PROOF

First we show that a particle inside r_s cannot remain at rest.

Recall that two neighbouring events cannot lie on the world line of a particle or a light ray if $ds^2 < 0$

(If so, then proper time, $\Delta\tau = \sqrt{ds^2}$, is imaginary)

Suppose a particle is at rest inside r_s

$$\Rightarrow dr = d\theta = d\phi = 0$$

BUT for $r < r_s$, $1 - \frac{2GM}{c^2 r} < 0 \Rightarrow ds^2 < 0$

CONTRADICTION

$$\therefore \boxed{dr \neq 0}$$

Do particles move inwards or outwards?

Solve for the NULL cones (i.e. lightlike intervals)

i.e. $ds^2 = 0$ (assuming $d\theta = d\phi = 0$)

we find $$c\frac{dt}{dr} = \frac{\pm 1}{\left(1 - \frac{2GM}{c^2 r}\right)}$$

i.e. Light cones "close up" as $r \to r_s$