Dept of Physics and Astronomy
Astronomy345M, 2006-07

Gravitation and Relativity II

Aims and Objectives

Chapter 2: **Static Models with Spherical Symmetry**

Aim: To understand the mathematical and physical principles which underpin the derivation of the Schwarzschild metric.

Objectives: By the end of the course students should be familiar with, and able to apply in the context of algebraic and numerical problems:

- The definition and properties of an orthogonal metric
- The general form of the metric for a static, spherically symmetric spacetime (S4)
- How to compute the Christoffel symbols for S4
- Given the components of the Ricci tensor for S4, how to derive the form of the Schwarzschild metric.

Chapter 3: **The Schwarzschild Metric and Classical Tests of GR**

Aim: To understand the classical predictions of GR – advance of pericentre, light deflection, gravitational redshift and time delay – and how they differ from Newtonian gravity.

Objectives: By the end of the course students should be familiar with, and able to apply in the context of algebraic and numerical problems:

- How to derive and solve the equation of motion for the Newtonian 2-body planetary orbit.
- How to derive the equivalent expression for GR.
- The approximate GR solution of a precessing ellipse, and its rate of precession
- How to derive and solve the quasi-Newtonian light deflection formula for a photon passing close to a point mass
- How to derive the equivalent expression for GR, and in particular the origin of the extra factor of two
- The equation defining the Einstein ring radius of a gravitational lens, and the numerical value of this radius for galactic and cosmological cases
- How to derive an expression for gravitational redshift within the framework of the Schwarzschild metric
- How to derive an expression for the Shapiro effect, for the gravitational time delay in terms of the Newtonian gravitational potential

Chapter 4: **Einstein's Equations for Static Spherically Symmetric Stars**

Aim: To understand the GR description of the interior structure of a star, and how it differs from the classical, Newtonian description.

Objectives: By the end of the course students should be familiar with, and able to apply in the context of algebraic and numerical problems:

- Given the components of the Einstein tensor and energy-momentum tensor for the interior of a spherically symmetric star, how to solve Einstein's equations to obtain the Oppenheimer-Volkoff equation
- How the OV equation reduces to the equation of hydrostatic equilibrium, for the case of a weak gravitational field
- How to solve the OV equation exactly for a star of constant density
- Limits on the radius of static stars within the framework of GR, and their extension to non-static stars via Buchdahl's Theorem.

Chapter 5: **Gravitational Radiation**

Aim: To understand the origin of gravitational waves as metric perturbations of a non-stationary spacetime, and to understand qualitatively the implications of this result for their detection and generation.

Objectives: By the end of the course students should be familiar with, and able to apply in the context of algebraic and numerical problems:

- The definition of a non-stationary spacetime and a weak gravitational field and 'nearly flat' spacetime
- The definition of background Lorentz and gauge transformations
- The approximate form of Einstein's equations in a nearly flat spacetime, expressed in the Lorentz gauge
- How the free-space solution of these equations is a wave equation propagating at the speed of light
- Qualitatively, how the adoption of the transverse traceless gauge reduces to only two the number of free parameters in the wave solution
- The effect of gravitational waves on the proper distance between test particles, for the 'plus' and 'cross' polarisation states
- How to derive the amplitude and frequency of gravitational waves produced by a binary neutron star system.

Chapter 6: **Black Holes**

Aim: To understand the physical nature of static black holes, within the framework of the Schwarzschild metric of GR.

Objectives: By the end of the course students should be familiar with, and able to apply in the context of algebraic and numerical problems:

- The nature of the Schwarzschild surface as a coordinate singularity, so that a test particle released from outside the Schwarzschild radius will reach it in a finite proper time (but infinite coordinate time as viewed by a distant observer)
- The behaviour of photons and material particles inside the event horizon, and in particular the geometry of null cones – meaning that

photons and material particles cannot escape once they have crossed the event horizon

- The effect of gravitational redshift on black holes, and the timescale over which they ‘switch off’ as their stellar progenitor collapses inside the event horizon
- The effect of frame dragging for a rotating black hole
- Qualitatively, the phenomenon of Hawking radiation and the Hawking lifetime of a black hole.

Chapter 7: **GR and Cosmology**

Aim: To understand how GR provides a natural theoretical framework for some of the key results of observational cosmology.

Objectives: By the end of the course students should be familiar with, and able to apply in the context of algebraic and numerical problems:

- How GR encapsulates the cosmological principle
- How to derive the Robertson-Walker metric, as the most general form for a spherically symmetric metric consistent with the cosmological principle
- Given Einstein tensor for the Robertson-Walker metric and the energy-momentum tensor for the cosmic fluid, how to derive the Friedmann equations.
- Qualitatively, how Einstein’s cosmological constant was introduced to provide a static universe solution, and why it failed.