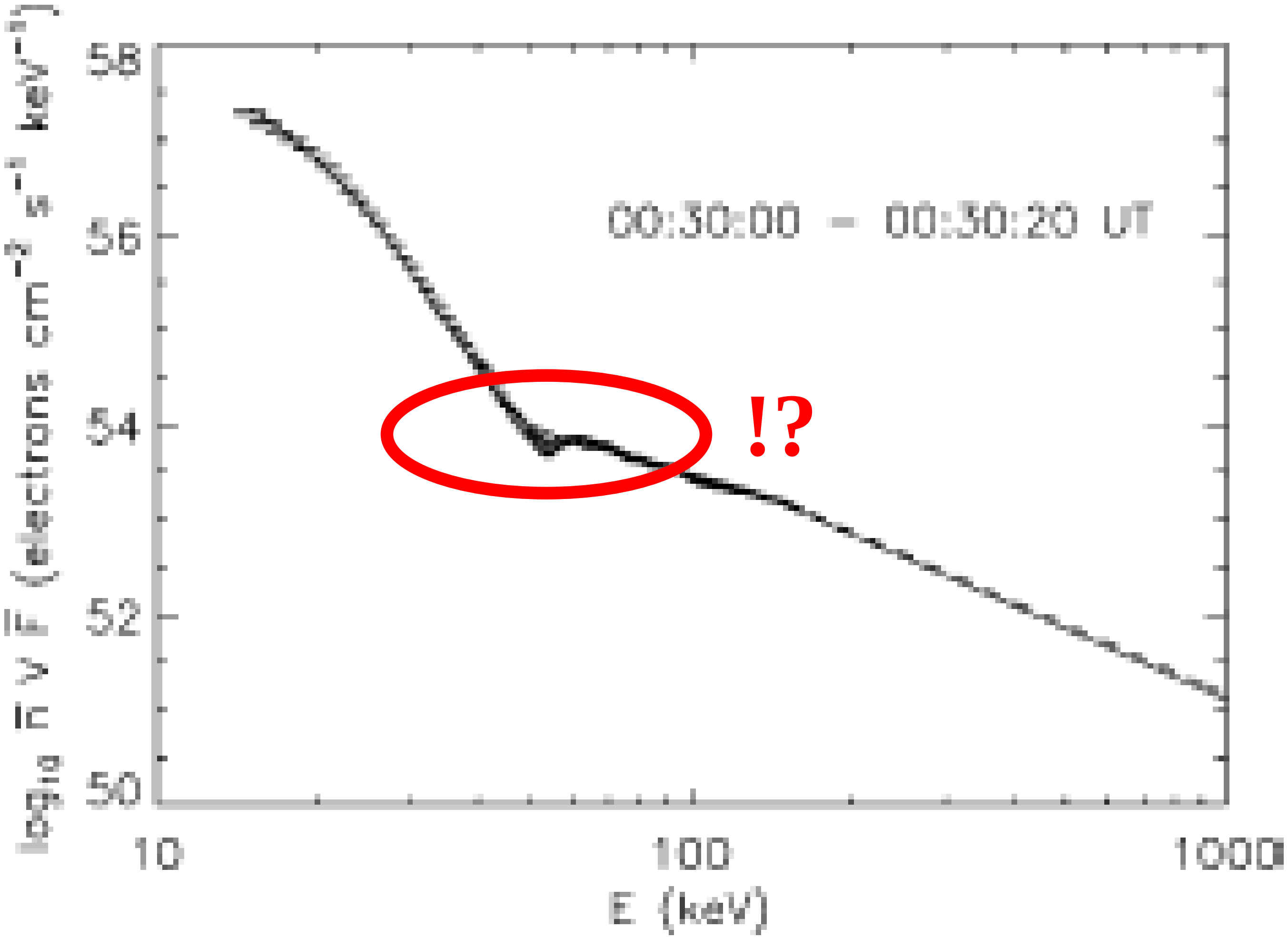
Why do we expect monotonically decreasing electron spectra in flares?

Gordon Emslie

Oklahoma State University

OSU

July 23, 2002 event (Piana et al.)



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$$I(\varepsilon) = \int_{\varepsilon} F(E)\, Q(\varepsilon, E)\, dE$$

$$dI(\varepsilon)/d\varepsilon = \int_{\varepsilon} F(E)\, [\partial Q(\varepsilon, E)/\partial\varepsilon]\, dE - F(\varepsilon) Q(\varepsilon, \varepsilon)$$

For reasonable forms of the cross section $Q(\varepsilon,E)$,

$$\partial\ Q(\varepsilon,E)/\partial\varepsilon < 0 \quad \forall E$$

So, ***photon spectra $I(\varepsilon)$ must be monotonically decreasing***

But this is ***not*** true for electron spectra $F(E)$!

$$\nu(\epsilon, E_o) = \int_0^t n\,Q(\epsilon, E)\,v\,dt = \int_\epsilon^{E_o} \frac{n\,Q(\epsilon,E)\,v\,dE}{|dE/dt|} = \int_\epsilon^{E_o} \frac{n\,Q(\epsilon,E)\,v\,dE}{n\,v\,|dE/dN|} = \int_\epsilon^{E_o} \frac{Q(\epsilon,E)}{|dE/dN|}\,dE$$

$$I(\epsilon) = \frac{A}{4\pi R^2} \int_\epsilon^\infty F_o(E_o)\,\nu(\epsilon, E_o)\,dE_o = \frac{A}{4\pi R^2} \int_{E_o=\epsilon}^\infty \int_{E=\epsilon}^{E_o} F_o(E_o)\,\frac{Q(\epsilon,E)}{dE/dN}\,dE\,dE_o$$

$$I(\epsilon) = \frac{1}{4\pi R^2} \int_{E=\epsilon}^\infty \frac{Q(\epsilon,E)}{|dE/dN|} \int_{E_o=E}^\infty A\,F_o(E_o)\,dE_o\,dE$$

$$I(\epsilon) = \frac{1}{4\pi R^2} \int_{E=\epsilon}^\infty \bar{n}V\,\overline{F}(E)\,Q(\epsilon,E)\,dE$$

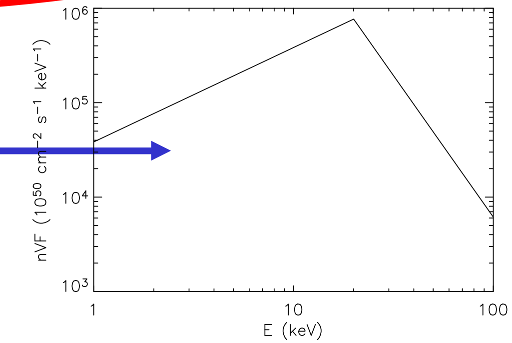
$$\bar{n}V\,\overline{F}(E) = \frac{1}{|dE/dN|} \int_{E_o=E}^\infty A\,F_o(E_o)\,dE_o = \frac{E}{K} \int_E^\infty A\,F_o(E_o)\,dE_o$$

$$\pi V \overline{F}(E) = \frac{E}{K} \int_E^\infty A\, F_o(E_o)\, dE_o$$

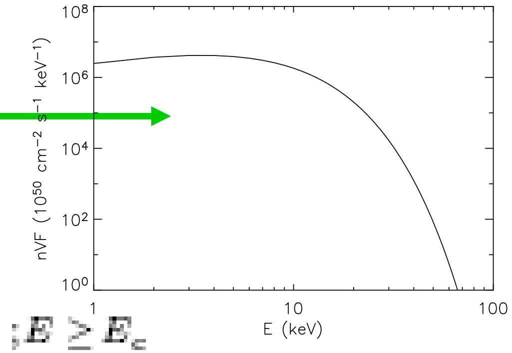
$$A\, F_o(E_o) = \begin{cases} (\delta - 1)\, \frac{\mathcal{F}}{E_c} \left(\frac{E}{E_c}\right)^{-\delta} & ; E \geq E_c \\ 0 & ; E < E_c \end{cases}$$

$$\int_E^\infty A\, F_o(E_o)\, dE_o = \begin{cases} \mathcal{F} \left(\frac{E}{E_c}\right)^{1-\delta} & ; E \geq E_c \\ \mathcal{F} & ; E < E_c \end{cases}$$

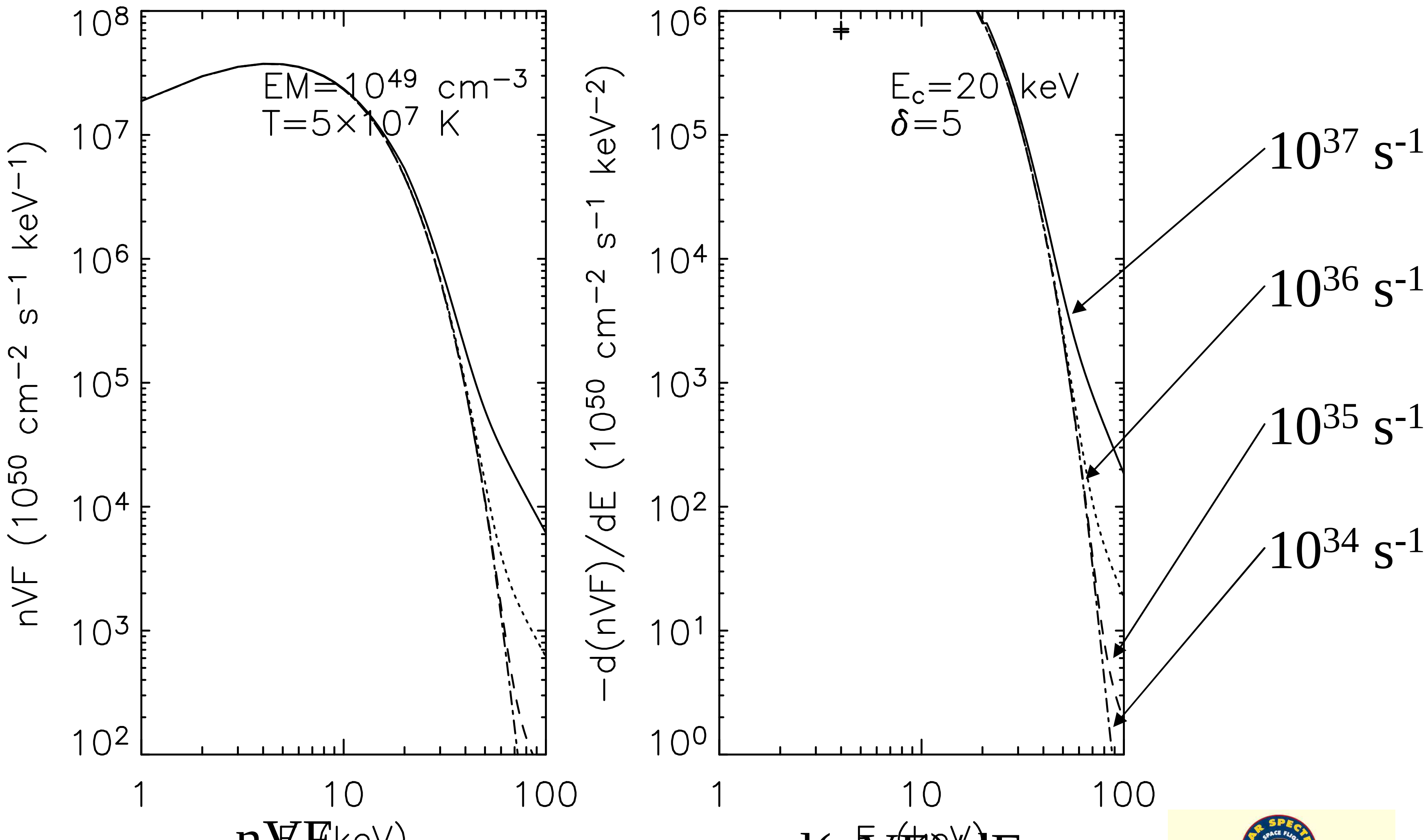
$$\left[\pi V \overline{F}(E)\right]_{\mathrm{nonthermal}} = \begin{cases} \left(\frac{\mathcal{F}}{K}\right) E_c^{\delta-1} E^{2-\delta} & ; E \geq E_c \\ \left(\frac{\mathcal{F}}{K}\right) E & ; E < E_c \end{cases}$$



$$\left[\pi V \overline{F}(E)\right]_{\mathrm{thermal}} = \sqrt{\frac{8}{\pi\, m_e}}\, EM\, \frac{E}{(kT)^{3/2}}\, e^{-E/kT}$$



$$\frac{d\left[\pi V \overline{F}(E)\right]}{dE} = \begin{cases} -(\delta - 2)\, \frac{\mathcal{F}}{K} \left(\frac{E}{E_c}\right)^{1-\delta} - \sqrt{\frac{8}{\pi\, m_e}}\, EM\, e^{-E/kT}\, \frac{1}{(kT)^{3/2}} \left(\frac{E}{kT} - 1\right) & ; E \geq E_c \\ \frac{\mathcal{F}}{K} - \sqrt{\frac{8}{\pi\, m_e}}\, EM\, e^{-E/kT}\, \frac{1}{(kT)^{3/2}} \left(\frac{E}{kT} - 1\right) & ; E < E_c \end{cases}$$



nVF

d(nVF)/dE

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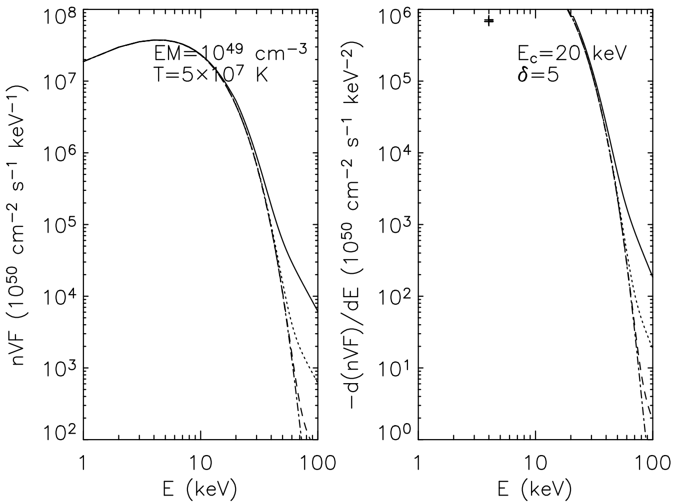
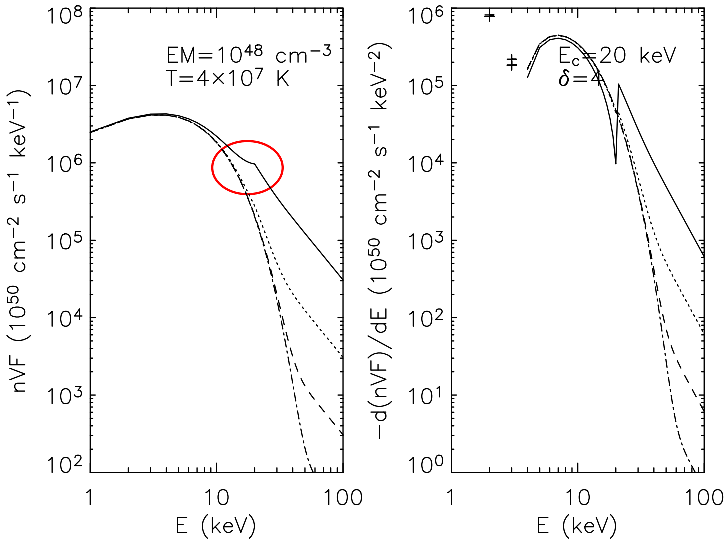
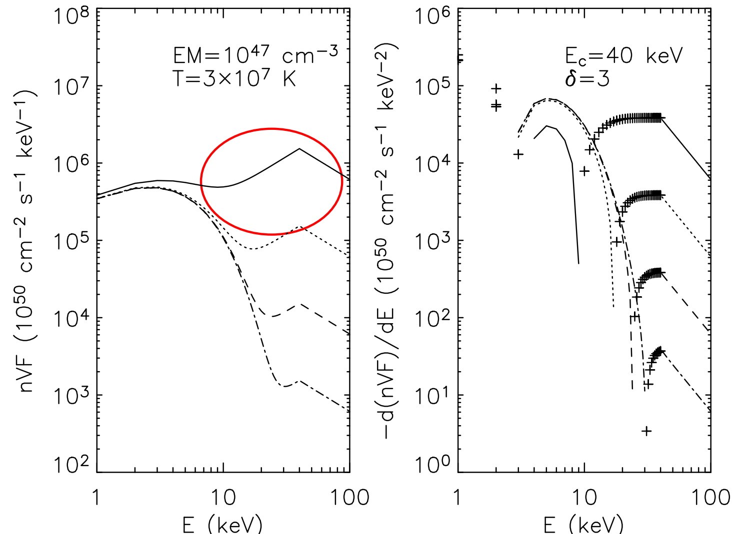
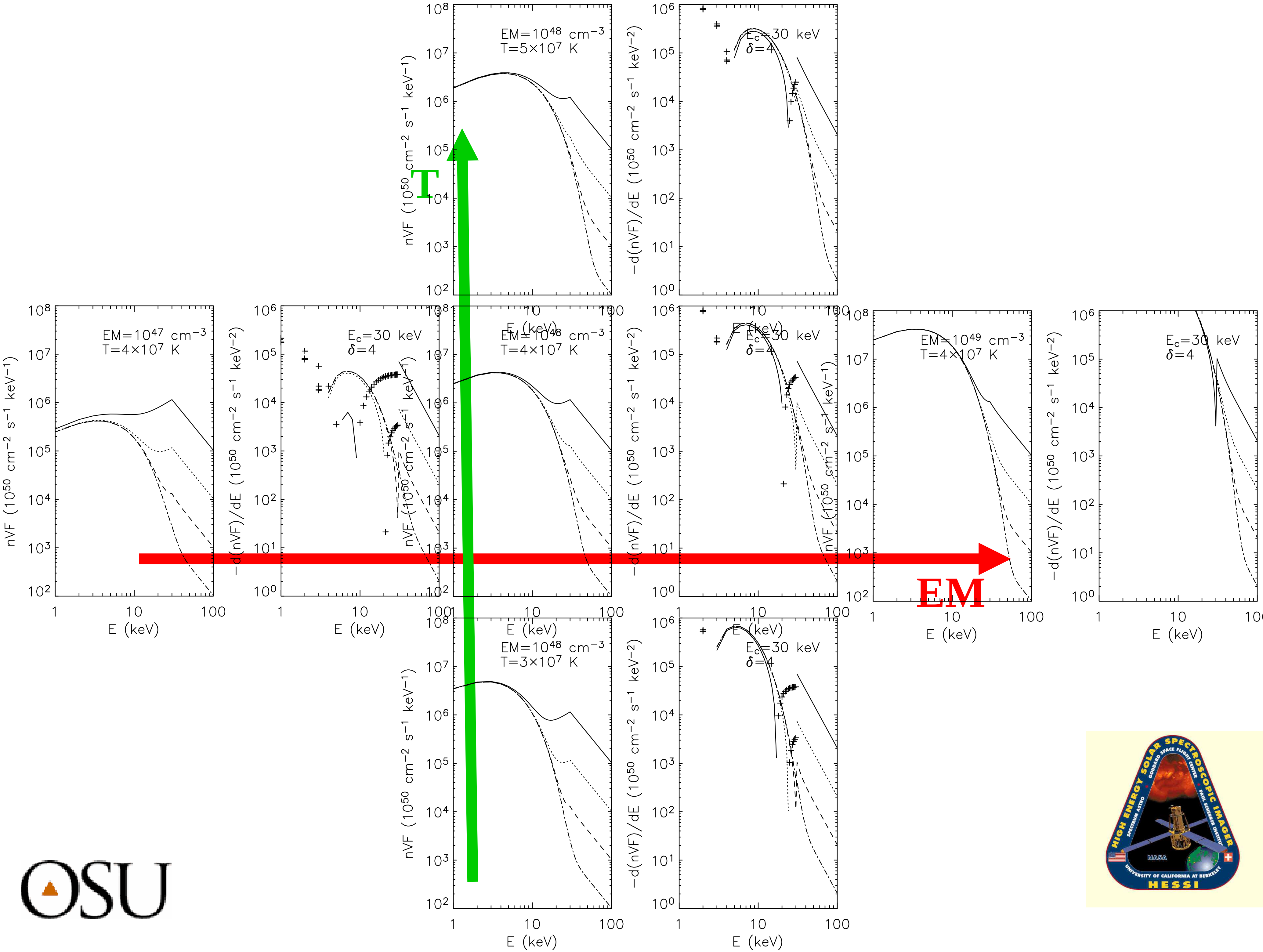


High EM, high T, low E_c, high δ



Low EM, low T, high E_c, low δ





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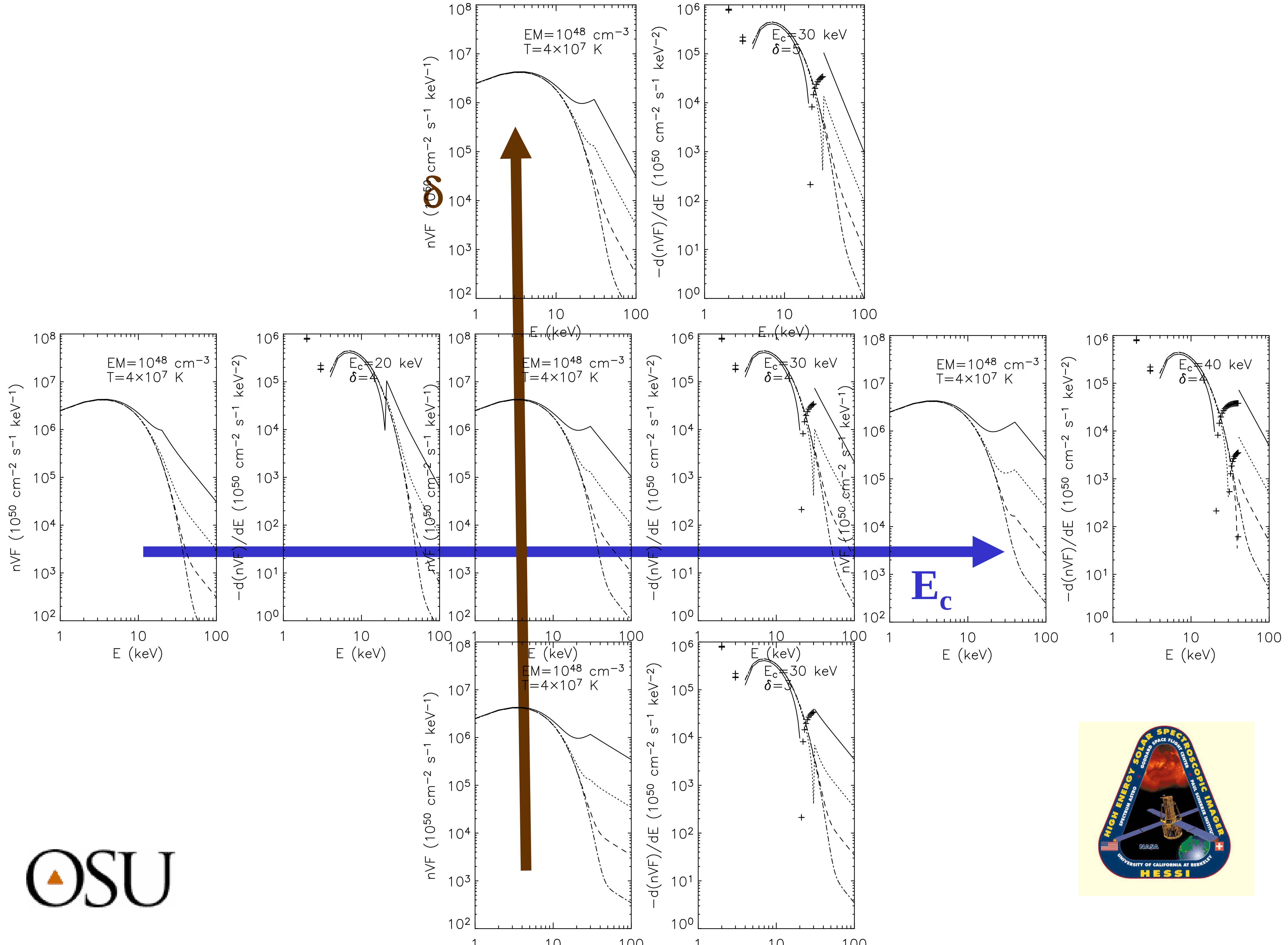
HIGH ENERGY SOLAR SPECTROSCOPIC IMAGER
HESSI



EM=10^48 cm^-3
T=4×10^7 K
E_c=30 keV
δ=5
E_c=20 keV
δ=4
E_c=30 keV
δ=4
E_c=40 keV
δ=4
E_c=30 keV
δ=3
nVF (10^50 cm^-2 s^-1 keV^-1)
-d(nVF)/dE (10^50 cm^-2 s^-1 keV^-2)
E (keV)
δ
E_c

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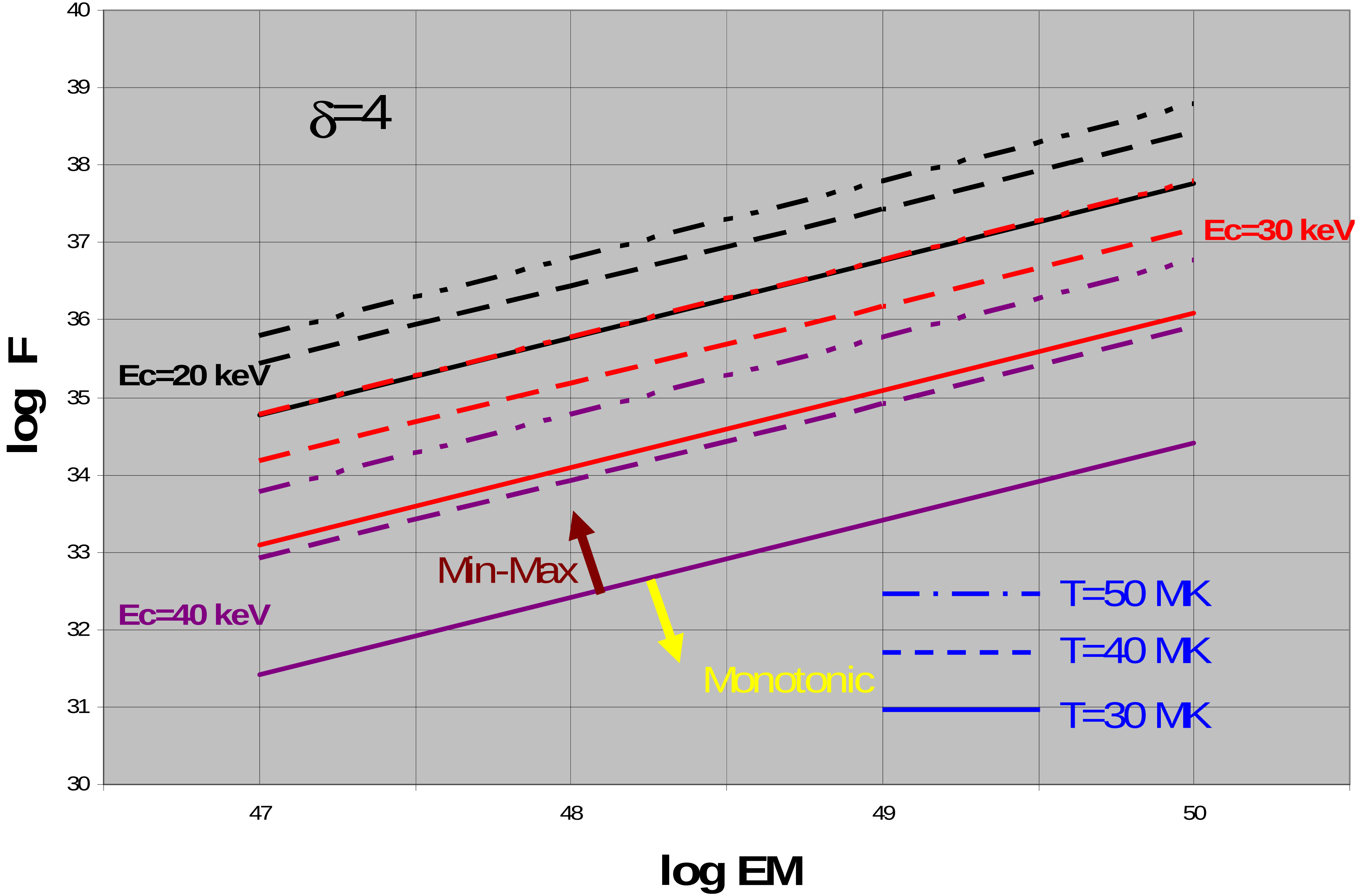
Summary



δ=4
Ec=20 keV
Ec=30 keV
Ec=40 keV
Min-Max
Monotonic
T=50 MK
T=40 MK
T=30 MK
log F
log EM
40
39
38
37
36
35
34
33
32
31
30
47
48
49
50

Conditions for a dip/hump

$$\frac{d\left[\pi V \overline{F}(E)\right]}{dE} = \begin{cases} -(\delta-2)\frac{\mathcal{F}}{K}\left(\frac{E}{E_c}\right)^{1-\delta} - \sqrt{\frac{8}{\pi m_e}}\, EM\, e^{-E/kT}\frac{1}{(kT)^{3/2}}\left(\frac{E}{kT}-1\right) & ;E \geq E_c \\ \boxed{\frac{\mathcal{F}}{K} - \sqrt{\frac{8}{\pi m_e}}\, EM\, e^{-E/kT}\frac{1}{(kT)^{3/2}}\left(\frac{E}{kT}-1\right)} & ;E < E_c \end{cases}$$

For a hump in F(E) to be present, we must generally have

$$F_{36}/EM_{48} > 7000\ T_7^{-3/2}\ \exp[-1.16E_c/T_7]$$

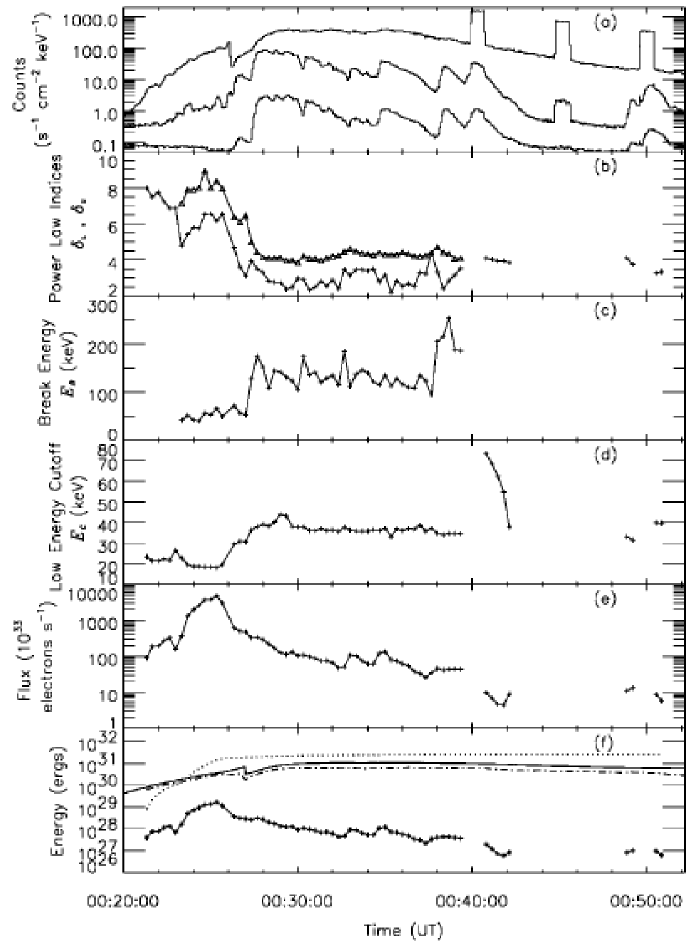
- Very sensitive to value of E_c
- Value of δ not that significant, since local thermal δ is large at $E_c >> kT$

Check

For July 23, 2002 event, Holman et al. (2003) get

$$EM_{48} = 41$$

$$T_7 = 3.7$$



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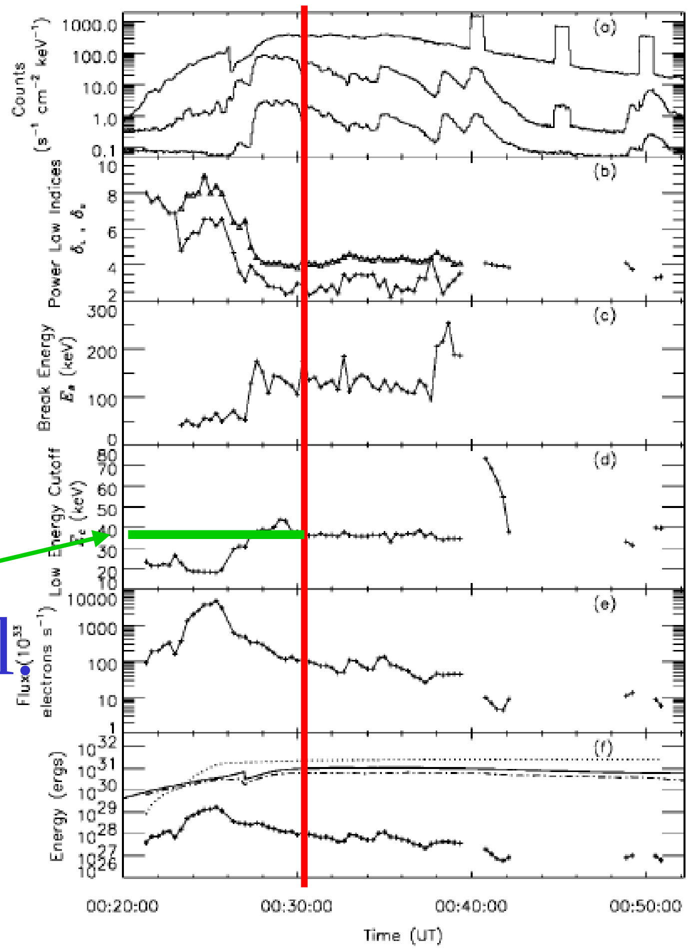
Using value of E_c from Holman et al. fit ($E_c \sim 34$ keV) gives minimum value of F_{36} = <u>0.95</u>



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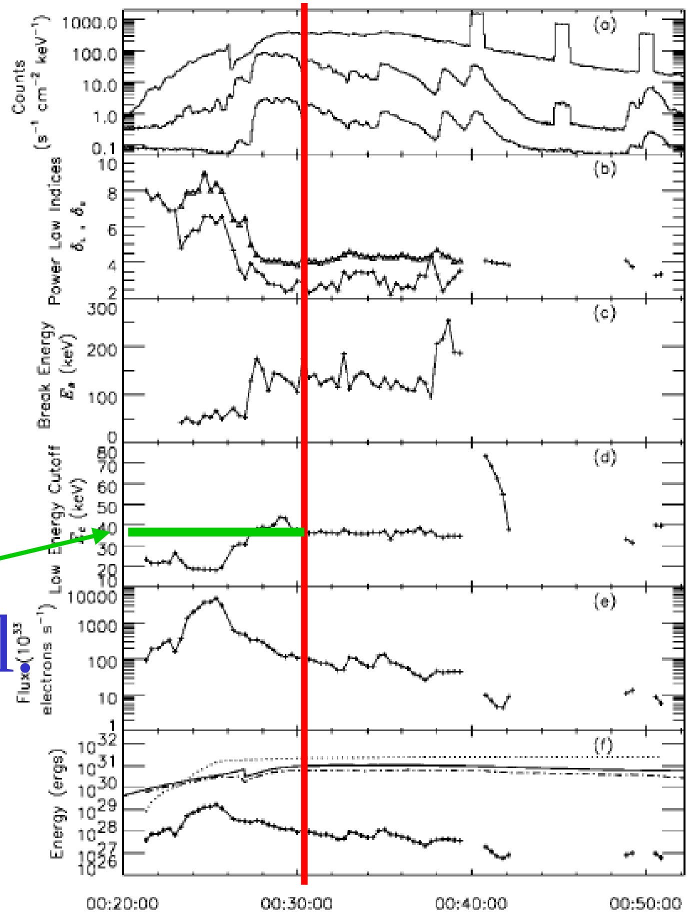
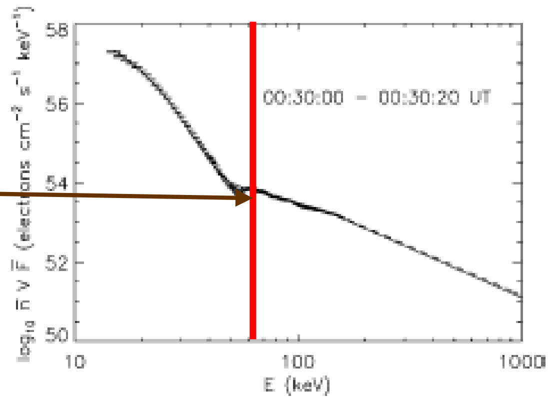
Using value of E_c from maximum in F ($E_c \sim 55$ keV) gives minimum value of F_{36} = <u>0.0013</u>





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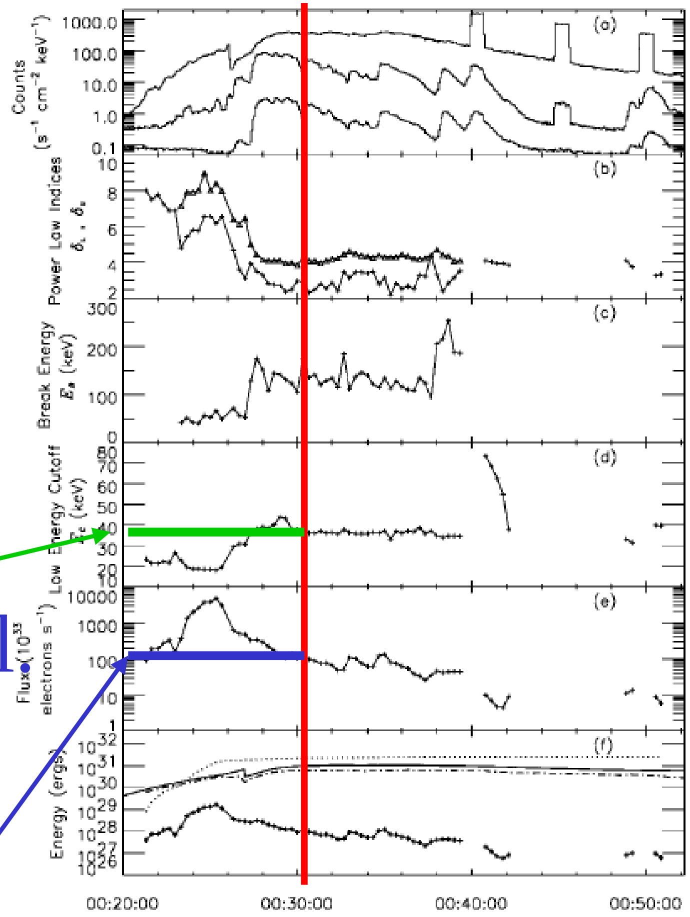
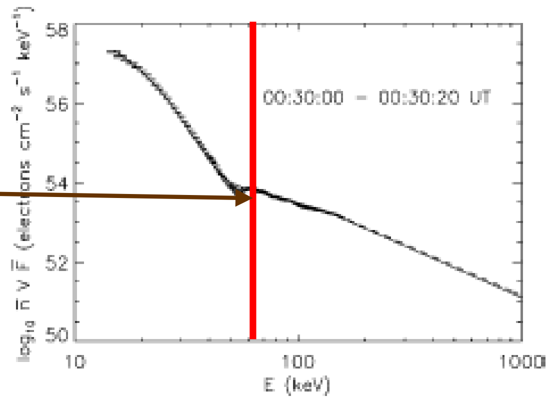
Using value of E_c from maximum in F ($E_c \sim 55$ keV) gives minimum value of F_{36} = <u>0.0013</u>

Actual value of F_{36} is ~ <u>0.1</u>

→ hump not unexpected!





Caveat

E_c may not represent an absolute cutoff:

- Form of nVF below $E = E_c$ may be different
- Value of E_c from Holman et al. fitting routine may not be applicable

Conclusions

- Humps in mean source electron spectra are not unexpected
- Dips may not be an albedo artifact
- Existence (or non-existence) of hump places constraints on F, EM and T