Statistical Astronomy I: A3/A4 Lectures, October 2000

COURSE CONTENT (10 lectures)

Sec. 1: Mathematical Building Blocks

1.1 *The theory of probability.* Probability as the relative frequency of outcomes. Law of Addition. Conditional Probability. Law of multiplication.

1.2 *Statistical independence.*

1.3 *Probability distributions.* Discrete random variables; Poisson distribution. Continuous random variables. Cumulative distribution function. Uniform and normal distributions.

1.4 *Expectation and other measures of a distribution.* Expected, or mean, value. Median. Mode. Variance. Skewness. Kurtosis. Variance of a function of random variable.

1.5 *Variable transformations.*

1.6 *Probability integral transform.*

1.7 *Multivariate distributions.* Joint PDF of two or more random variables. Marginal distributions. Conditional distributions. Bayes' theorem.

1.8 *Statistical independence revisited.*

1.9 *The bivariate normal distribution.*

Sec. 2: Statistical Building Blocks

2.1 *The sampling distribution.*

2.2 *Parameter estimation.* Definition of a statistic. Estimators. Bias of an estimator. Risk of an estimator. The sample mean. The law of large numbers. The central limit theorem.

2.3 *The principle of maximum likelihood.* Likelihood functions. Maximum likelihood estimators.

2.4 *The principle of least squares.* Least squares fits to straight lines. Linear regression.

2.5 *Goodness of fit.* The χ^2 distribution.

2.6 *Fitting general models.*

Sec. 3: Hypothesis Tests

3.1 *Simple hypothesis tests.* Test statistics. Null and alternative hypotheses. Critical regions.

3.2 **Incorrect decisions**. Type I and II errors. Power of a hypothesis test.

3.3 *Level of significance.*

3.4 *Two-tailed tests.*

3.5 *Goodness of fit for discrete distributions.* The binomial distribution.

3.6 *Are two distributions equal?.* The Kolmogorov-Smirnoff test.

3.7 *The student's 't' test.*

3.8 *Testing the difference of means.*

3.9 *Testing the ratio of variances.*

3.10 *Testing hypotheses on the correlation coefficient.*

Sec. 4: Point and Interval Estimation

4.1 *Defining confidence intervals.*

4.2 *Interpreting confidence intervals.*

Martin Hendry, October 2000.

Introduction

An experimental science such as astronomy involves measuring many diverse types of physical quantities (e.g. apparent magnitudes and diameters, colours, parallaxes, wavelengths). These quantities can often, in turn, be used to infer the values of other physical quantities which we *cannot* measure directly (e.g. luminosities, intrinsic sizes, temperatures, distances, velocities). This inference process generally involves making certain model assumptions. For example, to infer the distance of a star of a given spectral type, we might assume that **all** stars of that spectral type have the same absolute magnitude. We can then deduce the star's distance by combining the measured *apparent* magnitude with the assumed *absolute* magnitude.

The inference process is often made difficult because the physical quantities which we observe cannot be measured with arbitrary precision. In addition, the model assumptions which we make will only be valid to some given level of precision. For example, stars of the same spectral type will not have **exactly** the same absolute magnitude, and moreover the observed apparent magnitude of a given star will be subject, at some level, to a **measurement error**. These two effects mean that our inferred distance for the star will also be uncertain.

The aim of this course will be to develop the necessary tools to model the errors and uncertainties which arise in this inference process. The theory which describes the mathematical nature and behaviour of errors is known as **probability theory**. The branch of mathematics which describes how errors affect the process of inferring physical quantities from observational data is known as **statistics**. In Sections 1 and 2, we will study the 'building blocks' of probability and statistics respectively. In Section 3 we will then discuss in detail how we can use statistics to test our physical theories and determine, for example, which of two competing theories (or 'models') is in better agreement with a particular set of observational data. We call this process **hypothesis testing**. Finally in Section 4 we will consider a method of estimating inferred quantities which is very widely used in the astronomy and astrophysics literature (and indeed throughout the literature of almost any quantitative science); the method of estimation based on **confidence intervals**.