# 13. Right Ascension and Declination

In order to usefully map the position of the stars, we need to assign them co-ordinates that remain essentially fixed, i.e. that do not change with the diurnal motion. With HA-dec, we were half way there: δ is constant over a diurnal rotation. However, because the co-ordinate system is fixed relative to the *Earth*, H still varies as the Earth rotates. What we need is a co-ordinate system that is fixed relative to the *sky*. Thus, as the Earth rotates, the co-ordinate grid also rotates at the same rate as the sky, so the positions of the stars relative to the co-ordinate grid remain constant.
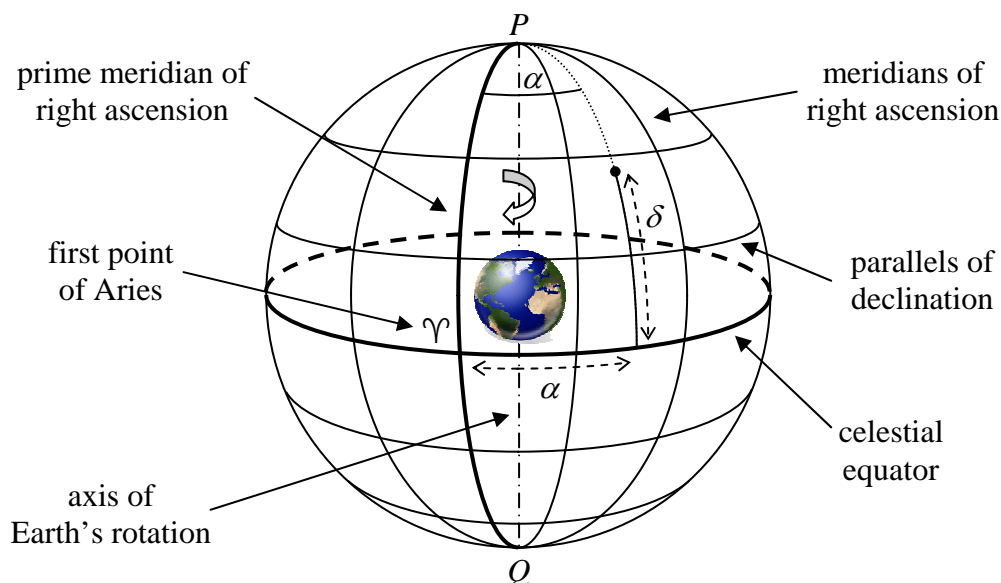
Consider a geocentric, fixed celestial sphere with the Earth rotating at its centre. Since the stars are stationary relative to this celestial sphere, we can assign them fixed co-ordinates in the same way that we assigned fixed latitudes and longitudes to locations on the surface of the Earth.

Just as with HA-dec, the celestial equator will be the projection of the Earth's equator onto this celestial sphere, with the north and south celestial poles as the poles of the co-ordinate system. Parallels of declination are defined in exactly the same way as in the HA-dec system.

With declination δ as the equivalent of terrestrial latitude, we also need an analogue for terrestrial longitude: this is the **right ascension**, α. Great half-circles passing through the north and south celestial poles form the meridians of right ascension.

Just as we use the Greenwich meridian as the prime meridian on the Earth, we need a prime meridian for the celestial sphere. For this we use the meridian passing through the point labelled ♈. This is the point of the **vernal equinox** (northern hemisphere spring equinox), also known as the *first point of Aries*. (Because this point actually wanders very slowly with time, the vernal equinox is in fact now located in the constellation of Pisces.) ♈ lies on the celestial equator.

The *declination* of a point is defined to be the arc length distance of the point from the celestial equator, measured along the meridian of right ascension passing through that point. As before, δ is zero on the celestial equator, and is positive towards the north and negative towards the south. The *right ascension* of the point is defined to be the spherical angle between the meridian passing through the point and the meridian passing through the vernal equinox. α is zero at the point of the vernal equinox and increases towards the east. (This runs opposite to the direction of increasing hour angle so that the diurnal rotation causes consistent effects in both co-ordinate systems, as we will see shortly.)

Together, α and δ form the RA-dec co-ordinate system. How will this co-ordinate system appear for the Earth-bound observer? Consider again the observer on the surface of the Earth at latitude ϕ. As discussed in Section 8, the very small stellar parallax allows us to switch freely between geocentric and topocentric co-ordinate systems, so for the terrestrial observer the declination behaves exactly like the declination in the HA-dec system. (This is why we use the same terminology for both.)

The meridians of right ascension appear similar to the meridians of hour angle, since they both run between the celestial poles. However, whereas the meridians of hour angle were fixed relative to the observer, the meridians of right ascension now *rotate* with the diurnal motion of the sky, completing one full rotation each sidereal day.
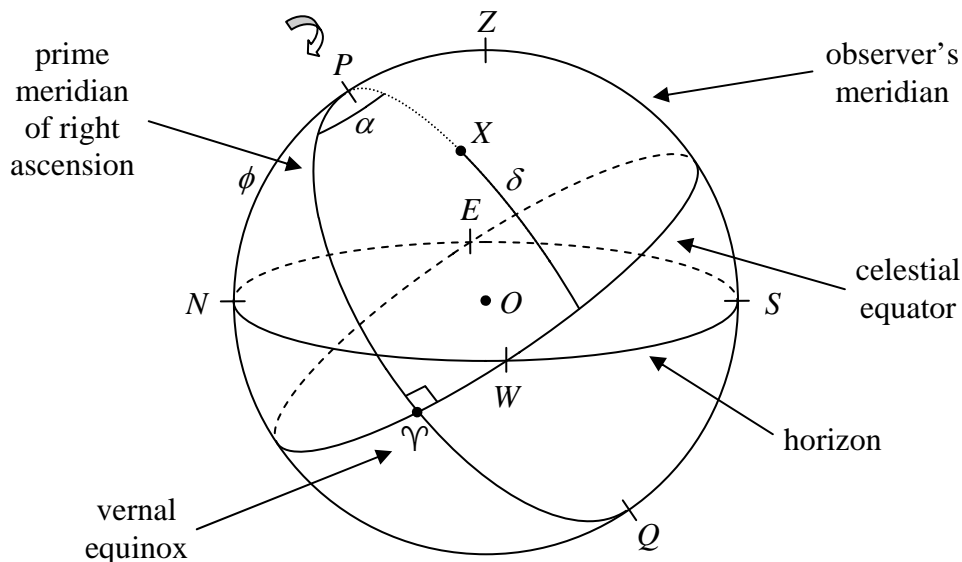
Right ascension behaves much like hour angle, but whereas hour angle was measured relative to the observer's meridian, which is fixed, right ascension is measured relative to the meridian of the vernal equinox, which rotates fully around the sky once per sidereal day. Thus, both the α and δ co-ordinates of a celestial object remain fixed, despite the diurnal motion.

Since the whole co-ordinate system appears to be steadily rotating from the perspective of the observer, it is natural also to express α as a *time*: using our conversion of $15° = 1^h$ as before, an object which is 15° further east than object *X* on the celestial sphere will have a right ascension value 1 hour greater than that of *X*, and so will rise 1 hour later, transit 1 hour later, set 1 hour later, and so on. During this time, object *X* will have increased its hour angle by 1^h: thus, the diurnal motion causes the hour angle of moving objects to increase in a positive manner, and also causes the right ascension of the objects seen passing a fixed reference point to increase in a positive manner.

It is to provide this consistency that hour angle and right ascension are measured in opposite directions around the sphere, since someone riding along with the moving meridians of right ascension would see the meridians of hour angle appear to move 'backwards' from their perspective.

Advantages of RA-dec:
- Right ascension and declination co-ordinates of an object do not change with the diurnal motion.
- RA-dec co-ordinates can be used to permanently catalogue the location of stars and deep-sky objects such as nebulae, star clusters, galaxies, etc.

Disadvantages of RA-dec:
- RA-dec co-ordinates are harder to visualise and measure than alt-az co-ordinates.
- RA-dec co-ordinates are harder to interpret in terms of position in the sky and object visibility than HA-dec co-ordinates.
- RA-dec co-ordinates are still not fixed for objects which exhibit annual motion, i.e. the Sun, planets and other solar system objects.

14. Converting between HA-dec and RA-dec

The RA-dec co-ordinate system allows us to refer to the positions of celestial objects in a way which doesn't depend on the time. However, this co-ordinate grid will appear in different positions for a terrestrial observer depending on the time of day, since it moves with the Earth's rotation. In order to determine where an object actually appears in the sky for an observer, we need to be able to convert the fixed RA-dec co-ordinates to the corresponding HA-dec co-ordinates for the observer, which will depend on the observer's location and the time of observation.

To do this, we make use of the concept of **Local Sidereal Time**.

Definition: The *Local Sidereal Time* at a given location is defined to be the current hour angle of ♈.

Definition: The *Greenwich Sidereal Time* is defined to be the current hour angle of ♈ as measured from Greenwich.
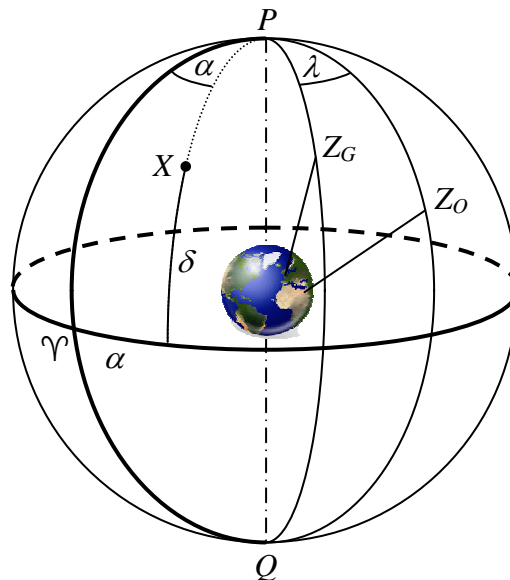
Consider an observer on the Earth's surface at point O, which is at a longitude of λ. The point Z_O is the zenith for the observer at O. Therefore, the arc PZ_OQ is the observer's meridian for the observer. The angle Z_OP♈ is the hour angle of the vernal equinox as measured from the observer's location. Therefore, for this observer the local sidereal time is given by
$$\text{LST} = Z_OP\,♈$$

Similarly, Z_G is the zenith for Greenwich and PZ_GQ is the observer's meridian for Greenwich. So the Greenwich sidereal time will be given by
$$\text{GST} = Z_GP\,♈$$

The angle Z_GPZ_O is the longitude λ of O. So
$$Z_OP\,♈ = Z_GP\,♈ + Z_GPZ_O$$
$$\text{LST} = \text{GST} + \lambda \qquad\qquad [14.1]$$

The local sidereal time at a particular location on Earth is the Greenwich sidereal time plus the location's longitude.

The angle Z_OPX will be the hour angle H of the star at position X, as measured from the observer's location. The angle ΥPX will be the right ascension α of the star. So

$$Z_OP\,\Upsilon = \Upsilon PX + Z_OPX$$
$$LST = \alpha + H \qquad\qquad [14.2]$$

For any celestial object, the object's right ascension plus the object's current hour angle is equal to the local sidereal time at the observing site. For an object at transit (i.e. $H = 0$), the local sidereal time equals the object's right ascension.

Equation (14.2) provides the link between the RA-dec and the HA-dec co-ordinate systems: if the value of α is known for an object (from a catalogue, say), and LST can be obtained from a *sidereal clock*, then H can be determined for the object. (Recall that the value of δ is the same in both co-ordinate systems.)

A sidereal clock keeps *sidereal time*, completing its full 24 hour cycle in $23^h\ 56^m\ 4^s$ (i.e. it runs slightly fast). Ordinary clocks keep civil time (basically, the hour angle of the Sun, or solar time, plus 12 hours), with a 24^h day, since human activity is most usefully synchronised with the daily movement of the Sun and the corresponding hours of daylight. Therefore, sidereal time 'drifts' relative to solar time: they are synchronised on the day of the vernal equinox (usually on or around 21st March), and are displaced relative to each other by half a sidereal day 6 months later, and a full sidereal day 12 months later. Therefore, a year is 365¼ solar days and 366¼ sidereal days long.

As can be seen from equation (14.1), each longitudinal location on Earth will have a unique local sidereal time, depending on its precise longitude. At Greenwich, LST = GST, and at a longitude of -180°, LST lags GST by half a sidereal day. (The familiar civil *time zones* are similar in principle but are based on solar time, and all locations in each zone – usually 15° wide in longitude so differing from each other by 1^h – use the same civil time.)