} \rho \overline{\upsilon^2}} \qquad (5.8)$$

At a temperature of **absolute zero**, i.e. $0\,\mathrm{K}$, all gas motions cease. Gas pressure drops to zero.

Section 6: Hydrostatic Equilibrium

The pressure (and hence the density and temperature) is not constant throughout a planetary atmosphere. A balance is maintained between the outward **pressure force** and the inward **gravitational force**.

We call this balance **hydrostatic equilibrium**.

Let's assume (as we did in section 2 for the interior of a planet) that the density of gas in the atmosphere is **spherically symmetric**.

We can then derive an expression for how the pressure changes as a function of height in the planet's atmosphere.

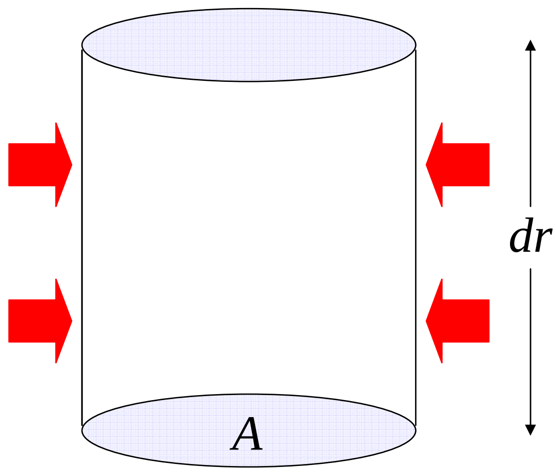
(we do this using calculus, forming a **differential equation***)*

Consider a small cylinder of gas in the planet's atmosphere, the bottom of which is a distance r from the centre of the planet.

Let the area of the cylinder be A and its height be dr.

Suppose the cylinder contains a mass m of gas.

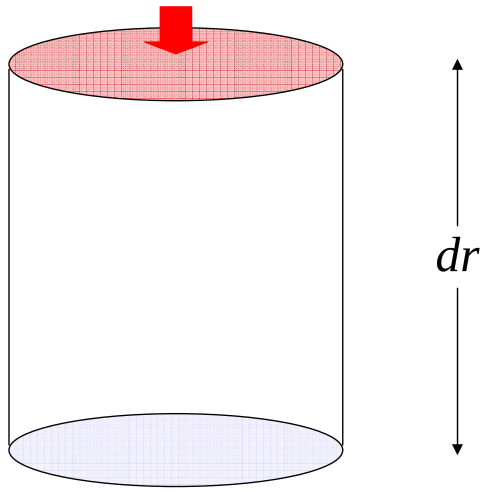
What forces will be exerted on this cylinder by the rest of the atmosphere?...



The horizontal forces on the walls of the cylinder will cancel out

Upper face:

Downward force, due to pressure exerted by gas above the cylinder *



$$F_{\text{upper}} \;=\; -A\,P(r+dr) \qquad (6.1)$$

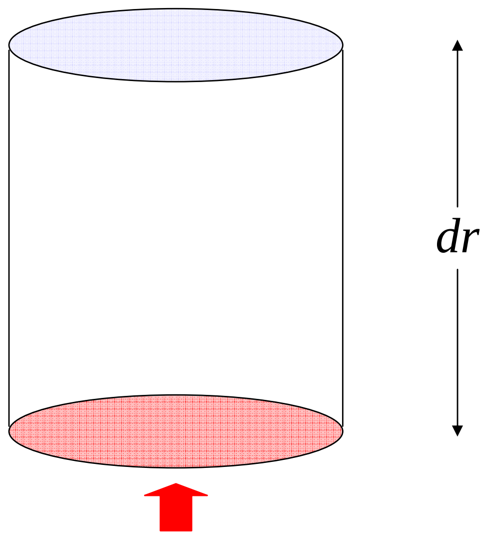
Notes: (1) we are taking upwards as positive

(2) we are using here the relation pressure = force per unit area

* There will also be a downward force due to the weight of atmosphere above the cylinder, but we don't consider that here since it will also apply to the lower face.

Lower face:

Upward force, due to pressure of the gas below the cylinder, and *downward* force, due to the weight of the gas in the cylinder.



$$F_{\text{lower}} \;=\; A\,P(r) \;-\; \frac{G M(r) m}{r^2} \qquad (6.2)$$

Notes: (1) $M(r)$ is the mass contained within radius r from the planet's centre

(2) The gravitational force term in eq. 6.2 is an approximation, since the cylinder does not all lie at distance r from the centre.

This is OK provided $dr << r$

We can re-write eq. (6.2) as $$F_{\text{lower}} = A\,P(r) - m\,g \tag{6.3}$$

We can also write the mass of gas in the cylinder as **density x volume**: $$m = \rho\,A\,dr \tag{6.4}$$

Substituting into eq. (6.3) gives

$$F_{\text{lower}} = A\,P(r) - A\rho g\,dr \tag{6.5}$$

To keep the cylinder static, we require that there be no **net force** on it, i.e. $$F_{\text{lower}} + F_{\text{upper}} = 0$$

So $$AP(r+dr) - AP(r) + A\rho g\,dr = 0 \tag{6.6}$$

Dividing by $A\,dr$ and re-arranging

$$\frac{P(r+dr) - P(r)}{dr} = -\rho g \tag{6.7}$$

In the limit as $dr \to 0$ the LHS is the **derivative** of $P(r)$ with respect to r i.e. the rate of change of pressure with radius.

Finally, then, we have

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\rho g \tag{6.8}$$

Also referred to as the ***pressure gradient***

Since the density and gravitational acceleration are both **positive**, this means that

$$\boxed{\frac{dP}{dr} < 0} \qquad (6.9)$$

i.e. $P(r)$ *decreases* with increasing radius.

How fast?...

We define the **pressure scale height** via

$$\boxed{\frac{1}{H_P} = -\frac{1}{P(r)}\frac{dP}{dr}} \qquad (6.10)$$

If we make the assumption that H_P is **constant**, then we can find an expression for the pressure as a function of radius.

Re-arranging eq. (6.10)

$$\boxed{\frac{dP}{P} = -\frac{dr}{H_P}} \qquad (6.11)$$

This is a **differential equation**. We solve it by integrating both sides.

$$\int \frac{dP}{P} = -\int \frac{dr}{H_P} = -\frac{1}{H_P}\int dr$$

By assuming the scale height is constant, we can take it out of the integral

i.e.

$$\boxed{\log P = -\frac{r}{H_P} + \text{constant}} \qquad (6.12)$$

Natural logarithm

The constant can be fixed by the pressure at $r = 0$, say $P = P_0$

$$\boxed{P(r) = P_0 \exp\left(-\frac{r}{H_P}\right)} \qquad (6.13)$$

Eq. (6.13) makes sense for e.g. the Sun, which is gaseous throughout.

For a planet like the Earth, with a solid interior, we can write

$$r = \underbrace{R}_{\text{Radius of surface}} + \underbrace{h}_{\text{Height above surface}}$$

We then fix the constant to be the pressure at the surface, $h = 0$

$$\boxed{P(h) = P_S \exp\left(-\frac{h}{H_P}\right)} \qquad (6.14)$$

The Earth's surface pressure is defined as **1 atmosphere**

From eqs. (6.8) and (6.10) $\quad H_P = \dfrac{P}{\rho g}$

For an **ideal gas**, from eq. (5.5) $\quad P = \dfrac{\rho k T}{\mu m_H}$

So $$\boxed{H_P = \frac{kT}{\mu m_H g}} \qquad (6.15)$$

As T **increases**, so does H_P, i.e. the atmosphere extends **further**.

As μ, g **increase**, H_P **decreases**. i.e. atmosphere **less** extended.

Substituting in eq. (6.14) $$\boxed{P(h) = P_S \exp\left(-\frac{\mu m_H g h}{kT}\right)} \qquad (6.16)$$

Section 7: Escape of a Planetary Atmosphere

The atoms or molecules in a planet's atmosphere are constantly moving. If they are moving fast enough, they can **escape**.

From eq. (3.7), this requires

$$\upsilon > \upsilon_{\text{escape}} = \sqrt{\frac{2GM_P}{R_P}} \tag{7.1}$$

But the particles will have a **distribution** of speeds - some will exceed the escape speed, while others will not.

When a sufficient fraction of the particles exceed the escape speed, the planet will effectively 'lose' its atmosphere.

A good 'rule of thumb' is:

> A particular component of a planet's atmosphere will be lost if, for that component, $\upsilon_{\text{rms}} > \frac{1}{6}\upsilon_{\text{escape}}$

υ_{rms} is the '**root mean square**' speed, the square root of the mean square speed we met in Section 5.

$$\upsilon_{\text{rms}} = \sqrt{\overline{\upsilon^2}} \tag{7.2}$$

We can use the results of Section 5 to relate the escape criterion to **temperature**, using:

$$\tfrac{1}{3} m \upsilon_{\text{rms}}^{2} = kT \tag{7.3}$$

So a particle of mass m will escape if

$$\boxed{kT = \tfrac{1}{3} m \upsilon_{\text{rms}}^{2} > \tfrac{1}{3} m \tfrac{1}{36} \upsilon_{\text{escape}}^{2} = \frac{1}{54} \frac{G M_P m}{R_P}} \tag{7.4}$$

We define the **escape temperature**

$$\boxed{T_{\text{escape}} = \frac{1}{54} \frac{G M_P m}{k R_P}} \tag{7.5}$$

The more massive the planet, the hotter it must be before a given atmospheric component is lost.

Section 8: Key Features of the Jovian Planets

Jovian planets: Jupiter, Saturn, Uranus and Neptune

Terrestrial planets: Mercury, Venus, Earth and Mars

We can summarise the differences between them:

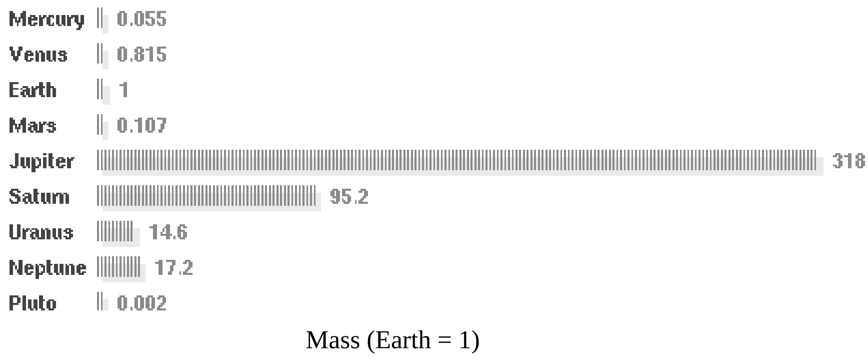
See Chapter 6, Table 6.2 Astronomy Today

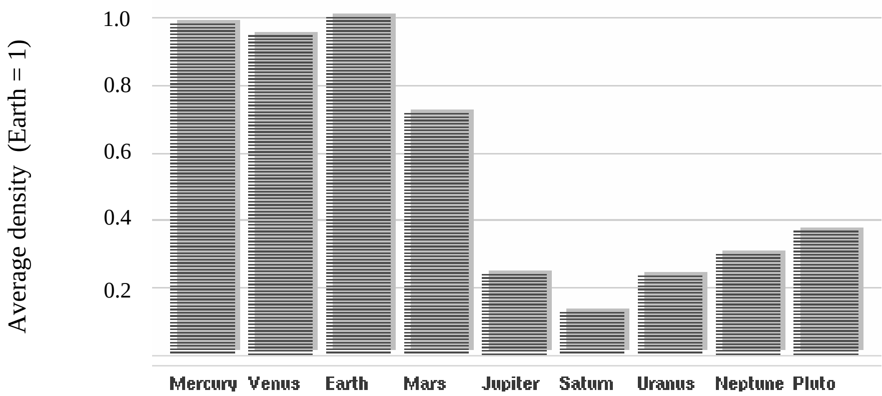
See SSP2 Lectures

Terrestrial Planets	Jovian Planets
Lower mass, smaller radii	Higher mass, larger radii
Near the Sun	Distant from the Sun
Higher surface temperature	Lower surface temperature
Higher average density	Lower average density
H and He depleted	Abundant H and He
Solid surface	Gaseous / Liquid *
Slower rotation period	Rapid rotation period
No rings	Many rings
Few satellites	Many satellites

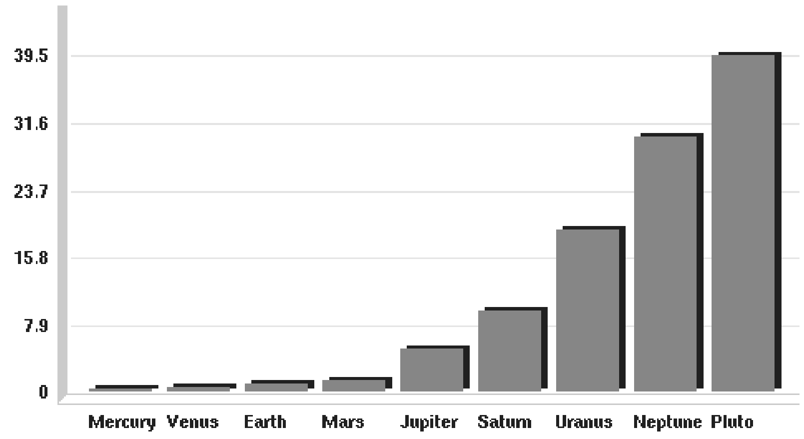
* Rocky core deep inside

Comparative masses and densities of the planets

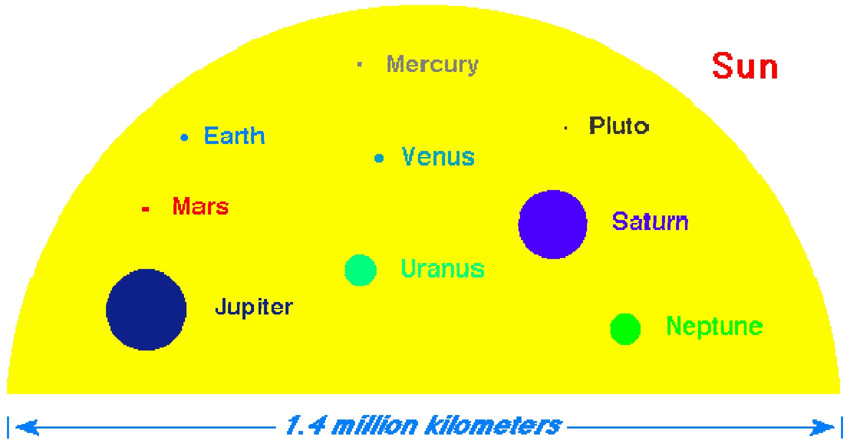




Comparative orbit sizes and diameters of the planets



Average distance from the Sun (AU)



Abundance of H and He

We can use the results of Section 7 to estimate the temperature required for hydrogen and helium to escape from a planetary atmosphere:

$$T_{\text{esc}} = \frac{1}{54} \frac{G M_P \, \mu \, m_H}{k \, R_P}$$

$$= \frac{6.673\times10^{-11} \times 5.976\times10^{24} \times 1.674\times10^{-27} \left(M_P / M_{\text{Earth}}\right) \mu}{54\times1.381\times10^{-23} \times 6.378\times10^{6} \left(R_P / R_{\text{Earth}}\right)} \text{ K}$$

$$\boxed{T_{\text{escape}} = \frac{140 \left(M_P / M_{\text{Earth}}\right) \mu}{\left(R_P / R_{\text{Earth}}\right)} \text{ K}} \qquad (8.1)$$

- For molecular Hydrogen, $\mu = 2$ so the escape temperature for the **Earth** is **280 K**

  This explains why the Earth has not retained its atmospheric molecular hydrogen.

- When the solar system was forming, the inner part was too hot to retain lighter elements, such as H and He; these are absent from all terrestrial planet atmospheres. *(See also SSP2)*

- For, e.g. molecular Nitrogen, $\mu = 28$ so the escape temperature for the Earth is **3920 K**

  So the Earth's atmosphere *can* retain its molecular nitrogen.

Plugging in the numbers for the Jovian planets, for molecular **Hydrogen**; these escape temperatures are so high that the Jovian planets will *not* have lost their atmospheric hydrogen.

Planet	Radius (Earth=1)	Mass (Earth=1)	T_{esc}
Jupiter	11.209	317.8	7939 K
Saturn	9.449	95.16	2820 K
Uranus	4.007	14.53	1015 K
Neptune	3.883	17.15	1237 K

Internal structure of Jupiter

Upper atmosphere:

90% H_2
10% He
0.2% CH_4, ammonia, water

Lower atmosphere:

High pressure, density 'squeezes' H_2

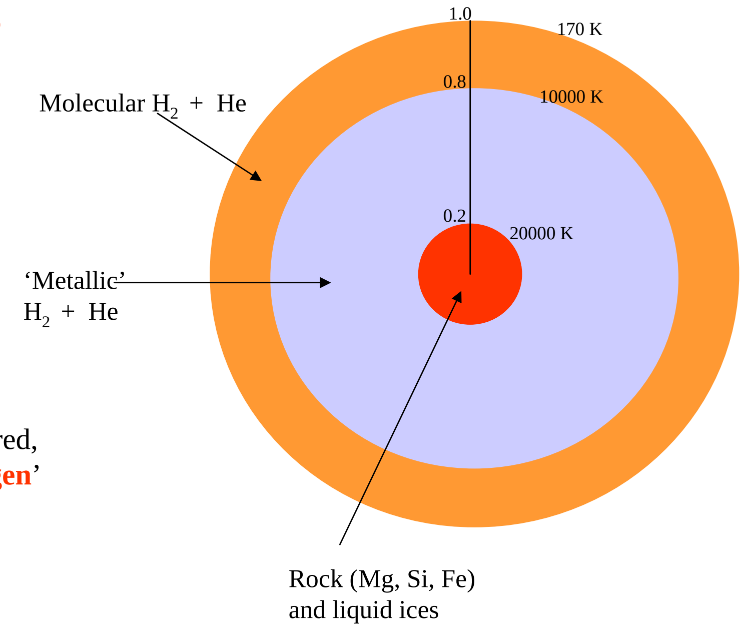
Molecular bonds broken; electrons shared, as in a metal – '**liquid metallic hydrogen**'

Core:

Dense, 'soup' of rock and liquid 'ices' (water, methane ammonia) of about 15 Earth masses

Evidence of **internal heating** – gravitational P.E. released during planetary formation (collapse of gas cloud)

[see SSP2 and A1Y Stellar Astrophysics]



Metallic hydrogen gives Jupiter a **strong magnetic field** (19000 times that of the Earth)

See Chapter 11, Astronomy Today

Internal structure of Saturn

Upper atmosphere:

97% H_2
3% He
0.2% CH_4, ammonia, water

Lower atmosphere:

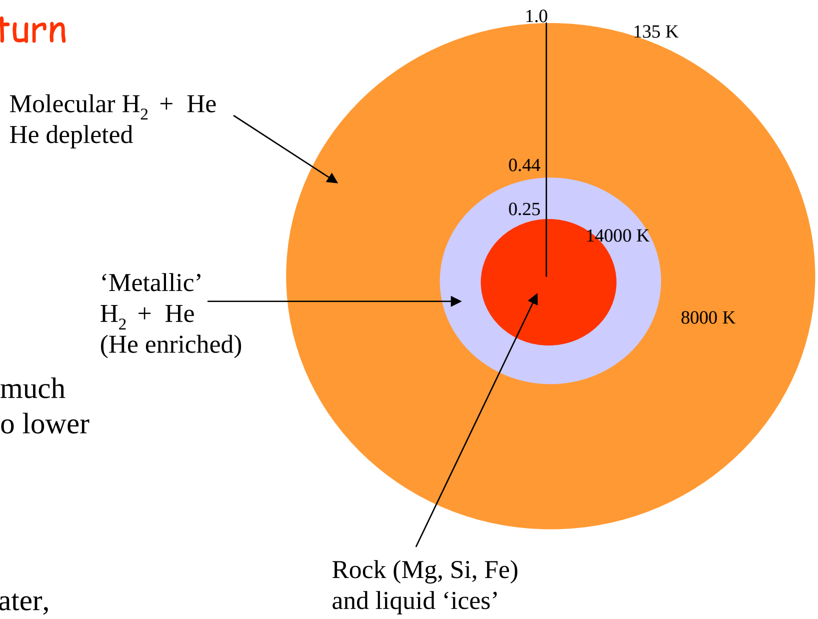
'**liquid metallic hydrogen**' (but at much greater depth than in Jupiter – due to lower mass and density)

Core:

Dense, 'soup' of rock and 'ices' (water, methane ammonia) of about 13 Earth masses

Internal heating **not entirely explained** by planetary formation; **extra heating** from release of P.E. as **heavier He sinks**.

Effect more pronounced for Saturn, as outer atmosphere cooler to begin with



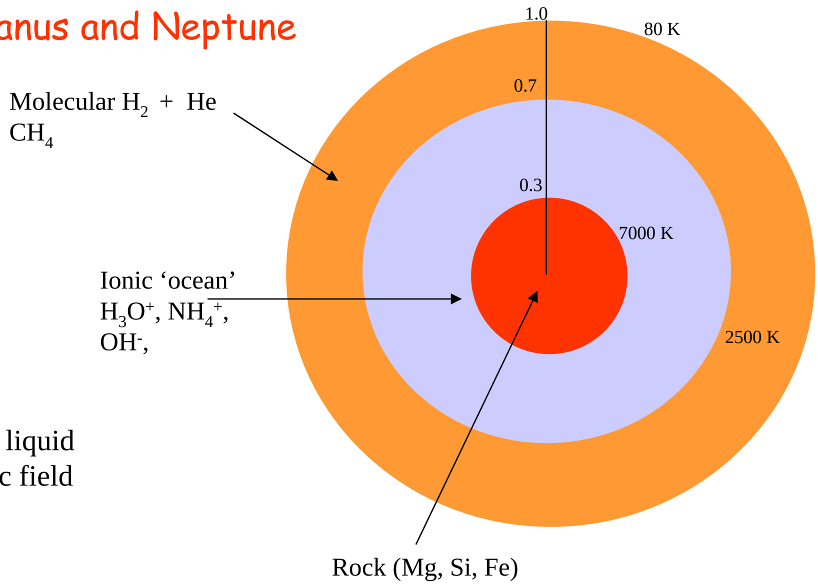
Metallic hydrogen gives Saturn a **strong magnetic field** (but weaker than Jupiter's)

See Chapter 12, Astronomy Today

Internal structure of Uranus and Neptune

Upper atmosphere:

Uranus	Neptune
83% H_2	74% H_2
15% He	25% He
2% CH_4	1% CH_4



Lower atmosphere:

Pressures **not high enough** to form liquid metallic hydrogen; weaker magnetic field due to ionic 'ocean'

Core:

Dense, 'soup' of rock, also about 13 Earth masses

Internal heating also important – particularly for Neptune (similar surface temperature to Uranus, despite being 1.5 times further from the Sun)

Cores of Uranus and Neptune form much higher (70% to 90%) fraction of total mass, compared with Jupiter (5%) and Saturn (14%)

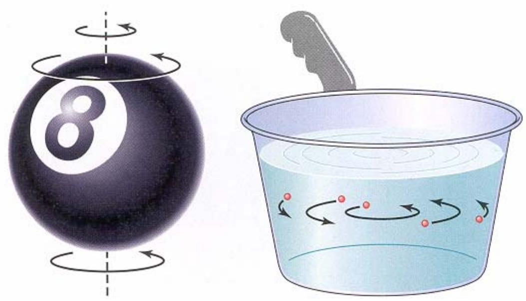
See Chapter 13, Astronomy Today

Rotation of the Jovian Planets

Jupiter, Saturn, Uranus and Neptune rotate very rapidly, given their large radii, compared with the terrestrial planets.

Also, the Jovian planets rotate ***differentially*** – not like a solid body (e.g. a billiard ball) but as a fluid (e.g. grains of rice).

We see this clearly on Jupiter: cloud bands and belts rotate at different speeds

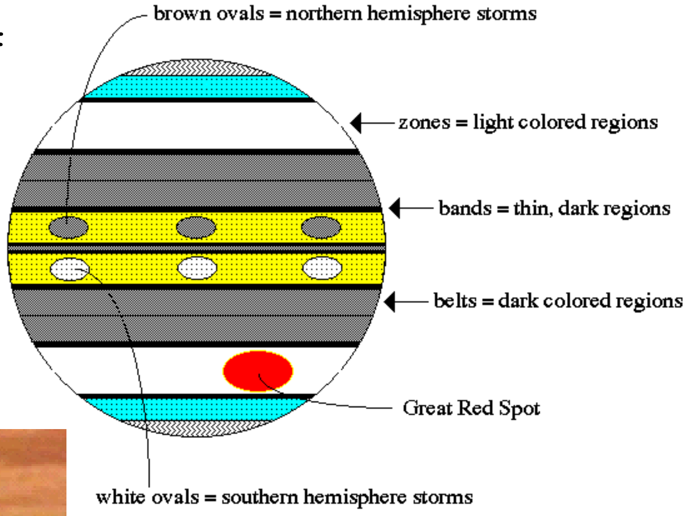


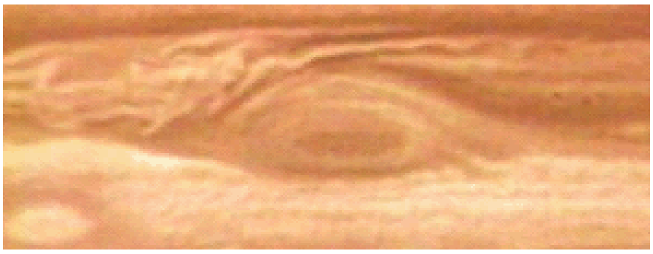
Planet	Rotation Period *
Mercury	58.6 days
Venus	243 days
Earth	24 hours
Mars	24 h 37 m
Jupiter *	9 h 50 m
Saturn *	10 h 14 m
Uranus	17 h 14 m
Neptune	16 h 7 m
Pluto	6.4 days

* At Equator

On Jupiter we also see that the cloud belts contain oval structures. These are storms; the most famous being the **Great Red Spot**. This is a hurricane which has been raging for hundreds of years. It measures about 40000km by 14000km

Winds to the north and south of the Great Red Spot blow in opposite directions; winds *within* the Spot blow **counterclockwise**, completing one revolution in about 6 days.

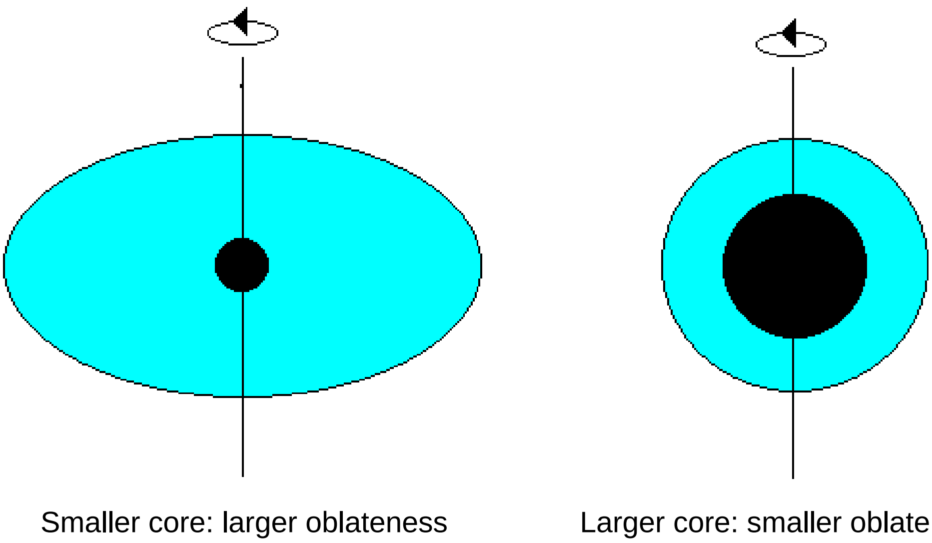




The Jovian planets are also significantly flattened, or **oblate**, due to their rapid rotation and fluid interior.

The effect is most pronounced for Jupiter and Saturn, which have relatively smaller cores

e.g. Jupiter's polar diameter = 133708km (6.5% less than equatorial diameter)



Section 9: Ring Systems of the Jovian Planets

All four Jovian planets have **RING SYSTEMS**. e.g. **Saturn's rings** are easily visible from Earth with a small telescope, and appear solid.

The rings consist of countless lumps of ice and rock, ranging from ~1cm to 5m in diameter, all independently orbiting Saturn in an incredibly thin plane – less than 1 kilometre in thickness.

(Diameter of the outermost ring – 274000 km. If Saturn's rings were the thickness of a CD, they would still be more than 200m in diameter!)

James Clerk Maxwell proved that Saturn's rings couldn't be solid; if they *were* then **tidal forces** would tear them apart. He concluded that the rings were made of 'an indefinite number of unconnected particles'

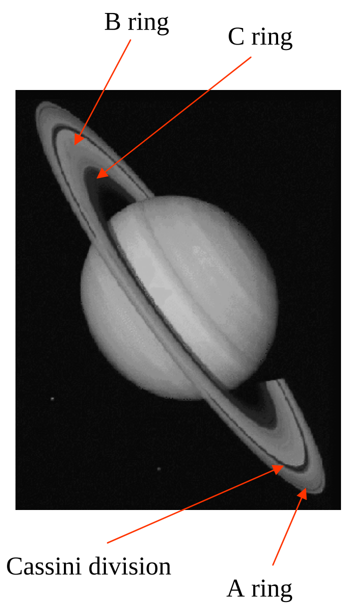
Saturn's rings are bright; they reflect ~80% of the sunlight that falls on them. Their ice/rock composition was confirmed in the 1970s when **absorption lines** of water were observed in the **spectrum** of light from the rings.

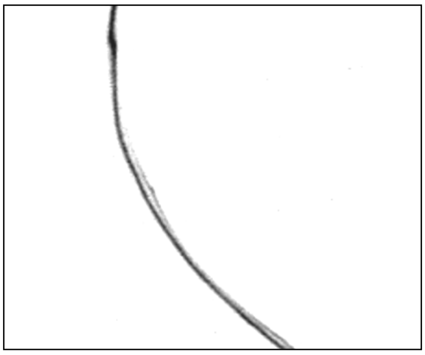
(See A1Y Stellar astrophysics for more on spectra and absorption lines)

Ground-based observations show only the **A**, **B** and **C** rings.

In the 1980s the **Voyager** spacecraft flew past Saturn, and observed thousands of **'ringlets'** – even in the **Cassini Division** (previously believed to be a gap).

They also discovered a D ring, (inside the C ring), and very tenuous E, F and G rings outside the A ring, out to ~5 planetary radii.

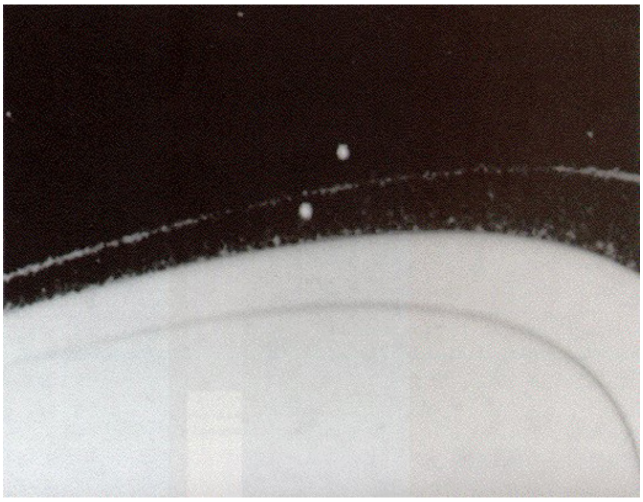




The F ring shows braided structure, is very narrow, and contains large numbers of micron-sized particles.

The structure of the F ring is controlled by the two '**shepherd moons**' – Pandora and Prometheus – which orbit just inside and outside it.

The gravitational influence of these moons confine the F ring to a band about 100km wide



Ring Systems of the other Jovian Planets

- **Jupiter**'s ring system is much more tenuous than Saturn's. It was only detected by the Voyager space probes. The ring material is primarily dust, and extends to about 3 Jupiter radii.

- **Uranus**' rings were discovered in 1977, during the **occultation** of a star, and first studied in detail by Voyager 2 in 1986

  There are 11 rings, ranging in width from 10km to 100km. The ring particles are very dark and ~1m across. Some rings are 'braided', and the thickest ring has shepherd moons. There is a thin layer of dust between the rings, due to collisions.

- **Neptune**'s rings were first photographed by Voyager 2 in 1989.

  There are 4 rings: two narrow and two diffuse sheets of dust. One of the rings has 4 'arcs' of concentrated material.

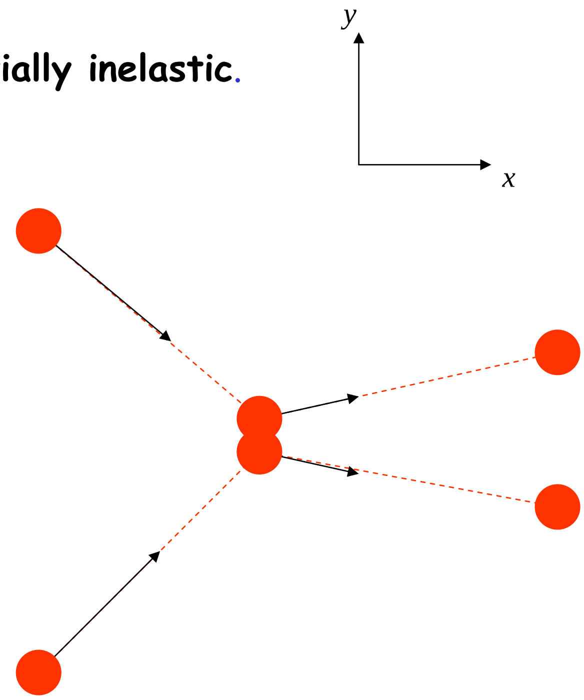
Why are the ring systems so thin?

Collisions of ring particles are **partially inelastic**.

Consider two particles orbiting e.g. Saturn in orbits which are slightly tilted with respect to each other.

Collision reduces difference of y components, but has little effect on x components

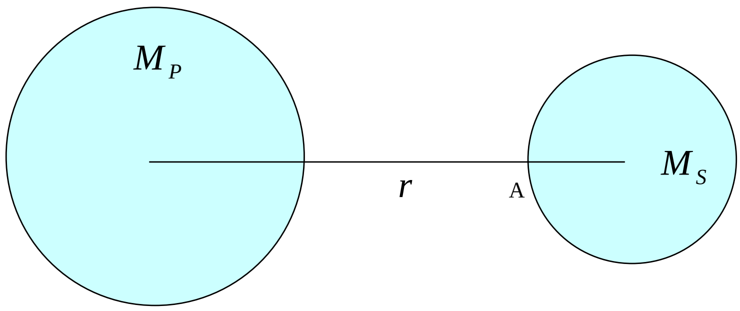
$\Rightarrow$ this thins out the disk of ring particles



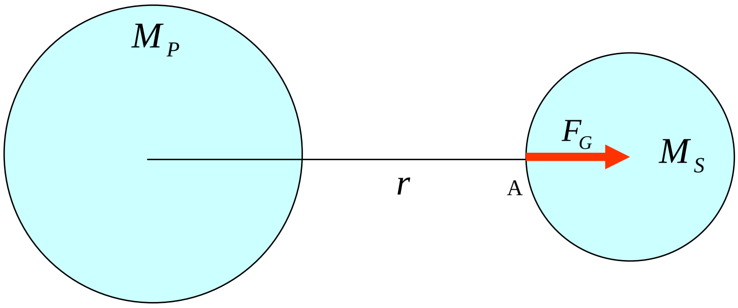
Section 10: Formation of ring systems

The ring systems of the Jovian planets result from **tidal forces**. During planetary formation, these prevented any material that was too close to the planet clumping together to form moons. Also, any moons which later strayed too close to the planet would be **disrupted**.

Consider a moon of mass M_S and radius R_S, orbiting at a distance (centre to centre) r from a planet of mass M_P and radius R_P .



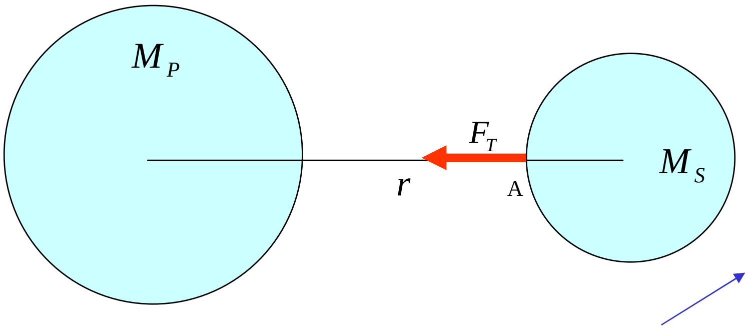
(Assume that the planet and moon are spherical)



(Assume that the planet and moon are spherical)

Force on a unit mass at A due to gravity of **moon alone** is

$$\boxed{F_G = \frac{G\,M_S}{R_S^{\,2}}} \qquad (10.1)$$



Tidal force on a unit mass at A due to gravity of **planet** is

$$\boxed{F_T = \frac{2G\,M_P\,R_S}{r^3}} \qquad (10.2)$$

This follows from eq. (4.3) putting $\Delta = R_S$

We assume, as an order-of-magnitude estimate that the moon is **tidally disrupted** if

$$\boxed{F_T > F_G} \qquad (10.3)$$

In other words, if

$$\boxed{\frac{2G\,M_P\,R_S}{r^3} > \frac{G\,M_S}{R_S^{\,2}}} \qquad (10.4)$$

This rearranges further to

$$\boxed{r \;<\; 2^{1/3}\left(\frac{M_P}{M_S}\right)^{1/3} R_S} \qquad (10.5)$$

We can re-cast eq. (10.5) in terms of the *planet's* radius, by writing **mass = density × volume**.

Substituting $$M_P = \tfrac{4\pi}{3}\,\overline{\rho}_P\, R_P^{\;3}$$

$$M_S = \tfrac{4\pi}{3}\,\overline{\rho}_S\, R_S^{\;3}$$

So the moon is tidally disrupted if $$\boxed{r \;<\; 2^{1/3}\left(\frac{\overline{\rho}_P}{\overline{\rho}_S}\right)^{1/3} R_P} \qquad (10.6)$$

More careful analysis gives

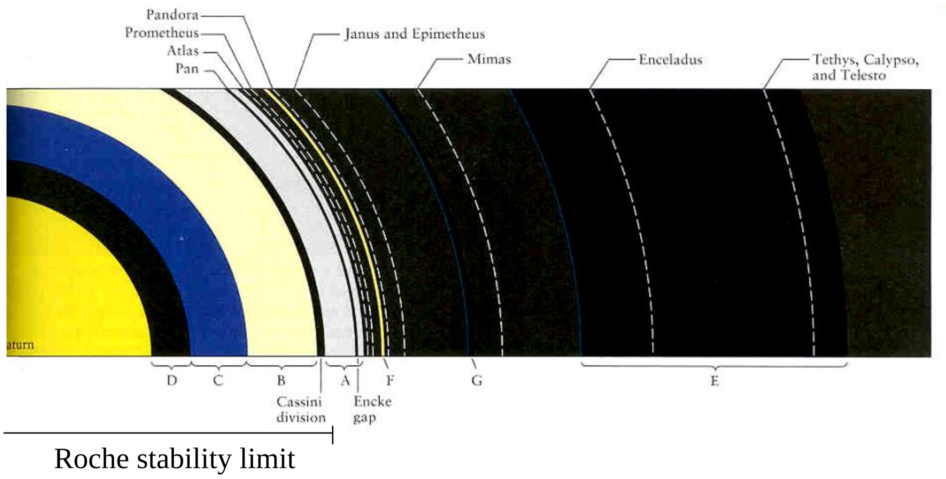
the **Roche Stability Limit** $$\boxed{r < 2.456\left(\frac{\overline{\rho}_P}{\overline{\rho}_m}\right)^{1/3} R_P} \qquad (10.7)$$

e.g. for **Saturn**, from the Table of planetary data $\overline{\rho}_P \approx 700\,\mathrm{kg\,m^{-3}}$

Take a mean density typical of the other moons $\overline{\rho}_m \approx 1200\,\mathrm{kg\,m^{-3}}$

This implies $$\boxed{r_{RL} = 2.456\times\left(\frac{700}{1200}\right)^{1/3}\times R_P \quad = \quad 2.05\,R_P} \qquad (10.8)$$

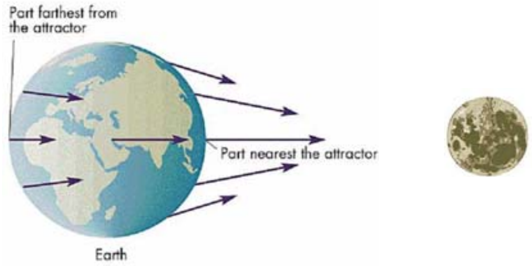
Most of Saturn's ring system ***does*** lie within this Roche stability limit. Conversely ***all*** of its moons lie further out!



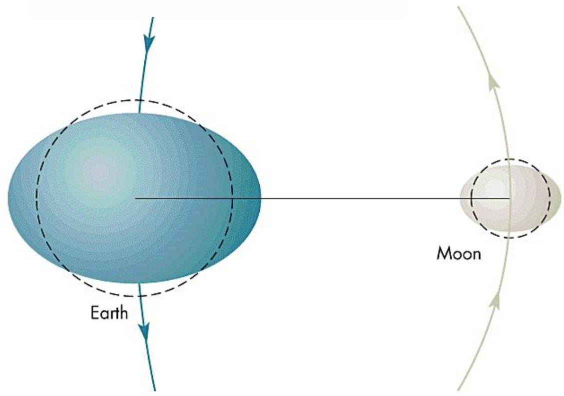
Section 11: More on Tidal Forces

Tidal forces also have an effect (albeit less destructive) *outside* the Roche stability limit.

Consider the Moon's tide on the Earth (and vice versa).



The tidal force produces an oval bulge in the shape of the Earth (and the Moon)



There are, therefore, **two** high and low tides every ~25 hours.

(Note: not every 24 hours, as the Moon has moved a little way along its orbit by the time the Earth has completed one rotation)

The Sun also exerts a tide on the Earth.

Now, $F_T \propto \dfrac{M_P}{r^3}$ so

$$\boxed{\frac{F_{T,\mathrm{Sun}}}{F_{T,\mathrm{Moon}}} = \frac{M_{\mathrm{Sun}}}{M_{\mathrm{Moon}}}\left(\frac{r_{\mathrm{Moon}}}{r_{\mathrm{Sun}}}\right)^3} \qquad (11.1)$$

and $M_{\mathrm{Sun}} = 1.989\times10^{30}\ \mathrm{kg}$ $\quad M_{\mathrm{Moon}} = 7.35\times10^{22}\ \mathrm{kg}$

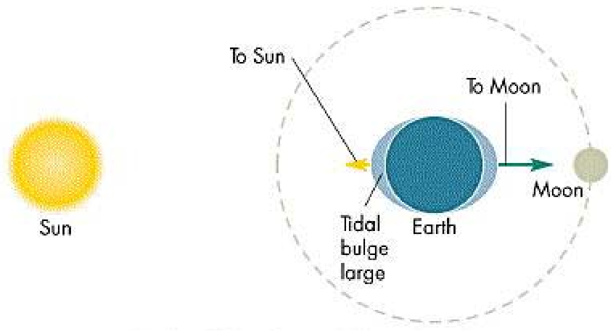
$r_{\mathrm{Sun}} = 1.496\times10^{11}\ \mathrm{m}$ $\quad r_{\mathrm{Moon}} = 3.844\times10^{8}\ \mathrm{m}$

so that

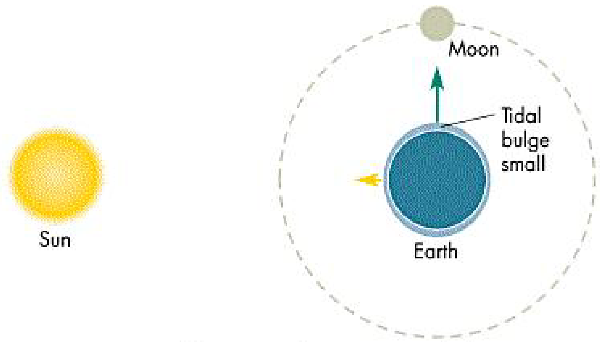
$$\boxed{\frac{F_{T,\mathrm{Sun}}}{F_{T,\mathrm{Moon}}} = \frac{M_{\mathrm{Sun}}}{M_{\mathrm{Moon}}}\left(\frac{r_{\mathrm{Moon}}}{r_{\mathrm{Sun}}}\right)^3 \approx 0.5} \qquad (11.2)$$

The Sun and Moon exert a tidal force similar in magnitude. The size of their combined tide on the Earth depends on their *alignment*.

Spring tides occur when the Sun, Moon and Earth are aligned (at Full Moon and New Moon). High tides are much higher at these times.

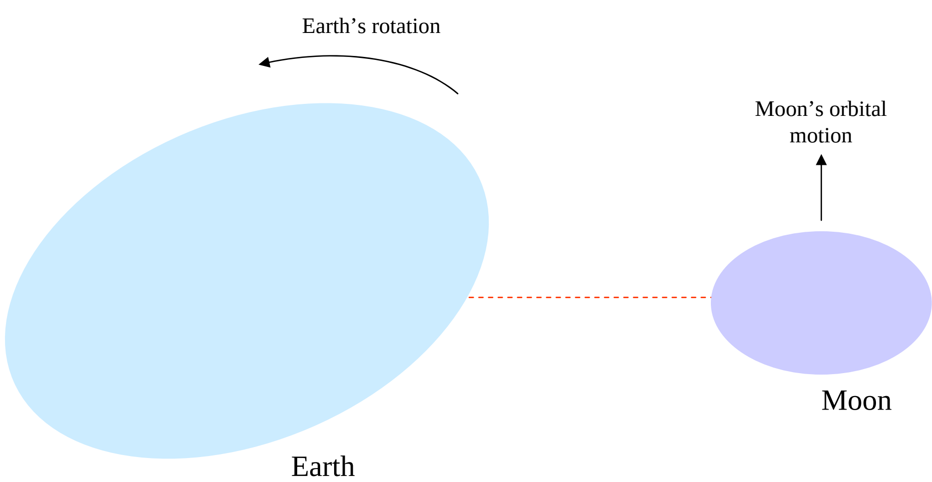


Neap tides occur when the Sun, Moon and Earth are at right angles (at First Quarter and Third Quarter). Low tides are much lower at these times.



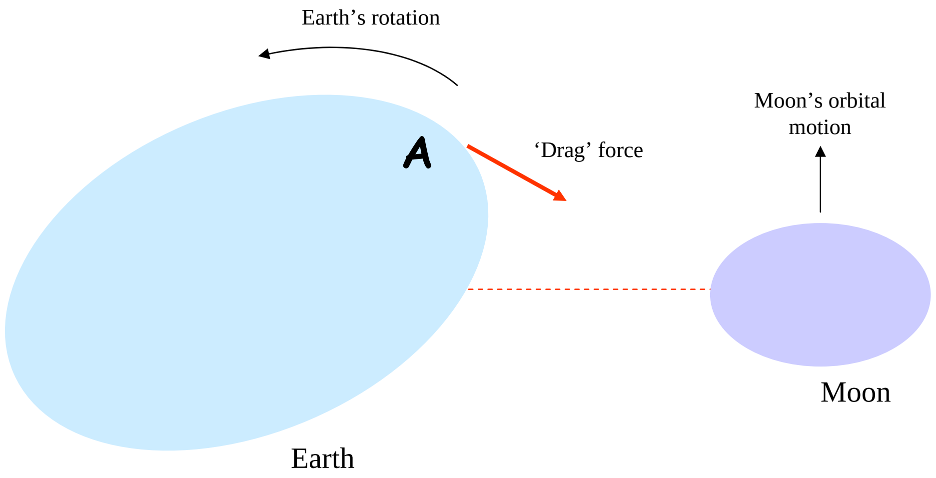
Even if there were no tidal force on the Earth from the Sun, the Earth's tidal bulge would **not** lie along the Earth-Moon axis. This is because of the Earth's rotation.

The Earth's rotation carries the tidal bulge ahead of the Earth-Moon axis. (The Earth's crust and oceans cannot instantaneously redstribute themselves along the axis due to friction)



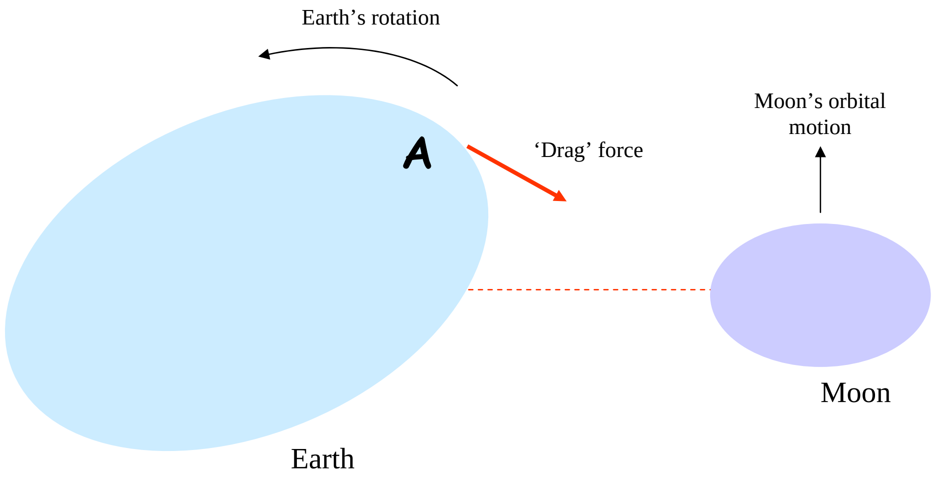
The Moon exerts a drag force on the tidal bulge at A, which slows down the Earth's rotation.

The length of the Earth's day is increasing by 0.0016 sec per century.



At the same time, bulge A is pulling the Moon forward, speeding it up and causing the Moon to spiral outwards. **This follows from the conservation of angular momentum**.

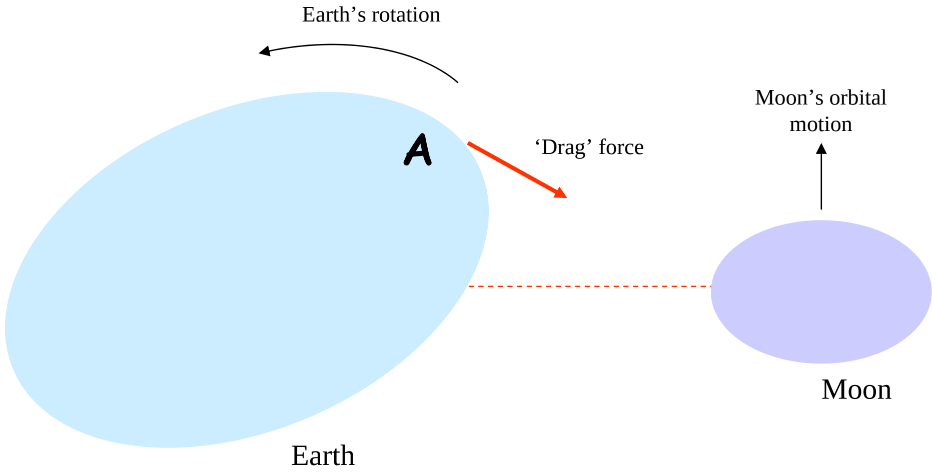
The Moon's semi-major axis is increasing by about 3cm per year.



- Given sufficient time, the Earth's rotation period would slow down until it equals the Moon's orbital period – so that the same face of the Earth would face the Moon at all times.

  (This will happen when the Earth's "day" is 47 days long)

  In the case of the Moon, this has *already happened !!!*



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  (This will happen when the Earth's "day" is 47 days long)

  In the case of the Moon, this has *already happened !!!*

- Tidal locking has occurred much more rapidly for the Moon than for the Earth because the Moon is much smaller, and the Earth produces larger tidal deformations on the Moon than vice versa.

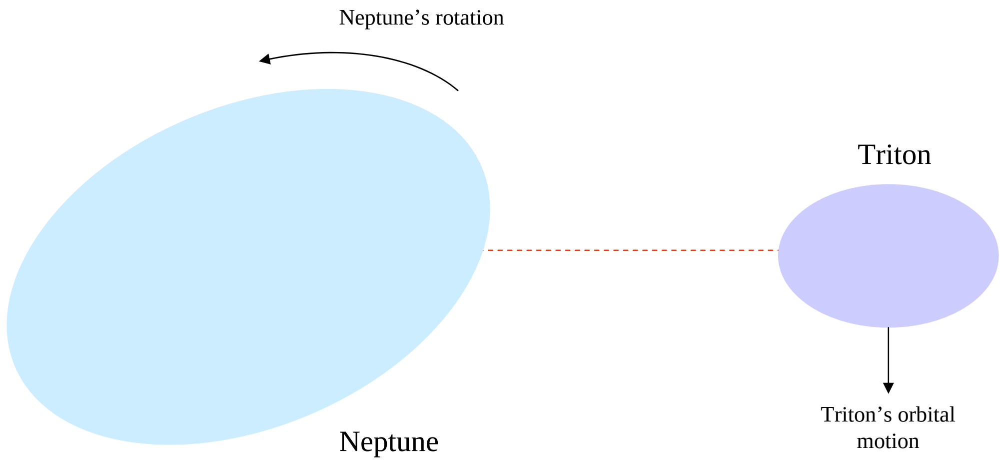
- The Moon isn't exactly tidally locked. It 'wobbles' due to the perturbing effect of the Sun and other planets, and because its orbit is **elliptical**. Over about 30 years, we see 59% of the Moon's surface.

- Many of the satellites in Solar System are in synchronous rotation, e.g.

Mars:	Phobos and Deimos
Jupiter:	Galilean moons + Amalthea
Saturn:	All major moons, except Phoebe + Hyperion
Neptune:	Triton
Pluto:	Charon

- Pluto and Charon are in mutual synchronous rotation: i.e. the same face of Charon is always turned towards the same face of Pluto, and vice versa.

- Triton orbits Neptune in a **retrograde** orbit (i.e. opposite direction to Neptune's rotation).



In this case Neptune's tidal bulge acts to *slow down* Triton. The moon is spiralling toward Neptune (although it will take billions of years before it reaches the Roche stability limit)

Section 12: The Galilean Moons of Jupiter

Tidal forces have a major influence on the Galilean Moons of Jupiter

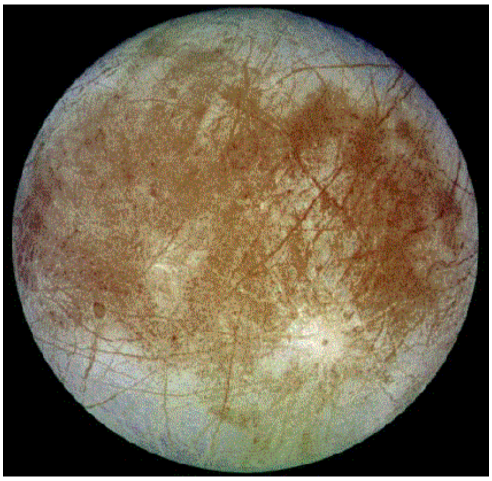
Name	Diameter (m)	Semi-major axis (m)	Orbital Period (days)	Mass (kg)
Io	3.642×10^{6}	4.216×10^{8}	1.769	8.932×10^{22}
Europa	3.120×10^{6}	6.709×10^{8}	3.551	4.791×10^{22}
Ganymede	5.268×10^{6}	1.070×10^{9}	7.155	1.482×10^{23}
Callisto	4.800×10^{6}	1.883×10^{9}	16.689	1.077×10^{23}
The Moon	3.476×10^{6}	3.844×10^{8}	27.322	7.349×10^{22}
Mercury	4.880×10^{6}	—	—	3.302×10^{23}

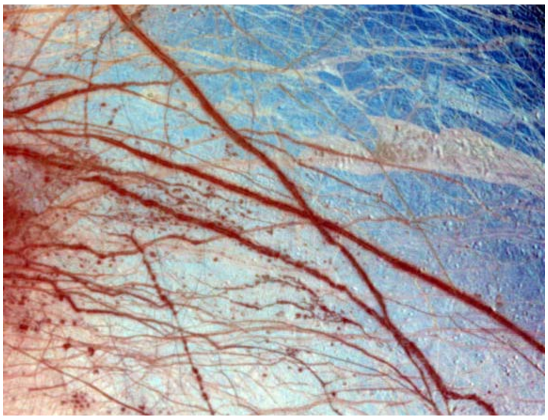
The orbital periods of Io, Europa and Ganymede are almost exactly in the ratio **1:2:4**. This leads to **resonant effects** :

- The orbit of Io is perturbed by Europa and Callisto, because the moons regularly line up on one side of Jupiter. The gravitational pull of the outer moons is enough to produce a small eccentricity in the orbit of Io. This causes the tidal bulges of Io to 'wobble' (same as the Moon) which produces large amount of **frictional heating**.

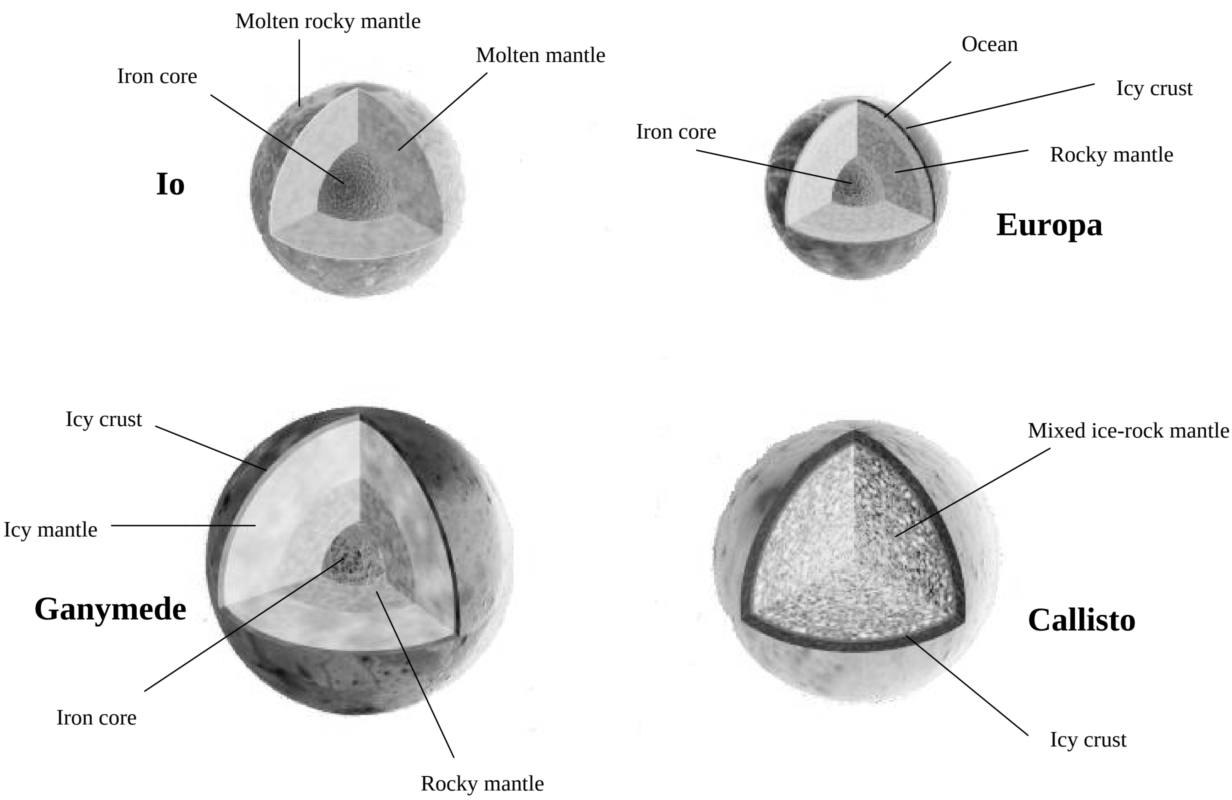
- The surface of Io is almost totally **molten**, yellowish-orange in colour due to sulphur from its continually erupting **volcanoes**.

- Tidal friction effects on Europa are weaker than on Io, but still produce striking results. The icy crust of the moon is covered in 'cracks' due to **tidal stresses**, and beneath the crust it is thought frictional heating results in a thin **ocean layer**





Interior structure of the Galilean Moons



Structure of the Galilean Moons

- Their mean density **decreases** with distance from Jupiter
- The fraction of ice which the moons contain **increases** with distance from Jupiter

This is because the heat from 'proto-Jupiter' prevented ice grains from surviving too close to the planet. Thus, Io and Europa are mainly rock; Ganymede and Callisto are a mixture of rock and ice.

- The **surface** of the Moons reflects their formation history:

Io: surface continually renewed by volcanic activity. No impact craters

Europa: surface young (< 100 million years), regularly 'refreshed' – hardly any impact craters

Structure of the Galilean Moons

- Their mean density **decreases** with distance from Jupiter
- The fraction of ice which the moons contain **increases** with distance from Jupiter

This is because the heat from 'proto-Jupiter' prevented ice grains from surviving too close to the planet. Thus, Io and Europa are mainly rock; Ganymede and Callisto are a mixture of rock and ice.

- The **surface** of the Moons reflects their formation history:

Ganymede: Cooled much earlier than Io and Europa. Considerable impact cratering; also 'grooves' and ridges suggest history of tectonic activity

Callisto: Cooled even earlier; extensive impact cratering