Astronomy 2 – Special Relativity
Tutorial Question for week 5

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2 November 2000

Question

Show that the speed, relative to the platform, of the trains in figure 1 of part 3 of the lecture notes is $v = 1/3$. Remember that the train carriage is 6 m long, and that the clocks are showing times in units of metres. Hint: what two events are simultaneous in the platform frame?

Show that the time shown in figure 2 is correct.

Model answer

Let events ① and ② be the events 'rear clock shows 3' and 'front clock shows 1' respectively. These therefore have coordinates $(t'_1, x'_1) = (3\,\mathrm{m}, -3\,\mathrm{m})$ and $(t'_2, x'_2) = (1\,\mathrm{m}, 3\,\mathrm{m})$. These events are simultaneous in the platform frame, so $t_1 = t_2$. From the inverse LT, Eqn. (3.12), we therefore have

$$\gamma(t'_1 + vx'_1) = \gamma(t'_2 + vx'_2)$$

(in units where $c = 1$). Rearranging, and inserting the numerical values gives $v = 1/3$.

The carriage has length, in its rest frame, of $2L_0 = 6\,\mathrm{m}$. By length contraction, the length of the carriage, as measured in the platform frame, is $2L = 2L_0/\gamma$. Let event ③ be the event located at the rear clock, when it is a distance L further along the platform (ie, in the location in figure 2 of the notes). In the platform frame, this event has coordinates (t_3, x_3).

Since the carriage is travelling at speed v, we must have

$$t_3 = t_1 + \frac{L}{v},$$

and

$$x_3 = x_1 + L,$$

and we wish to know what the coordinate t'_3 is (since this is the time shown on the moving clock in figure 2). From the LT, we know

$$\begin{aligned} t'_3 &= \gamma(t_3 - vx_3) \\ &= \gamma\left(t_1 + \frac{L}{v} - v(x_1 + L)\right) \\ &= \gamma(t_1 - vx_1) + \gamma L\left(\frac{1}{v} - v\right) \\ &= t'_1 + L_0\left(\frac{1}{v} - v\right). \end{aligned}$$

Putting in the numbers, we therefore find that $t'_3 = 11$, as shown on the diagram.