Astronomy 2 – Special Relativity
Tutorial question for week 9

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Question

A rocket starts at rest at a space station, and accelerates along the x-axis at a constant rate α in its own frame S'. The rocket's acceleration 4-vector is therefore $A' = (0, \alpha, 0, 0)$. Using the (inverse) Lorentz transformation equations, calculate the components of the rocket's acceleration vector in the space station's frame, and confirm that $\mathbf{A} \cdot \mathbf{A} = \alpha^2$ in this frame also. Write down the rocket's velocity 4-vector in this frame (use units in which $c = 1$), and confirm that $\mathbf{U} \cdot \mathbf{A} = 0$. Differentiate this with respect to the proper time, to obtain

$$\frac{\mathrm{d}\mathbf{U}}{\mathrm{d}\tau} = \gamma \left(\frac{\mathrm{d}\gamma}{\mathrm{d}t}, \frac{\mathrm{d}}{\mathrm{d}t}(\gamma v), 0, 0 \right).$$

Hence deduce that $\gamma v = \alpha t$, and thus that

$$\frac{1}{v^2} = \frac{1}{\alpha^2 t^2} + 1.$$

If, at time t, the rocket sets off a flashbulb which has frequency f' in its frame, use the Doppler formula to show that the light is observed, at the spacestation, to have frequency

$$f = f'(\sqrt{1 + \alpha^2 t^2} - \alpha t).$$

What is this factor if the flashbulb is set off at time $t = 3/(4\alpha)$?

[Version 1.1, 2000/11/30 16:29:38]