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High Energy Astrophysics II: Example Sheet 4

This example sheet contains a mixture of numerical and algebraic problems, drawing upon material from both HEA1 and HEA2.

1. Ultra high energy cosmic rays appear to arrive at the Earth isotropically, after being deflected by the Galactic magnetic field, with $B = 10^{-10}\,\mathrm{T}$. Using the formula for the gyroradius of a highly relativistic proton, estimate the energy (in eV) at which this gyroradius becomes greater than the radius of the Galaxy (equal to 10 kpc). Compare your answer with an estimate of the kinetic energy of a tennis ball, served at 100 km/h.

2. Calculate the velocity of a 2 keV electron, and the temperature of a thermal distribution of electrons with this root mean square velocity. Thus, calculate the typical number of inverse Compton scatterings in this thermal distribution required to boost the frequency of a CMBR photon from 100GHz to 150GHz.

3. Verify that, if the observed flux of the 511 keV annihilation line from within 5 degrees of the Galactic Centre is 9.9 photons $\mathrm{m^{-2}\,s^{-1}}$, this implies a line luminosity of $L_{GC} \sim 7 \times 10^{29}\,\mathrm{W}$ and a positron injection rate of $\sim 10^{43}\, e^{+}\mathrm{s^{-1}}$. (Take the distance to the Galactic Centre to be 8.5 kpc). Assuming that the line emission comes from a spherical volume, estimate the mean density of positrons in this region required to match the implied injection rate. Explain why this annihilation radiation from the Galactic Centre is unlikely to be from massive stars, core-collapse supernovae or cosmic ray interactions with the interstellar medium. Verify that, in order to explain the origin of this radiation as Type Ia supernovae, we require a SNIa rate of roughly 0.5 per century.

4. Explain why observations of classical X-rays require the use of rockets or satellites, while hard X-rays may be observed using balloon platforms. The star Sco X-1 delivers a number flux of 10^2 hard X-rays per square metre per second at the Earth's atmosphere. If the distance of Sco X-1 is 700 pc, estimate the luminosity of Sco X-1 in hard X-rays.

5. The galaxy cluster Abell 901 emits X-rays in the energy range 0.1 – 2.4 keV, predominantly due to thermal bremsstrahlung from the intra-cluster gas, which is at a mean temperature of 3×10^{7} K. The energy flux of X-rays in this energy range detected at the Earth is $3.1\times10^{-17}\,\mathrm{W\,m^{-2}}$ and the cluster lies at a distance of 720 Mpc. Assuming that Abell 901 is a spherical cluster of radius 22 kpc, and the intra-cluster gas is an isothermal plasma of fully ionised hydrogen, estimate the number density of protons in the intra-cluster gas.

6. It is given that the luminosity of an ultra-relativistic electron, emitting synchrotron radiation in a constant magnetic field, is proportional to the square of its Lorentz factor, γ. Show that, for this electron, γ decays with time according to the equation

$$\gamma(t) = \gamma_0 (1 + C\gamma_0\, t)^{-1}$$

where γ_0 is the Lorentz factor of the electron at time $t = 0$ and C is a constant.

7. The Chandra satellite observes a classical X-ray flux of $3\times10^{-16}\,\mathrm{Wm^{-2}}$ from the X-ray transient MXB 159-29. Show that this flux implies a classical X-ray luminosity of

$$L = 3.6\times10^{24}\, d^{2}\ \mathrm{W}$$

where d is the distance, in kiloparsecs, of MXB 159-29. The spectrum of MXB 159-29 can be fitted by a blackbody with $kT = 0.3\,\mathrm{keV}$. Hence, estimate the distance of MXB 159-29, assuming that MXB 159-29 is a neutron star of radius 10km.

8. Explain clearly the differences between Thomson scattering, Compton scattering and Inverse Compton scattering. Under what conditions can the Inverse Compton cross-section be approximated by the Thomson cross-section? A homogenous volume, V, is filled with ultra-relativistic electrons of energy $\gamma m_e c^2$ and number density n_e, and Cosmic Microwave Background photons of frequency, v, and energy density U_v. Show that the timescale for each electron in the volume to lose its energy, via Inverse Compton scattering of the CMB photons, is given by

$$\tau \approx \frac{9\pi\,\varepsilon_0^2\, m_e^3\, c^5}{2\gamma U_v\, e^4}$$

Calculate this timescale for 1 GeV electrons.