Astronomy A3/A4H

Statistical Astronomy I: Example Sheet 2

1. The random variables X and Y have joint pdf given by:-

$$p(x,y) = e^{-y}, \quad 0 < x < y < \infty, \quad \text{zero elsewhere}$$

   (a) Determine the marginal pdf of X and Y

   (b) Determine the conditional pdf of X given Y and Y given X

   (c) Are X and Y independent random variables?

2. The random variables, X_1 and X_2, have joint pdf given by:-

$$p(x_1,x_2) = 12x_1x_2(1-x_2), \quad 0 < x_1 < 1, 0 < x_2 < 1, \quad \text{zero elsewhere}$$

   Show that X_1 and X_2 are statistically independent

3. If the random variables, X_1 and X_2 have joint pdf given by:-

$$p(x_1,x_2) = 2e^{-x_1-x_2}, \quad 0 < x_1 < x_2, 0 < x_2 < \infty, \quad \text{zero elsewhere}$$

   show that X_1 and X_2 are statistically dependent

4. Let X have the pdf given by:-

$$p(x) = x^2/9, \quad 0 < x < 3, \quad \text{zero elsewhere}$$

   Find the pdf of $Y = X^3$

5. Let X have the pdf given by:-

$$p(x) = 2xe^{-x^2}, \quad 0 < x < \infty, \quad \text{zero elsewhere}$$

   Find the pdf of $Y = X^2$

6. Let X have a uniform pdf over the interval $(-\pi/2, \pi/2)$.

   (a) Show that $Y = \tan X$ has pdf (known as the *Cauchy* distribution) given by:-

$$p(y) = \frac{1}{\pi(1+y^2)}, \quad -\infty < y < \infty$$

   (b) Determine the mean and variance of Y

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