



DEPARTMENT OF  
PHYSICS AND ASTRONOMY

# ASTRONOMY 2

# Laboratory Handbook

Session 2004-2005



# Astronomy 2 Laboratory

|                          |                                                                                 |
|--------------------------|---------------------------------------------------------------------------------|
| Laboratory Head          | Dr Martin Hendry, Room 607 ext 5685                                             |
| Laboratory Demonstrators | TP1: Ms Jennifer Toher, Mr John Veitch<br>TP2: Mr Ross Galloway, Mr John Veitch |
| Laboratory Technician    | Mr Matt Trainer                                                                 |
| Observatory Technician   | Mr Colin Hunter                                                                 |

## 2.1 Timing & Location

The class meets on six Friday afternoons in each of Teaching Periods 1 and 2 at the Observatory at Acre Road, off Maryhill Road. There will be a catch-up lab available after each term's allocation, so that students can finish off any incomplete experiments. Attendance is from 2.30 to 5.30 p.m. The dates for each term will be notified in the A2Z Timetable.

## 2.2 Objectives of the course

The aim of the class is to introduce various techniques of astronomical measurement and of data analysis. It is intended that the class will develop further your skills for making measurements, for analysis of data, for working co-operatively with others and for the presentation of written reports. Specifically, the practical sessions are designed to

- illustrate and expand upon material presented in the lecture courses, and, where required, introduce new material relevant to the education of a professional astronomer.
- demonstrate the physical principles underlying observational astronomy, from data collection to data processing and interpretation
- impart skills in the deployment and use of instrumentation, astronomical software and techniques appropriate to detection and measurement of various physical and astronomical quantities
- assist the clear, concise and accurate reporting of a performed experiment

Note that the labs are not merely an adjunct to the lectures, but are instead a valid, independent teaching medium in which new concepts can be introduced in a practical way. Given the timing of the labs, some material may be required to be learned in advance of its presentation in the lectures. The best way to tackle such labs is to prepare before starting the exercise, and so students may wish to find out in advance to which experiment they will next be assigned.

## 2.3 Laboratory Assessment

### *Working in pairs*

You will work in pairs and are required to complete four experiments, with three afternoons allowed for each. It is expected that each member of a pair will co-operate with the other to ensure that the

work is shared equitably and carried out efficiently. This will help develop the personal skills and attitudes which are required to operate within a group.

There have been occasional cases in the past in which a student has only attended on one of the three afternoons allocated to an experiment but has handed in a record or report for the experiment and claimed credit for having carried out all of the work. This is clearly not satisfactory and it will be necessary for each student to demonstrate that he or she was present during the carrying out of an experiment and participated fully in it before being able to claim credit for it. However, due allowances will be made for illness or other unavoidable circumstances which might cause absence from the practical class.

### *Demonstrators*

There will be two demonstrators and the lab head (Martin Hendry) at each session. The laboratory technician (Matt Trainer) and observatory technician (Colin Hunter) will also be present and can offer advice on particular equipment etc. Please don't hesitate to ask if there is anything unclear about the experiments. The staff will come round each experiment periodically to find out how things are going and may ask to see your lab record, so please keep it up to date. The lab head will judge whether or not an experiment has been completed on the basis of your lab notes; the record books will be marked by the lab head or the demonstrators. You may be allowed to proceed to another experiment before this happens, depending on the prevailing circumstances, but your aim should always be to complete the record of your previous experiment, and get it marked, *before* moving on to the next one.

### *Laboratory Records*

**You each must have a suitable lab book, namely a bound A4 book including graph paper.** The experiments performed have to be written up fully as lab records, using your lab book. This book will be used to record ALL data, to graph results and to record conclusions. Although it does not have to be pristine, it must however describe clearly all measurements undertaken, including a full disclosure of all data and equipment used, and must state equally clearly the results and conclusions of the experiment you performed. Sufficient theory should be included to make the account clear and understandable; you may wish to make additional notes relevant for the laboratory report. There is sufficient time allocated within the laboratory sessions to enable you to complete a lab record; normally you will not proceed to another experiment without presenting a definitive record of an experiment. Remember the key points:

1. Read the lab sheets, and make sure you know what you are required to do; ask a demonstrator if it's not clear.
2. Write a brief description of the experimental objectives in your lab book before you start to carry out any experiments or record any data!
3. Record your data thoroughly, and plot all graphs as you take the data; this will help you identify poor data, and allow you to repeat measurements if necessary.
4. Make sure your graph fits the paper: scan through the likely range of measurements, and scale the graph accordingly.
5. **IMPORTANT:** Quantify the uncertainties (errors) of your measurements and your final results
6. Make sure you complete all aspects of the experiment
7. Present your results clearly, and summarise the performance of the experiment.

## Marking Scheme

The marking scheme for a lab record is usually as follows:

| Mark | Guide to Lab Record Grading Criteria                                                                                                                                                                                                                            |
|------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 0-3  | lab not completed or largely incomplete; inadequate performance.                                                                                                                                                                                                |
| 4-5  | lab substantially complete, adequate results, but below average interpretation of results; rushed attempt, with for example, poor quality data not recognised as such                                                                                           |
| 6-7  | lab completed, with a competent set of results, and a reasonable attempt at interpreting the data and quantifying errors                                                                                                                                        |
| 8-10 | a very good lab performance producing a sound result; clear understanding evident, in addition to careful data taking and a thorough quantification of errors. Placing results in a wider context, and optimal performance of the experiment earn higher marks. |

Note that the neatness or otherwise of the lab record will not feature significantly in the assessment; as long as the lab is legible, the criteria above will be applied to the content, rather than the presentation. If you have any queries about the mark you received for a particular experiment, don't hesitate to raise it with the lab head or one of the demonstrators.

## Laboratory Reports

Two of your completed experiments must be written up fully as reports as part of the “continuous assessment” element of the course; guidelines about what is expected from the reports are included in this document. One report will be required in each of Teaching Periods 1 and 2, to be handed in at the beginning of **Week 15 (24/01/05)** and **Week 23 (11/04/05)** respectively. Each lab report is marked out of 20.

## Laboratory Sheets

Detailed descriptions of each experiment are maintained as Laboratory Sheets, available from the demonstrators, and from the laboratory website. Since the experiments are continually being updated and improved, the lab sheets are held centrally to avoid inappropriate notes being distributed. Please retain your lab sheets for future reference, but don't use them to record data!

## 2.4 Experiments

The A2Z laboratory experiments change from year to year, depending on the size of the class and the availability of observational equipment and computing resources. The general philosophy is that, over the course of the year, all students should acquire a range of experience by carrying out experiments that develop different skills. These skills include:-

- Data acquisition and recording
- Operating experimental hardware and computer simulation software
- Carrying out quantitative data analysis (using specialist software, where appropriate)
- Designing experimental procedures to test specific hypotheses
- Presenting and interpreting quantitative results
- Assessing critically possible sources of experimental or observational error

Below is a brief summary of the range of A2Z experiments available at the Garscube Observatory:

### 1. The Cepheid Period-Luminosity Relation and the Extragalactic Distance Scale

This is a computer based experiment and involves estimating the periods of a sample of Cepheid variable stars, with the aim of verifying the *Period-Luminosity* relation for these stars. It demonstrates simple methods for obtaining the period from time series data, and illustrates the concept of the *standard candle* – used in estimating extragalactic distances.

### 2. Photometry and Spectroscopy of the Pleiades

This is a computer based experiment in which students simulate the operation of a 40cm telescope, and use it to measure apparent magnitudes and acquire spectra of stars in the Pleiades open cluster. These data are then used to construct a *colour-magnitude* diagram for the cluster. From this diagram students determine a *main sequence*, and compare this with the theoretical main sequence to estimate the distance of the Pleiades.

### 3. Detection of Extra-Solar Planets

In this computer based experiment students compare high resolution stellar spectra with spectral templates to determine the *Doppler shift* of a star, due to the gravitational pull of an unseen planet as they orbit their common centre of mass. From analysis of how the Doppler shift changes with time students can estimate the orbital period – and hence the mass – of the planet.

### 4. Filter Photometry and the Magnitude System

The aim of this experiment is to introduce and achieve familiarity with some of the concepts associated with the Johnson magnitude system and the measurements of stellar magnitudes. Specifically, it introduces *bolometric correction*, *colour index* and *colour excess* via a laboratory simulation of stellar observations.

### 5. Stellar Classification and Interstellar Reddening

This experiment is an internet-based spectral classification exercise. Students construct *colour-colour* diagrams from real data for several open clusters, and classify the spectral type of a sample of cluster stars. Analysis of the colour-colour diagrams is then carried out to estimate the interstellar reddening of each open cluster.

## **6. Atomic Lines**

In this experiment students carry out a detailed analysis of spectral lines from discharge lamps using a diffraction grating spectrometer. Students examine several lamps, calibrating the spectrometer, before attempting to identify the composition of a lamp containing a mixture of different elements.

## **7. Centimetre Optics**

Here microwave apparatus is used to explore transmission, reflection, refraction, diffraction, polarisation and interference of electromagnetic waves at wavelengths much greater than optical. The results are relevant to radio interferometry, as well as conventional visible optics.

## **8. Poisson Statistics**

In this experiment students will use a Geiger counter to investigate the levels of natural radioactivity at the Garscube observatory. Data will be collected continuously and recorded on computer. Students will then analyse the counts data using statistical software, and will test the hypothesis that radioactive counts follow a Poisson distribution.

## **9. Modelling Planetesimal Collisions**

Students will use a household brick, broken into pieces with a sledgehammer, to simulate the collision processes that occurred in, e.g., the very early Solar System, the formation of the asteroid belt and Saturn's rings. Students will study the distribution of fragment sizes and compare them with the *power-law* distribution predicted by theory.

## **10. Measuring Asteroid Rotation Periods**

In this experiment students will measure the brightness variations of light reflected from the irregular surface of a simulated asteroid. From analysis of the observed light curve, students will estimate the rotation period of the asteroid.

*Note that the experiments are continually being reviewed and improved, and equipment is regularly being serviced and upgraded. Consequently some of the above experiments may be temporarily unavailable, some others may be slightly changed, and some new ones may be added.*

## **2.5 Coffee Break**

There will be a 20 minute coffee/tea break in every lab, usually around 4pm. Drinks and biscuits are on sale at the Observatory. There is also a small shop close to the Observatory (turn right at the junction with Acre Road) if you want to purchase other items.

## 2.6 Safety

Students must be aware at all times of the need to work safely in the Observatory. All students are required to read:

1. the Code of Safe Practice for Undergraduates (see <http://www.physics.gla.ac.uk/~rferrier/code.html> )
2. the guidelines on Safety in the Observatory, reproduced below.

Students will be required to sign that they have received these documents, and have agreed to abide by them. (In recent years some experiments have required students to use the Observatory Darkroom for photographic development work, although these experiments – using photographic plates – have generally become obsolete with the introduction of modern electronic detectors. Additional guidelines on Safety in the Darkroom will be given out to students as and when they are required).

## 2.7 Safety in the Observatory

The attention of students is drawn to the following points. Many of these are pure common sense; that does not diminish their significance but makes non-observance of them more reprehensible.

**The First Aid Officer is Colin Hunter, in the Workshop tel ext. 30.**

**The First Aid Box is also located in the Workshop.**

1. Care must be taken when using any electrical or electronic equipment. This is particularly necessary when using high voltage apparatus, e.g. photomultipliers.
2. Under no circumstances may the sun be viewed directly through any optical aid, however small - telescope, coelostat, etc. Even momentary direct exposure of the eye to focused solar radiation will lead to permanent damage to eyesight, even blindness. So **never** look directly at the sun through a telescope.
3. Equipment in the workshop is not to be used by students. In fact students should regard the workshop as out of bounds unless they have the express permission of the laboratory supervisor or whoever is on duty at the time, or in an emergency request for first aid.
4. Telescope / Coelostat Domes
  - (i ) Telescope Dome: go up and down the stairs carefully, avoiding the upper floor beam with you head.
  - (ii) Coelostat Dome: in gaining access to the roof, make sure the ladder is secure. Take care with respect to the restricted headroom. Hard hats are available and should be worn.
  - (iii) Make sure that the hooks holding the hatches are secure, and do not use the hatches as a handrail. The hatches must not be left open when the telescope/coelostat is being used or adjusted.
  - (iv) Keep your hands clear of the rail when the dome-turning motor is in operation. In case of difficulty press the **red** button to stop the dome-turning motor.
6. Please note the location of the **fire exits** in the Garscube Observatory:
  - (i) in the library
  - (ii) at the end of the corridor between laboratories E and F.

7. Please note: the **first aid box** is in the **workshop**; the first aid officer is **Colin Hunter**, workshop technician.

8. Every experiment has an accompanying instruction sheet. Comments on safety relevant to that experiment are included in the sheet. Study the sheet carefully before beginning the experiment.

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## 2.8 GUIDELINES FOR THE WRITING OF REPORTS

The writing of laboratory reports should be treated as an important part of the work of the practical class. Not only does it describe the work which you have carried out and allows the assessment of its quality, but it also provides an essential training in the writing of papers and reports for any subsequent professional career. It is just as an important communications skill as having the ability to present a clear and well reasoned talk and should be treated as such.

Laboratory reports should normally be word-processed using facilities either at the observatory or in the departmental open access cluster. A combination of Word for Windows and Excel should provide the author with an appropriate range of tools (such as text & mathematics typesetting, data tables, graphs etc) for a professional presentation of experimental work undertaken in the laboratory.

It is difficult to provide precise rules for the writing of the laboratory reports for this class because of the wide diversity of experiments but there are some well defined guidelines which can be used and which may serve you well elsewhere at university and after you leave. In the paragraphs below is a suggested layout which satisfies the crucial requirements for a good report:

The first and most important of these is that the report must be written in a way that allows an understanding of the experiment by a person who has no detailed knowledge of the experiment but who has appropriate basic knowledge of the subject. The question to ask yourself is "Could another student in my class carry out this experiment from my report? Am I providing all the necessary information?." This is the essential feature of any satisfactory report but it does not mean that you can simply reproduce the instruction sheets provided for the experiment. Along with the results obtained, this feature of your report demonstrates the extent to which the experiment was understood.

### **Abstract or Overview**

The report should usually begin with a brief but sufficiently complete description of any underlying theory so that the reader can understand the basic principles of what was done. This can assume a reasonable basic knowledge of Astronomy and will be very brief in some cases but it is normally an essential preliminary to the description of the experiment itself. It also serves the important purpose of getting the readers mind into tune with the theory and methods involved in the experiment.

### **Experimental Apparatus and Method**

The next part will usually contain a description of the apparatus and the way in which the experiment was performed. There is no need to give a list the pieces of apparatus involved but this part should be sufficiently complete that, even without any prior knowledge of the apparatus or experiment, the reader will understand clearly what was done. There is no point in saying, for example, "The fringe positions were measured using the microscope" when neither the fringes or microscope have already

been introduced. Diagrams are often essential and certainly should be used whenever possible to make the explanation clearer. Although it might seem that these requirements will demand a large section, there is also the need to have the report as brief as possible so that all unnecessary detail has to be minimised. The essential guideline should be "Would I understand what was done in the experiment from reading the report?" and this is what you must apply.

## **Results**

The results obtained and the analysis which gives the final results and conclusions should be in the next section. There should be enough numerical information to allow the reader to reproduce the analysis but there is no need to give the arithmetical details. It is reasonable to assume that the reader will have a calculator. Error analysis should be carried out whenever possible and, if appropriate, comments should be made about the most important sources of the uncertainties in the final results. At all times, the report should demonstrate that the experimental uncertainty in each measurement was considered and that all reasonable efforts were made to assess it and to minimise it.

## **Conclusions and Discussion**

The final part gives the conclusions and any relevant comments on the experiment, including its weaknesses and limitations. Putting your results into a broader context should be done here (eg do your results agree with published data, and if not, why not?).

Obviously, there can be many variations on what has been described. If the experiment has more than one part, the pattern above might be repeated separately for each. Some experiments are computer based. In this case, there might be a large section on the theory behind the analysis carried out, a very small or even no section on the apparatus followed by a large section on the analysis. Use your common sense and, again, always think about how easily you would be able to read the report and understand it.

*Do not simply reproduce word for word the contents of the lab sheets!*

**The normal style of reports is in the third person and this should be used. However, it should not be written as a set of instructions. It is to be a description of what you did and not a recipe.**

## 2.9 Error Analysis

It is vital in experimental work to be able to quantify the accuracy of measurements. This section is a brief introduction to error analysis, enabling students to place the estimation of accuracy on a rigorous foundation. The statistical issues summarised here will be discussed in depth with individual groups over the course of the twelve lab sessions, as particular problems and questions relating to errors arise.

### ***Repeated measurements of a single quantity***

Suppose a single number (e.g. a physical quantity, such as length, mass, temperature, pressure etc) is measured repeatedly, yielding the values  $x_1, x_2, \dots, x_n$  where  $n$  is the number of measurements undertaken, each assumed to be of equal reliability. In the limit where  $n \rightarrow \infty$ , the infinite set of measurements  $\{x_i\}$ , plotted as a histogram, tends to the underlying probability distribution curve (known more precisely as *probability density function*) for the quantity,  $x$ . In reality, of course, we have only a finite set of measurements (indeed we may only have a single measurement!) and it is assumed that each observation is randomly sampled from this underlying probability density function. The best estimate of the *true* value of the quantity derived from  $n$  measurements is then simply the mean,  $\bar{x}$ , where

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i .$$

The *precision* of your experiment can be quantified by the width (usually expressed in terms of a quantity known as the *standard deviation*) of the probability distribution curve for  $x$ . We could determine the standard deviation from the histogram in the limit as  $n \rightarrow \infty$ . For the practical case of a finite number of measurements we have to make do with an estimate of the standard deviation. This is generally given by the adjusted root mean square (rms) deviation,  $s_n$ :

$$s_n = \frac{1}{\sqrt{n-1}} \sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} .$$

The statistical assumption that our measurements are sampled randomly from the underlying probability density function ensures that  $\bar{x}$  tends to the true value of  $x$  as  $n \rightarrow \infty$ . To quantify how accurate is our estimate  $\bar{x}$ , we use the adjusted *standard error on the mean*,  $S_n$ :

$$S_n = \frac{s_n}{\sqrt{n}} .$$

The result of the experiment is then quoted as

$$x = \bar{x} \pm S_n$$

### ***Combinations of measurements***

Where the desired physical quantity is not measured directly, but is instead a function of several different measurements, then the best estimate of the true value and precision of the measurement must come from a combination of the components. The following simple examples show how to do the calculations.

(1) **simple scaling by a known constant:**  $z = ax$

Assuming the usual  $n$  measurements of  $x$ ,  $\{x_i\}$  which then give rise to  $n$  measurements of  $z$ ,  $\{z_i = ax_i\}$ , then the mean value of  $z$  is just  $\bar{z} = a\bar{x}$ , and the adjusted rms deviation of  $z$  is  $s_n(z) = a s_n(x)$ , with the obvious extension of notation.

(2) **summation:**  $z = x + y$ .

Suppose we have  $n$  measurements of  $x$ , and  $m$  measurements of  $y$ , yielding  $m \times n$  measurements of  $z$ . The mean value now is given by

$$\bar{z} = \bar{x} + \bar{y},$$

and the adjusted rms deviation is

$$s_{mn}(z) = \sqrt{s_n(x)^2 + s_m(y)^2}.$$

(Question: how will the above formulae change for the case of the *difference* of two quantities?)

(3) **general formula:**  $z = f(x)$

Assuming that  $f$  is ‘well behaved’, and the usual  $n$  independent measurements of  $x$  have been taken, then the best estimate of the mean value and adjusted rms deviation of  $\bar{z}$  is

$$\bar{z} = f(\bar{x}),$$

and

$$s_n(z) = f'(\bar{x}) s_n(x).$$

More general formulae exist for estimating errors on the product or quotient of two or more measured quantities; ask the Lab Head or demonstrators for advice.

### **Method of least squares**

Very often a physical quantity that can be expressed as a single number varies with the conditions under which it is measured – i.e. with the value of some other physical quantity – and the purpose of the experiment is to find the relationship between these quantities. We shall confine attention in what follows to the linear relationship  $y = ax + b$ , where  $x$  and  $y$  are the quantities that are measured, and  $a$  and  $b$  are the quantities that the experiment is designed to determine. (Astronomical examples of such relations include the Period-Luminosity relationship for Cepheid variable stars, which can be represented as a linear relation between log (period) and absolute magnitude – see your A1Y cosmology notes for more details and e.g. the A2Z laboratory on Cepheids for a practical illustration).

We could in principle find  $a$  and  $b$  from just two pairs of measured values  $(x,y)$ , solving the pair of simultaneous equations. However, this is unlikely to lead to accurate estimates of  $a$  and  $b$  since we can expect that the measured values of  $x$  and  $y$  will be subject to experimental error, or – equally important in astronomical contexts – what is termed *intrinsic scatter*, since we don’t expect e.g. Cepheid variables to follow the PL relation exactly. A more useful approach is to measure a set  $(x_1, y_1), \dots, (x_n, y_n)$  of pairs of values (each presumably with slightly different experimental error and/or intrinsic scatter) and use *all* of these pairs of data together to estimate  $a$  and  $b$ . Because of the experimental and intrinsic errors, these pairs of data will not all lie on a single straight line, but the method of least squares provides a way to determine the line which ‘best fits’ (i.e. passes closest to) the data. In its

simplest case, we assume that the measured values of  $x$  are error-free and each observed value  $y_i$  carries a random error,  $e_i$ , drawn from a probability distribution with (known) standard deviation  $\sigma_i$ ; i.e.  $y_i = ax_i + b + e_i$ .

We will consider two cases here:

- 1)  $\sigma_i$  is identically equal for each observation (unweighted least squares)
- 2)  $\sigma_i$  is different for each observation (weighted least squares)

### ***Unweighted Least Squares***

In this case the least squares estimates of  $a$  and  $b$  are given by the following formulae

$$a_{LS} = \frac{n \sum_{i=1}^n x_i y_i - \sum_{i=1}^n x_i \sum_{i=1}^n y_i}{n \sum_{i=1}^n x_i^2 - \left[ \sum_{i=1}^n x_i \right]^2}$$

$$b_{LS} = \frac{\sum_{i=1}^n x_i^2 \sum_{i=1}^n y_i - \sum_{i=1}^n x_i \sum_{i=1}^n x_i y_i}{n \sum_{i=1}^n x_i^2 - \left[ \sum_{i=1}^n x_i \right]^2}$$

and estimates of the errors on  $a$  and  $b$  are given by

$$S_n(a) = \sqrt{\frac{n}{n-2}} \frac{\sqrt{n} \sigma}{\sqrt{n \sum_{i=1}^n x_i^2 - \left[ \sum_{i=1}^n x_i \right]^2}}$$

$$S_n(b) = \sqrt{\frac{n}{n-2}} \frac{\sqrt{\sum_{i=1}^n x_i^2} \sigma}{\sqrt{n \sum_{i=1}^n x_i^2 - \left[ \sum_{i=1}^n x_i \right]^2}}$$

i.e. the standard errors on  $a$  and  $b$  are proportional to the standard deviation of the errors on the observed  $y$  values.

### **Weighted Least Squares**

In this case the least squares estimates of  $a$  and  $b$  are given by the following formulae

$$a_{LS} = \frac{\sum_i^n \frac{1}{\sigma_i^2} \sum_{i=1}^n \frac{x_i y_i}{\sigma_i^2} - \sum_{i=1}^n \frac{x_i}{\sigma_i^2} \sum_{i=1}^n \frac{y_i}{\sigma_i^2}}{\sum_i^n \frac{1}{\sigma_i^2} \sum_{i=1}^n \frac{x_i^2}{\sigma_i^2} - \left[ \sum_{i=1}^n \frac{x_i}{\sigma_i^2} \right]^2}$$

$$b_{LS} = \frac{\sum_i^n \frac{x_i^2}{\sigma_i^2} \sum_{i=1}^n \frac{y_i}{\sigma_i^2} - \sum_{i=1}^n \frac{x_i}{\sigma_i^2} \sum_{i=1}^n \frac{x_i y_i}{\sigma_i^2}}{\sum_i^n \frac{1}{\sigma_i^2} \sum_{i=1}^n \frac{x_i^2}{\sigma_i^2} - \left[ \sum_{i=1}^n \frac{x_i}{\sigma_i^2} \right]^2}$$

and estimates of the errors on  $a$  and  $b$  are given by

$$S_n(a) = \sqrt{\frac{n}{n-2}} \frac{\sqrt{\sum_i^n \frac{1}{\sigma_i^2}}}{\sqrt{\sum_i^n \frac{1}{\sigma_i^2} \sum_{i=1}^n \frac{x_i^2}{\sigma_i^2} - \left[ \sum_{i=1}^n \frac{x_i}{\sigma_i^2} \right]^2}}$$

$$S_n(b) = \sqrt{\frac{n}{n-2}} \frac{\sqrt{\sum_i^n \frac{x_i^2}{\sigma_i^2}}}{\sqrt{\sum_i^n \frac{1}{\sigma_i^2} \sum_{i=1}^n \frac{x_i^2}{\sigma_i^2} - \left[ \sum_{i=1}^n \frac{x_i}{\sigma_i^2} \right]^2}}$$

Note that if  $\sigma_i = \sigma$ , for all  $i$ , the above formulae reduce to those for the unweighted case.

### **Number of significant figures**

Assessing correctly the number of significant figures in error analysis is an involved procedure. However a good rule of thumb is to quote your final result with the same number of significant figures as the least number of significant figures occurring in any component part of a calculation. Hence if most quantities in a calculation are known to 5 sig figs, but your measuring instrument can deliver only 4, then the final result cannot be known to better than 4 sig figs, and should be quoted accordingly, with a suitably rounded error.