University of Glasgow

Department of Physics and Astronomy

Session 2006-07: A345M Gravitation & Relativity II

Welcome to Gravitation and Relativity II. In these 10 lectures we will apply the mathematical and physical machinery developed in Gravitation and Relativity I to explore how general relativity influences a range of astrophysical phenomena – from the interior structure of stars to the geometry of the Universe.

The good news (well, depending on your point of view!) is that I won't be introducing any more new mathematical methods in this course; GR-I has already equipped you with all the tools of tensor algebra and calculus that you'll need to make sense of the syllabus of GR-II.

The maybe not-so-good news is that we will be making extensive use of these mathematical tools throughout GR-II, so if you haven't yet had the chance to get on top of the material covered in GR-I then it's best not to wait too long before you start. To help with this, I will begin my GR-II notes by providing a summary of the key mathematical and physical ideas which you should have brought with you from GR-I. I will also make available example sheets (some of which were incorporated in Norman's examples) covering the material of GR-I, which I have used when I have taught GR-I in the past. These may help with your revision by giving you a larger pool of example questions with which to hone your tensor skills!

I also intend to provide detailed printed notes for each section of the course. These should not be seen as a substitute for attending the lectures themselves; nor should they discourage you from consulting other textbooks to supplement the lecture material. I will follow quite closely the relevant content of Bernard Schutz' textbook "A First Course in General Relativity" (henceforth referred to as 'Green Schutz') and it is highly recommended – although not essential – that you take a look at Green Schutz. As Norman has also pointed out, however, it's very useful in studying GR to consult as many different sources as possible.

Note that throughout the course I will generally adopt the same metric signature as Green Schutz – i.e. writing the Minkowski metric of special relativity (with $c = 1$) as:-

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

This signature may differ from the signature adopted by Dr Gray in GR-I, but if so this should be seen as a virtue rather than some sinister plot to confuse you. It is important to appreciate that the choice of metric signature is simply a convention, and that one should be comfortable with different conventions: e.g. $(-, +, +, +)$ and $(+, -, -, -)$. Note also that some authors introduce a new imaginary time coordinate, ict, in order to obtain a $(+, +, +, +)$ metric signature. What we learn from studying relativity is that space and time coordinates are merely frame-dependent labels, so we should not get too attached to a particular signature or to a particular choice of coordinate system. Indeed, when we consider later in GR-II the spacetime inside the event horizon of a black hole, we will see that the 'everyday' meanings of these coordinate labels can be completely misleading.

Martin Hendry

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Course Website: `http://www.astro.gla.ac.uk/users/martin/teaching/gr2.html`

GR-II: Course Contents

1. **Resumé of Gravitation and Relativity Part I**

   Foundations of GR; properties of manifolds; transformation laws for tensors; the metric tensor; covariant differentiation, parallel transport and geodesics; the energy-momentum tensor; the Riemann-Christoffel and Ricci tensors; conservation of energy and momentum; Einstein's field equations.

2. **Static Models with Spherical Symmery**

   Orthogonal metrics; spherically symmetric metrics in curved spacetime; evaluation of the Christoffel symbols and components of the Ricci tensor; derivation of the Schwarzschild metric.

3. **The Schwarzschild Metric and Classical Tests of General Relativity**

   Geodesics for the Schwarzschild metric; identification with planetary orbits; classical tests of General Relativity: advance of pericentre, gravitational light deflection – applications to gravitational lensing and binary pulsars.

4. **Einstein's Equations for Static Spherically Symmetric Stars**

   Derivation of the Oppenheimer-Volkhoff equation; outline of a general numerical solution and derivation of the exact solution for constant density.

5. **Gravitational Waves**

   Linearisation of Einstein's Equations for a weak gravitational field; establishment of the wave equation for gravitational radiation; example of plane gravitational radiation – its quadrupole nature and forms of polarisation; example of a binary star system.

6. **Black Holes**

   The infall of particles and photons towards the Schwarzschild horizon; behaviour of the coordinate time and radial coordinate inside and outside this horizon; new form of the metric and interpretation of the spacetime diagram; rotating Black Holes and Frame Dragging; Hawking radiation.

7. **GR and Cosmology (time permitting)**

   The cosmological principle and derivation of the Robertson-Walker metric; Einstein's cosmological constant.