Astronomy A3/A4H

Statistical Astronomy I: Example Sheet 5

(In questions involving Hypothesis tests, if the significance level is not explicitly stated then you should choose the level yourself)

1. The Baade-Wesselink method is used to estimate the distance, R, and radial peculiar velocity, U, of 9 galaxies containing type II supernovae with the following results.

R (kms^{-1})	8030	1054	3251	1856	3025	5493	6049	4733	3891
U (kms^{-1})	1312	-247	872	532	-427	1052	-138	47	926

Calculate the sample correlation coefficient, $\hat{\rho}$. Assuming R and U to have a bivariate normal pdf, test the hypothesis that the radial peculiar velocities and galaxy distance estimates are independent.

2. Let X and Y be random variables. Find expressions for the following in terms of the variance and covariance of X and Y.

   (a) var(aX) and var(aY), where a is a constant.

   (b) cov(aX, aY), where a is a constant.

   (c) cov(X,$X+Y$)

   (d) cov($X+Y$,$X-Y$)

   Show that var($X+Y$) = var(X) + var(Y) + 2 cov(X,Y)

   What property of the correlation coefficient, ρ, is indicated by your solution to (a) and (b)?

3. A particular model for galaxy formation predicts, from numerical simulations, a correlation coefficient, $\rho = 0.95$, between absolute magnitude and colour for spirals galaxies. This correlation is tested with a sample of 21 galaxies in a cluster of known distance. A sample correlation coefficient, $\hat{\rho} = 0.5$, was obtained from these data. Can we reject at the 1% level the null hypothesis that absolute magnitude and colour are uncorrelated? Using the same significance level, now test the null hypothesis that the *true* correlation coefficient, $\rho = 0.95$, as suggested by the numerical simulations.

4. For a sample of 24 dwarf nova outbursts the sample correlation coefficient of the outburst amplitude and duration is computed to be $\hat{\rho} = 0.75$. Using the normal approximation to the sampling distribution of $\hat{\rho}$, can we reject at the 5% significance level the null hypothesis that the *true* correlation coefficient, ρ, is as small as (a) $\rho = 0.6$, (b) $\rho = 0.5$?

5. The following cluster data are used to fit a linear model to the Tully Fisher relation between absolute magnitude, M, and log of the line width of the HI 21cm line, P, for spiral galaxies.

P	2.71	2.05	2.67	2.23	2.36	2.52	2.91	2.43	2.27	2.84
M	-21.1	-19.2	-20.6	-19.4	-20.0	-20.2	-21.5	-19.8	-19.2	-20.9

Use the method of least squares to determine the equation of the best-fit straight line under the linear model:-

$$M_i \quad = \quad a + b\,P_i + e_i \qquad (1)$$

Assuming that the residuals, e_i, are normally distributed with mean zero and dispersion, $\sigma = 0.17$, determine errors for the least squares estimates of a and b. Construct a χ^2 test to determine if these data give an acceptable fit to the linear model of equation (1) at the 1% and 5% significance levels.

6. The angular diameter, θ, of the expanding photosphere of a type II supernova is observed simultaneously with two different telescopes, A and B, at a number of epochs, with the following results (in arc seconds):-

A	101.8	102.8	111.0	113.5	114.4	114.8	114.5	116.2	120.2	123.5
B	99.2	103.1	114.8	111.6	110.1	110.3	110.7	114.3	117.6	119.2

It is suspected that, due to incorrect flat-fielding, the results from telescope B differ systematically from those of telescope A. The following model is constructed:-

$$\theta(A)_i \quad = \quad \alpha + \beta\,\theta(B)_i + e_i \qquad (2)$$

where α and β are constants and the errors, e_i, are normally distributed with mean zero and dispersion, $\sigma = 1.7$.

Determine least squares estimates, with errors, for α and β with this model. Use the χ^2 statistic to test the goodness of fit of the model to the data at the 1% significance level.

Use the F statistic to test the hypothesis that the linear relationship between $\theta(A)$ and $\theta(B)$ can be adequately described simply by a constant scale factor (i.e. with $\alpha = 0$) against the alternative hypothesis that the relationship also involves a non-zero systematic offset, α, as given by equation (2).

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