Astronomy A3/A4M, Physics P4M

Gravitation and Relativity II: What's Examinable?

Here is a list of what material I expect you to know – and, perhaps more importantly, *not* to know – from GR-II for the May/June 2007 degree exam. I've tried to make the list as complete and exhaustive as possible, and to avoid any ambiguities. If there are any 'grey areas' that you're still concerned about (i.e. what does he mean by *that*?) then please don't hesitate to ask.

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Chapter 2

- You should know what is meant by an **orthogonal metric** and be able to show that if the components of the metric tensor are orthogonal in one coordinate system they are not necessarily orthogonal in another
- You should be able to derive the results of Eqs. (2.12) – the form of the Christoffel symbols for an orthogonal metric. That means that you need to know the expression for the Christoffel symbols expressed in terms of the partial derivatives of the metric components – Eq. (1.22). See also supplementary notes on this, to help you remember where to put the indices!
- You **don't** need to remember the numerical values of the conversion factors for geometrised units, although you should know that they follow from setting $G = c = 1$
- You should be familiar with the form of a spherically symmetric spacetime – Eq. (2.22) – but you *don't* need to be able to derive this equation
- You should also know that the metric for a static spherically symmetric spacetime can be written in the form of Eq. (2.23), and thus you should be able to compute the Christoffel symbols for such a metric (Eqs. 2.24)
- You should be able to write down the Riemann-Christoffel tensor in component form – Eqs. (1.36) and (1.38) – and hence, by contracting, the Ricci tensor – Eq. (2.25). You do *not* need to remember the components of the Ricci tensor in the form of Eqs. (2.26) – (2.29)
- Given the components of the Ricci tensor in the form of Eqs. (2.26) – (2.29), you should be able to derive the Schwarszchild metric in the form of Eq. (2.46), reproducing the physical argument of Sect. 2.4

Chapter 3

- Given Eq. (3.1), you should be able to derive Eq. (3.11), the geodesic differential equation for r, for a material particle. You should also be able to derive Eq. (3.52) – the equivalent expression for a photon
- You **don't** need to be able to derive the Newtonian planetary orbit solution of Sect. 3.2, although you should know that the Newtonian case has solution of the form Eq. (3.16), so that you can derive Eq. (3.20) and spot the additional GR term of $3Mu^2$
- Given the Newtonian solution, Eq. (3.17), you should be able to substitute into Eq. (3.20) and obtain the approximate GR solution of Eq. (3.26) and show that Eqs. (3.28) – (3.30) are particular integrals. You should then be able to produce Eq. (3.35), showing that advance of perhelion occurs
- You **don't** need to be able to derive the Newtonian light deflection result of Eq. (3.48), although you should be able to derive the GR result – Eq. (3.61) – and be aware that it is twice the Newtonian value
- You should be able to derive the expression for the angular radius of the Einstein Ring – Eq. (3.67)
- You should be able to derive Eq. (3.75) for the gravitational redshift

- Given the Schwarzschild metric, you should be able to derive Eqs. (3.86) and (3.89), in the weak field limit. You should then be able to derive Eq. (3.95), for the Shapiro Effect time delay

Chapter 4

- You **won't** be asked to derive the curvature scalar – Eq. (4.11) – for the Schwarzschild metric
- Given the components of the Ricci tensor (Eqs. 4.2 – 4.5), the Einstein tensor (Eqs. 4.13 – 4.16) and the Einstein equations (Eqs. 4.26 – 4.28), you should be able to derive: Eq. (4.33); Eq. (4.35) and (finally) Eq. (4.45) – the Oppenheimer-Volkoff equation
- You should be able to solve the Oppenheimer-Volkoff equation for a constant density star, to obtain an expression for $P(r)$ and $m(r)$, and an expression for the central pressure, P_0

Chapter 5

- You should be able to show that a nearly flat spacetime is still nearly flat under a background Lorentz transformation and a gauge transformation
- You **won't** be asked to derive Eq. (5.29), for the Einstein tensor in a weak gravitational field, nor to derive Eq. (5.35), Einstein's equations for a weak gravitational field. What you should know, however, is that we can always find a gauge transformation such that Einstein's equations can be written in the form of Eq. (5.35)
- You should know that Einstein's equations in free space can be written in the form of a wave equation – Eq. (5.40) – thus establishing that gravitational waves, 'ripples' produced by the metric perturbations, propagate at the speed of light
- You **won't** be asked to derive any of the results relating to the form of the plane-wave amplitudes in the transverse-traceless gauge; you should know, however, that it is possible to write the amplitude components in the form of Eq. (5.62), so that the metric perturbation amplitude depends on just A_{xx} and A_{xy}
- You **won't** be asked to derive Eq. (5.70). You should, however, know that in the transverse-traceless gauge a test particle will remain 'at rest' as a gravitational wave passes – meaning that the coordinate system adjusts itself to remain attached to the particle
- Given Eq. (5.75), you should be able to derive Eqs. (5.85) and (5.86), differential equations for the geodesic deviation of a ring of test particles, and given Eqs. (5.87) and (5.88) show that these are solutions of these equations. You should also know that these solutions correspond to two distinct polarisation states
- You **don't** need to know how the polarisation states for gravitational waves fit into the general theory of quantum fields, and how the results for gravitons compare with those for neutrinos and photons
- You should know that the amplitude of gravitational waves – like all wave phenomena – is inversely proportional to the distance from the source, so that typical gravitational wave amplitudes are exteremely small
- You should know that, because of the conservation of linear and angular momentum, there can be no dipolar gravitational radiation. You **don't** need to be able to derive this result, however
- You should also know that spherically symmetric metric perturbations don't produce gravitational waves

Chapter 6

- Given the Schwarzschild metric, you should know that the elapsed proper time and coordinate time for a test particle to reach the Schwarzschild radius are finite and infinite respectively – i.e. using Eqs. (6.8) and (6.10)
- You should be able to show that no particle can be stationary inside the Event Horizon
- Given the coordinate transformation Eq. (6.13), you should be able to derive the null cones, Eq. (6.15), and thus show that even photons are carried to smaller coordinate radii inside the Event Horizon
- Given the Schwarzschild expression for the gravitational redshift, you should be able to derive Eq. (6.24), and thus show that a black hole will dim rapidly due to gravitational redshift
- You **won't** be examined on anything from Sect. 6.6 and 6.7 (although you may want to read them for fun!)