Theory and modelling of radio emission

Eduard Kontar

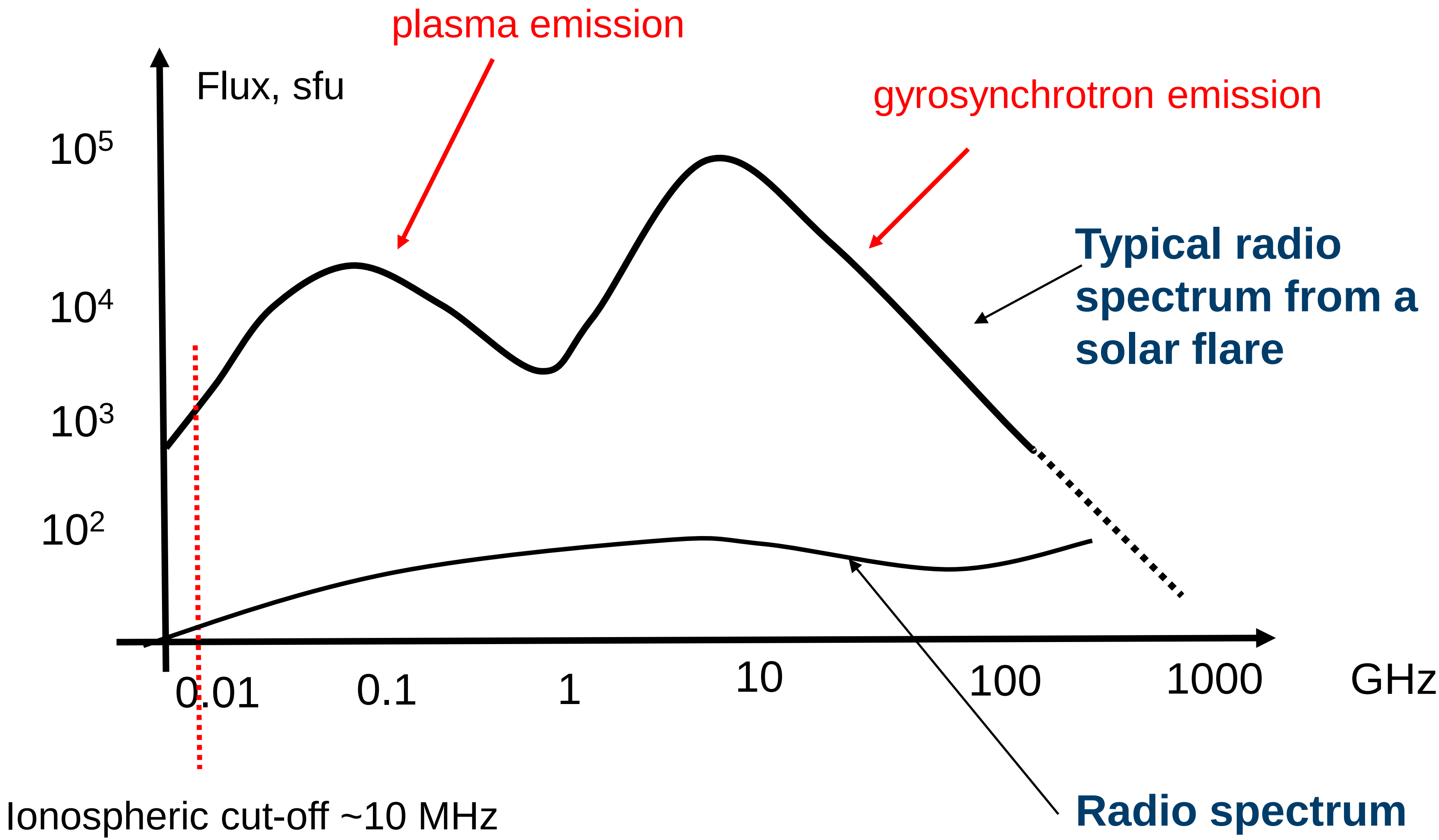
University of Glasgow, UK

2018 CESRA summer school
This presentation/publication has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 730562 [RadioNet]

Lectures outline

- Radio emission basics
- Solar radio emission: main properties
- Radio emission and energetic particles
- Kinetic equations and plasma waves
- Modelling of radio emission

Solar radio emission



1 sfu = 10^4 Jansky

Brightness temperature

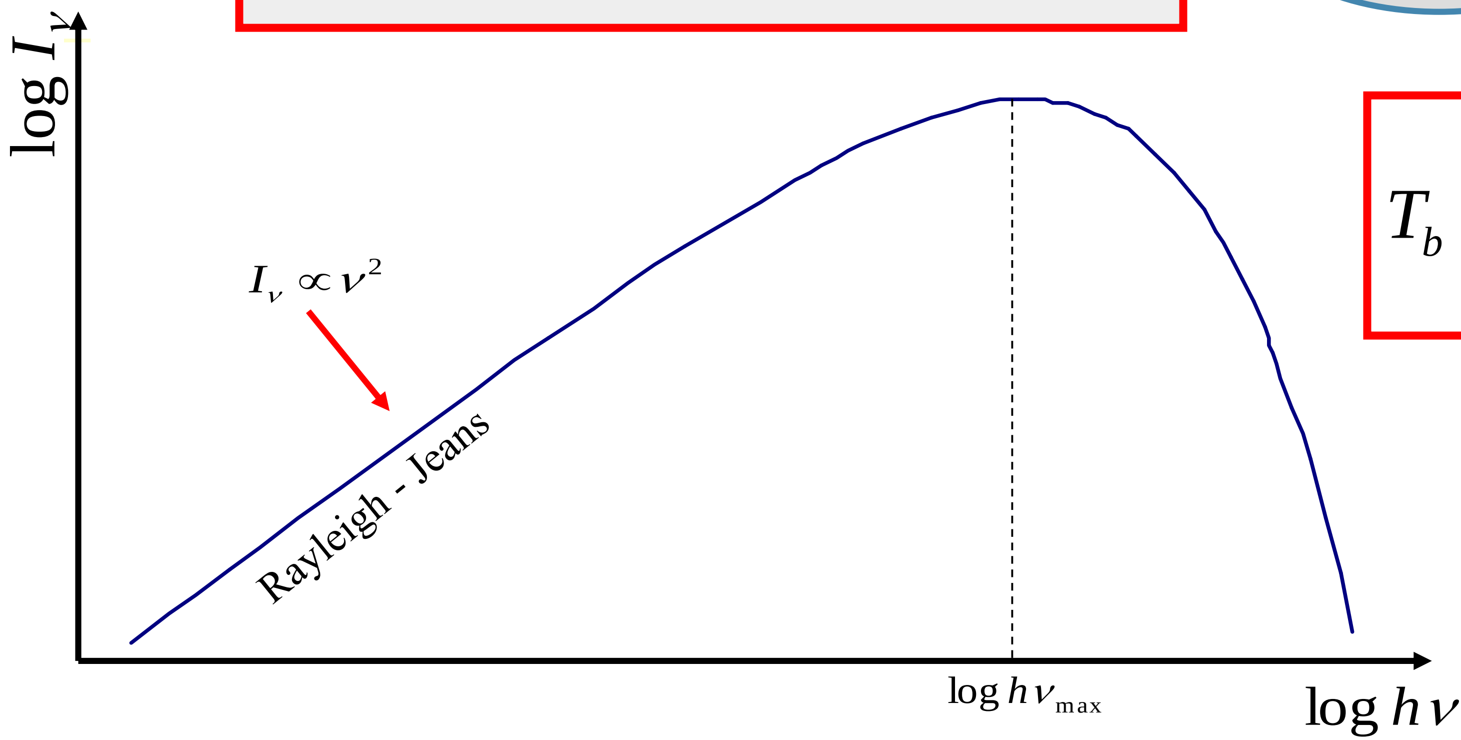
We can always make a definition, common in radio astronomy: Brightness temperature

At typical radio frequencies and temperatures $h\nu << kT \Rightarrow \exp\left(\frac{h\nu}{kT}\right) - 1 \approx \frac{h\nu}{kT}$

Hence

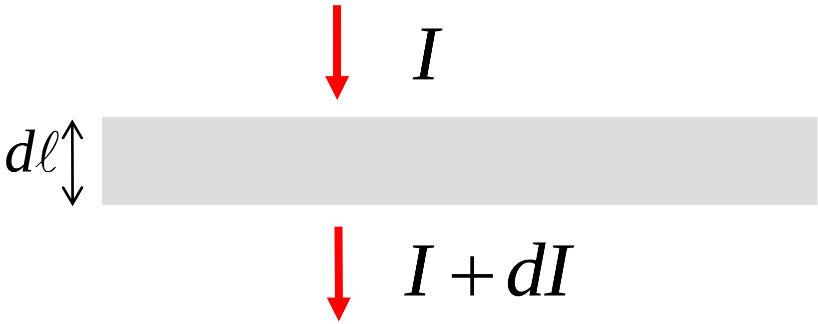
$$I_\nu = \frac{2h\nu^3}{c^2\left[\exp\left(\frac{h\nu}{kT}\right) - 1\right]} \cong \frac{2\nu^2 kT}{c^2}$$

Rayleigh – Jeans approximation



$$T_b = \frac{c^2 I_\nu}{2\nu^2 k}$$

Optically thick and optically thin emission



If we model the absorption in the slab as:

$$dI = -I\,\kappa\,d\ell$$

Absorption coefficient, which is not in general constant, but depends on depth and frequency in the atmosphere

The optical depth, denoted by τ , so that $$I_{obs} = I_0\, e^{-\tau}$$

- If $\tau = 0$ we describe the atmosphere as **"transparent"** and $I_{obs} = I_0$
- If $\tau << 1$ we describe the atmosphere as **"optically thin"** and $I_{obs} \approx I_0$
- If $\tau \geq 1$ we describe the atmosphere as **"optically thick"** and $I_{obs} << I_0$

For example, free-free absorption coefficient (Dulk, 1985):

$$\kappa(\nu) = 0.2 n_e^2 T^{-\frac{3}{2}} \nu^{-2} (\mathrm{cm}^{-1})$$

Radio emission mechanisms

Solar radio emission mechanisms

Free-free emission (collisions of electrons with protons and other particles*)*

Gyromagnetic emission *(cyclotron and gyrosynchrotron)*

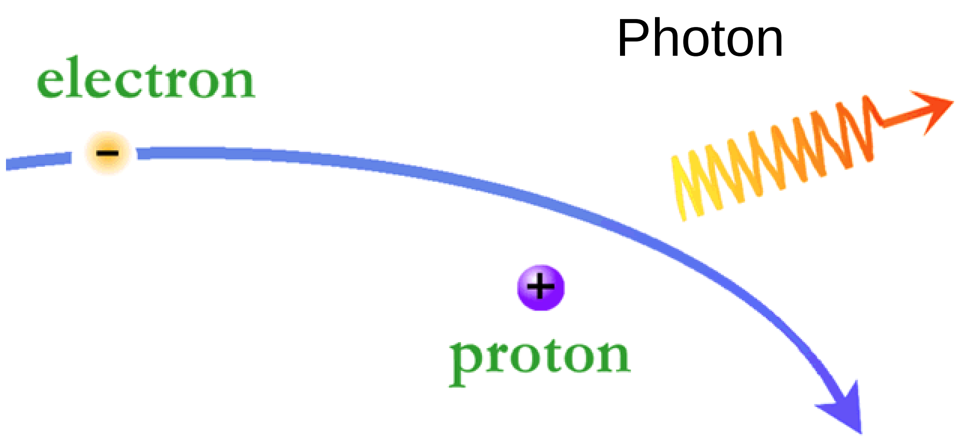
Coherent emission *due to wave-wave and wave-particle interaction*

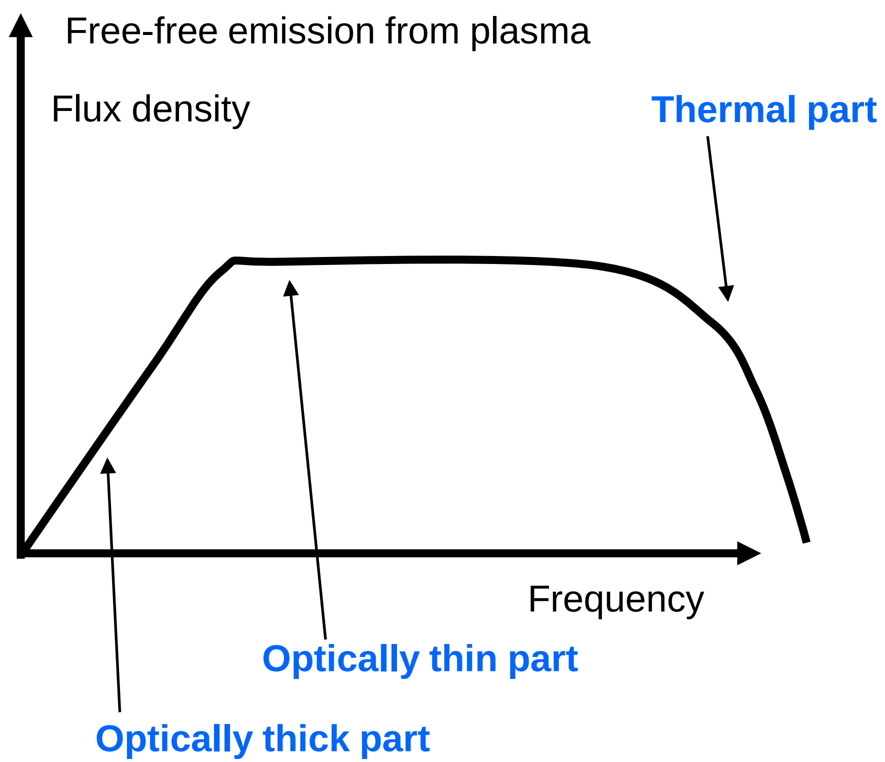
$$\nu_B = \frac{eB}{2\pi m_e c},$$ <= gyrofrequency

$$\nu_p = \sqrt{\frac{n_e e^2}{\pi m_e}},$$ <= plasma frequency

Free-free emission

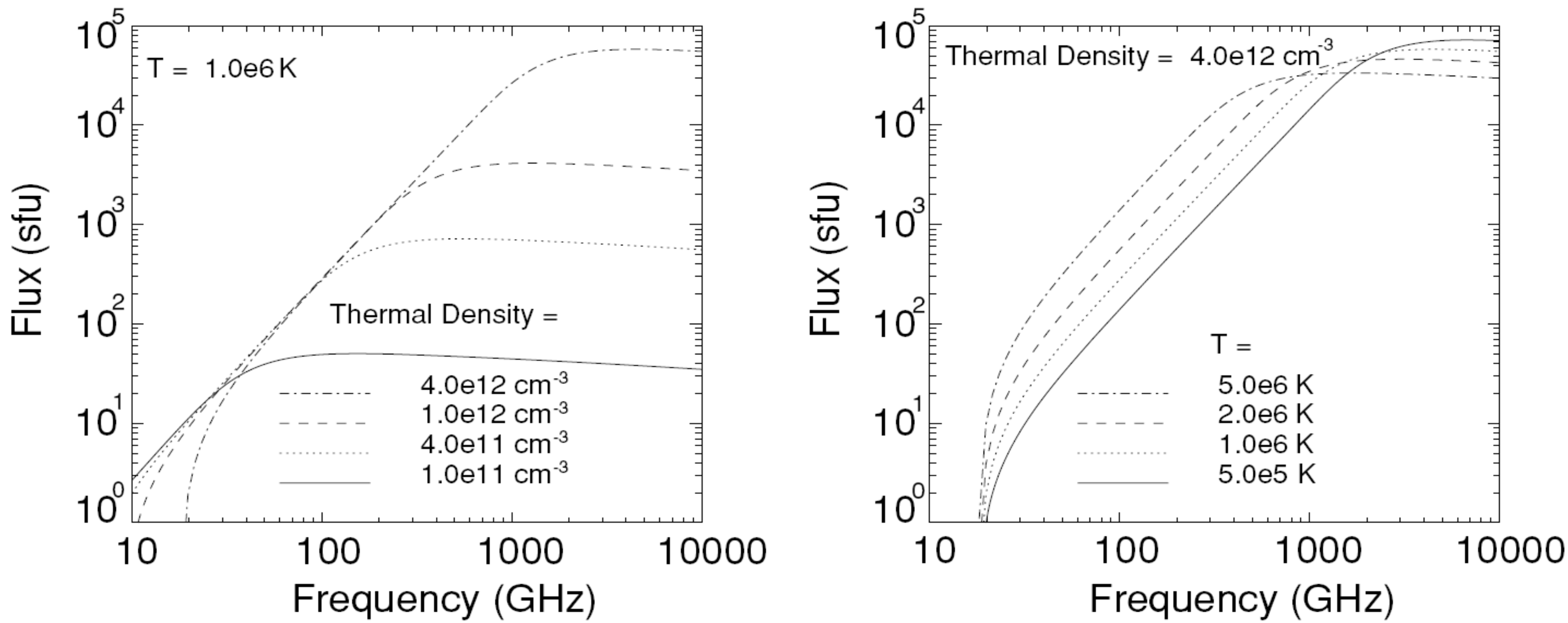
Photons are produced by **free-free transitions** of electrons – also known as **Bremsstrahlung** ('braking radiation')





Free-free emission

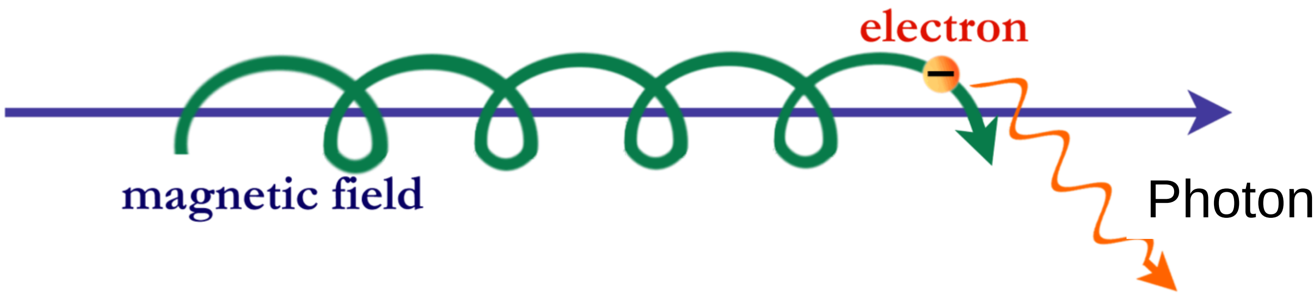
A rising spectrum from a compact (20”) source requires that the source is relatively **dense** (n_e~10^{11} cm^{-3}) and **hot** (T_e ~10 MK).



Thermal free-free radio spectra produced from a uniform cubic source with a linear size of 20’’ for $n_e = 10^{11}$ to 4×10^{12} cm^{-3} and T_e = 0.5–5 MK.
From Fleishman & Kontar, 2010

Cyclotron Radiation

Any constant velocity component parallel to the magnetic field line leaves the radiation unaffected (no change in *acceleration*), and electron spirals around the field line.



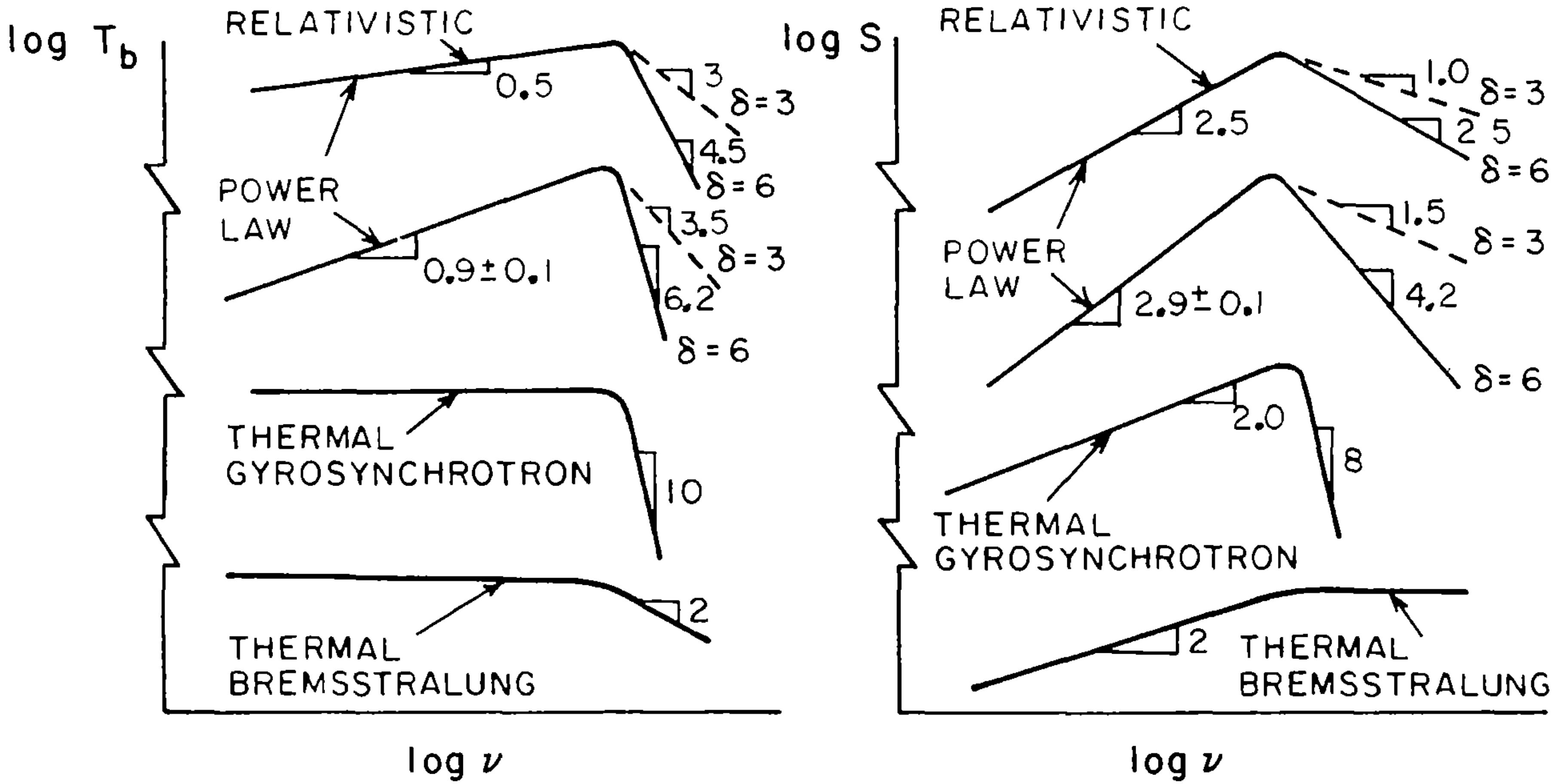
Electron cyclotron line has frequency

$$\nu_{\mathrm{B}} = \Omega_{\mathrm{e}}/2\pi = eB/2\pi m_{\mathrm{e}}c \approx 2.8 \times 10^{6} B.$$

In ultra-relativistic limit, this radiation is known as **synchrotron** – it is strongly Doppler shifted and forward beamed due to relativistic aberration.

In mildly or sub relativistic limit, this radiation is known as **Gyrosynchrotron**

Gyro-magnetic emission



Brightness Temperature and Flux density as a function of frequency for various emission mechanisms (*Dulk, 1985*)

Plasma emission mechanisms

Electron Beam

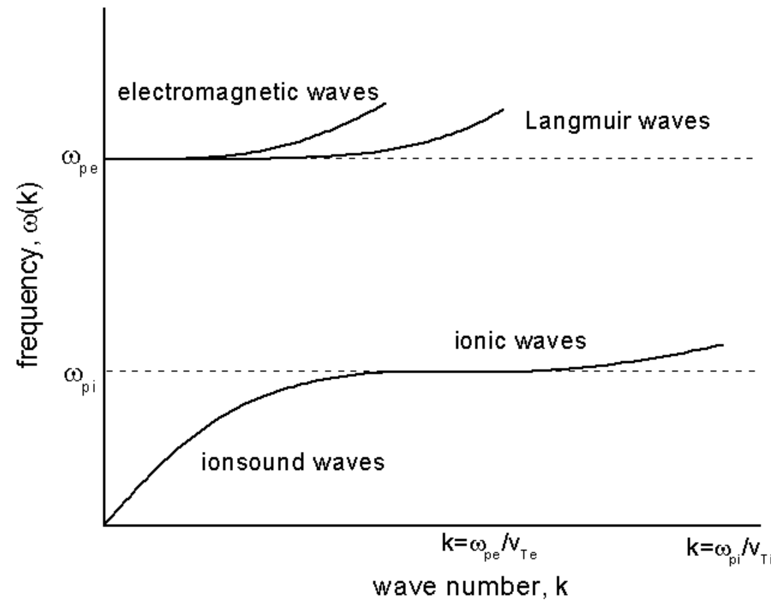
Langmuir waves

Wave-wave interactions

Fundamental Radio emission

Secondary waves

Harmonic radio emission

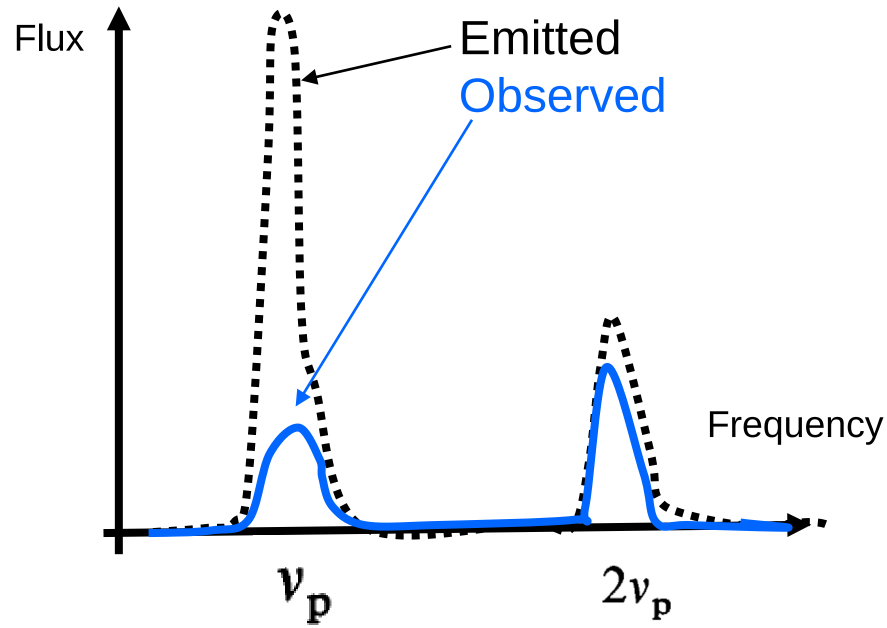


Coherent emission *due to wave-wave and wave-particle interaction*

$$\nu_{\mathrm{p}} = \omega_{\mathrm{p}}/2\pi = [n_{\mathrm{e}}e^2/\pi m_{\mathrm{e}}]^{1/2} \approx 9000\ n_{\mathrm{e}}^{1/2}$$

plasma frequency

Plasma emission mechanisms



Fundamental radio emission (at local plasma frequency)

1) Ion-sound decay L=T+S
2) Scattering off ions L+i=T+i

Harmonic radio emission (double plasma frequency)

1) Decay and coalescence
L =L'+S, L+L'=T
2) Scattering and coalescence
L+i=L+i', L+L'=T

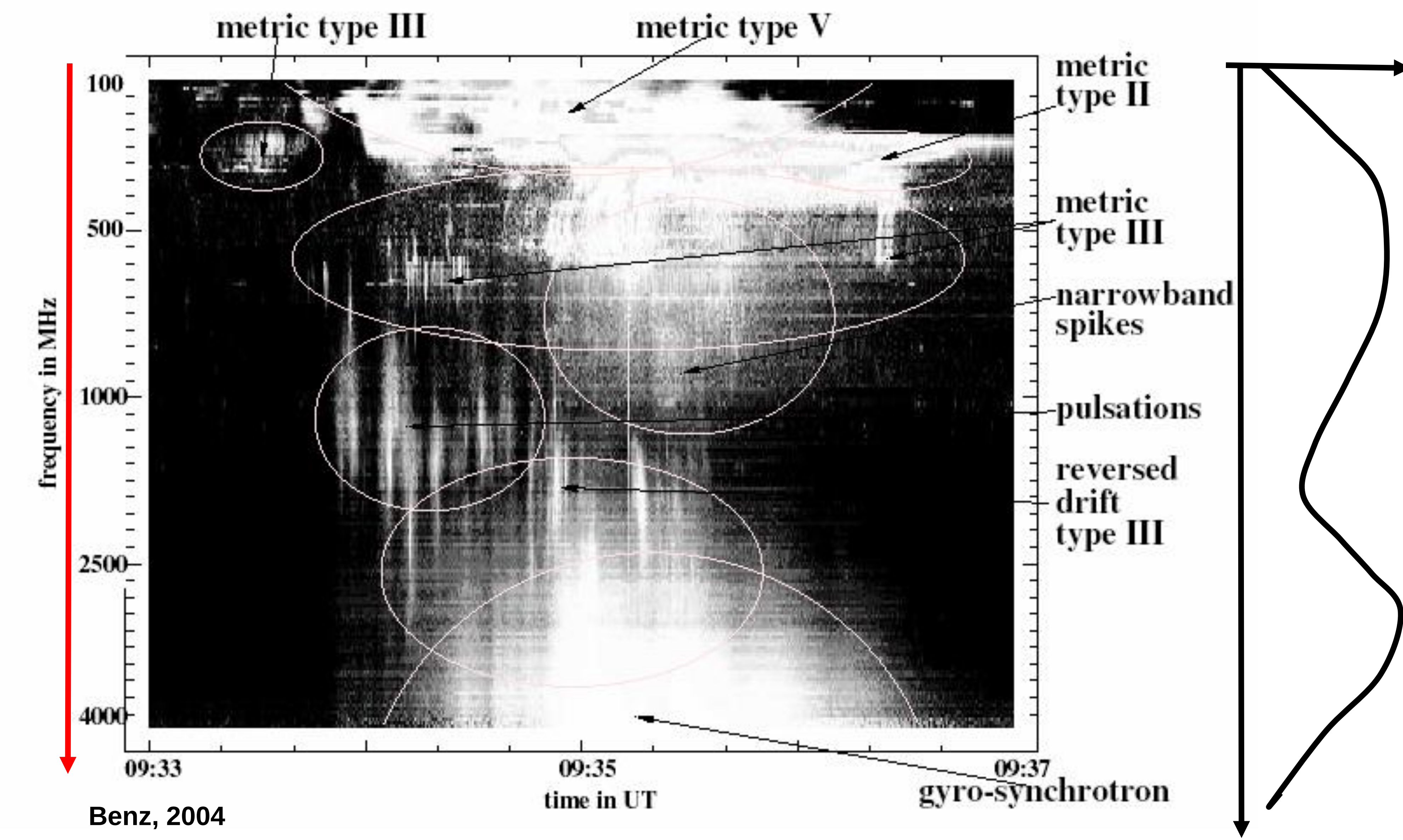
For each act of decay or coalescence we have the corresponding conservation laws for momentum and energy require:

$$\mathbf{k}' = \mathbf{k}'' + \mathbf{k}, \quad \omega(\mathbf{k})_{\sigma'} = \omega(\mathbf{k})_{\sigma''} + \omega(\mathbf{k})_{\sigma}$$

Radio emission from solar flares

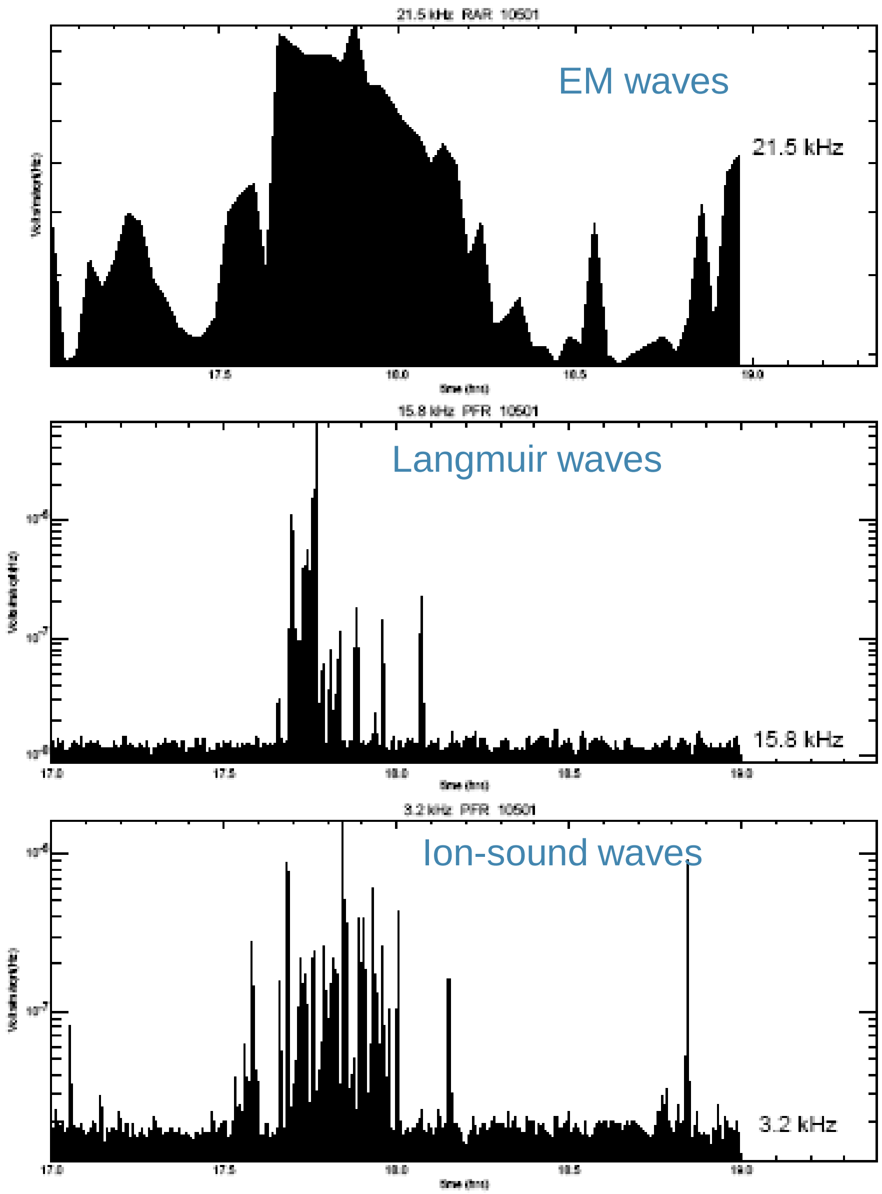
Solar radio emission is complex!

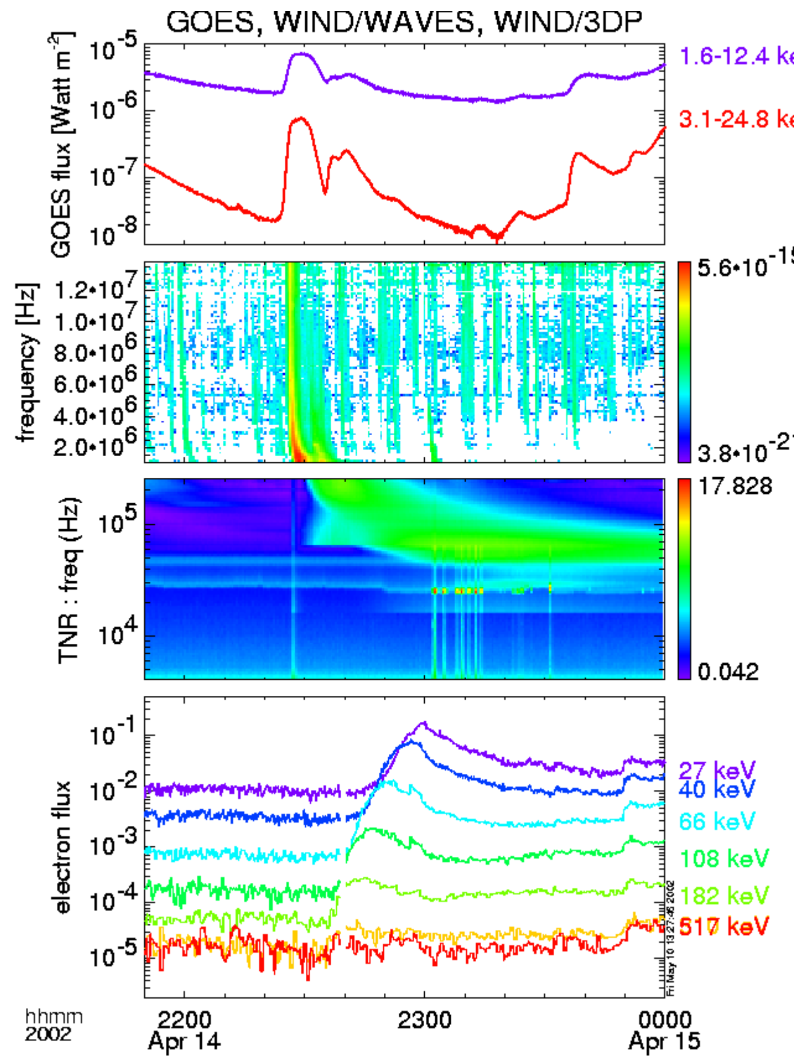
A typical dynamic spectrum of an active Sun



Benz, 2004

Observations near 1AU





Kinetic description of plasma

Test particles description: Exact solution for small number of individual particles while the rest of the particles are treated as an external slow varying media

Fluid description of plasma: plasma is assumed to be a continues media at ***L>>l***, where ***L*** is a scale of processes to consider, and ***l*** is the mean free path of a particle in a plasma. In classical fluid description ***L*** is the collisional length.

Classical kinetics is the study of the relationship between motion and the forces affecting motion introducing statistical tools for description.

… other methods or combination of the above

Kinetic description of plasma is based on *the distribution function* f**(t,r,p)** in phase space **(r,p)**.The value f(t,**r,p**)**drdp** is the average number of particle in the phase volume **drdp**, i.e. in the range (**r**,**r+dr**) and (**p**,**p+dp**). **Note:** the number of particles with $\mathbf{p}=\mathbf{p}_0$ and $\mathbf{r}=\mathbf{r}_0$ is equal to zero.

If we ignore collisions than each particle is a closed subsystem and the corresponding distribution function obeys *Liouville theorem* as a result of which we can write:

$$\frac{\mathrm{d}f(t,\mathbf{r},\mathbf{p})}{\mathrm{d}t}=0$$

where the time derivative is a along a trajectory in the phase space.

Using equations of motion we derive:

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + \mathbf{F}\frac{\partial f}{\partial \mathbf{p}} = 0.$$

The third term in this equation shows the influence of *an external field* on the the particles. When the interaction between particles cannot be neglected we have:

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + \mathbf{F}\frac{\partial f}{\partial \mathbf{p}} = \left(\frac{\partial f}{\partial t}\right)_c$$

where $\left(\frac{\partial f}{\partial t}\right)_c$ is integral of collisions and the equations of this type kinetic are called *kinetic equations*.

The term in the right hand side of kinetic equations is a source or a sink of particles in the phase space volume **drdp**.

The main force acting on a particle in a plasma is electromagnetic.

If the collisional term in the right hand side is a small value and the kinetic equations will take the form

$$\frac{\partial f_{e,i}}{\partial t} + \mathbf{v}\frac{\partial f_{e,i}}{\partial \mathbf{r}} + q_{e,i}\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}\right)\frac{\partial f_{e,i}}{\partial \mathbf{p}} = 0$$

where **E, B** are the average values of electric and magnetic field respectively. This equation was first derived by *Vlasov* in 1937.

This equation should be completed with the system of Maxwell equations,

$$\nabla\mathbf{B} = 0, \quad \nabla\mathbf{D} = 4\pi\rho, \quad \nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}, \quad \nabla\times\mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial\mathbf{D}}{\partial t}$$

and with the sources

$$\rho = \sum_j q_j \int f_j \mathrm{d}\mathbf{p}, \quad \mathbf{j} = \sum_j q_j \int \mathbf{v} f_j \mathrm{d}\mathbf{p}$$

Who is this person?

Perturbations and waves

We first linearize ***Vlasov equations*** by separating out zero and first order $f = f_0 + \delta f, \quad E = 0 + \delta E, \quad \text{etc.}$

Avoiding the terms of second order, we have

$$\frac{\partial \delta f_{e,i}}{\partial t} + \mathbf{v}\frac{\partial \delta f_{e,i}}{\partial \mathbf{r}} + q_{e,i}\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}\right)\frac{\partial f_{0,e,i}}{\partial \mathbf{p}} = 0$$

Since isotropic distribution function depends only on absolute value of **p**, for such function df/d**p** is parallel to **p**=m**v**, and thus the last term is zero.

We also assuming perturbations to vary as

$$\delta f, \delta E, \delta B, \text{etc.} \sim exp(i\mathbf{k}\mathbf{r} - i\omega t)$$

Waves in un-magnetized plasma

Together with Maxwell equations we have:

$$-i\omega\delta f_{e,i} + i\mathbf{v}\mathbf{k}\delta f_{e,i} - e\mathbf{E}\frac{\partial f_{0,e,i}}{\partial \mathbf{p}} = 0$$

$$i\mathbf{k}\mathbf{B} = 0, \quad i\mathbf{k}\mathbf{D} = 4\pi\rho, \quad i\mathbf{k}\times\mathbf{E} = \frac{i\omega}{c}\mathbf{B}, \quad i\mathbf{k}\times\mathbf{H} = \frac{4\pi}{c}\mathbf{j} - \frac{i\omega}{c}\mathbf{D}$$

$$\rho = -e\sum_j \int \delta f_j \mathrm{d}\mathbf{p}, \quad \mathbf{j} = -e\sum_j \int \mathbf{v}\delta f_j \mathrm{d}\mathbf{p}.$$

where plasma is considered neutral.

Recall from Electromagnetism: $\mathbf{D} = \varepsilon\mathbf{E}$

Solving the system of equations we can find ***dielectric tensor***:

$$\varepsilon_l(\omega,\mathbf{k}) = 1 - \sum_j \frac{4\pi e^2}{k^2}\int \mathbf{k}\frac{\partial f}{\partial \mathbf{p}}\frac{d\mathbf{p}}{(\mathbf{k}\mathbf{v}-\omega)}$$

$$\varepsilon_t(\omega,\mathbf{k}) = 1 - \sum_j \frac{2\pi e^2}{\omega}\int \mathbf{v}_\perp\frac{\partial f}{\partial \mathbf{p}_\perp}\frac{d\mathbf{p}}{(\mathbf{k}\mathbf{v}-\omega)}$$

Waves in un-magnetized plasma

$$\varepsilon_l(\omega,\mathbf{k}) = 1 - \sum_j \frac{4\pi e^2}{k^2}\int \mathbf{k}\frac{\partial f}{\partial \mathbf{p}}\frac{d\mathbf{p}}{(\mathbf{kv}-\omega)}$$

$$\varepsilon_t(\omega,\mathbf{k}) = 1 - \sum_j \frac{2\pi e^2}{\omega}\int \mathbf{v}_\perp\frac{\partial f}{\partial \mathbf{p}_\perp}\frac{d\mathbf{p}}{(\mathbf{kv}-\omega)}$$

dispersion relation

We want to solve this system of equations to find $\omega = \omega(k)$:

However, the integrals have a pole $\omega = kv$, , and the actual value depends on the path: above, below the pole or the average of two. We take the path above the pole i.e. add a small value $\omega \to \omega + i0$ This rule was suggested by ***Landau*** in 1946 and is named after him.

$$\int_{-\infty}^{\infty}\frac{f(z)dz}{z-i0} = \mathrm{PV}\int_{-\infty}^{\infty}\frac{f(z)dz}{z} + i\pi f(0)$$

Landau rule

Obviously integrals have a pole $\omega = kv$, but the actual value depends on the path: above, below the pole or the average of two. We take the path above the pole i.e. add a small value $\omega \to \omega + i0$ This rule was suggested by ***Landau*** in 1946 and is named after him.

$$\int_{-\infty}^{\infty} \frac{f(z)dz}{z - i0} = \mathrm{PV} \int_{-\infty}^{\infty} \frac{f(z)dz}{z} + i\pi f(0)$$

Principal Value

Comes from Residue

This gives us

$$\varepsilon(\omega, \mathbf{k})_l = 1 - \sum_j \frac{4\pi e^2}{k^2} \int \mathbf{k} \frac{\partial f}{\partial \mathbf{p}} \frac{d\mathbf{p}}{(\mathbf{kv} - \omega - i0)}$$

$$\varepsilon_t(\omega, \mathbf{k}) = 1 - \sum_j \frac{2\pi e^2}{\omega} \int \mathbf{v}_\perp \frac{\partial f}{\partial \mathbf{p}_\perp} \frac{d\mathbf{p}}{(\mathbf{kv} - \omega - i0)}$$

Maxwellian plasma

Let us apply Maxwellian distribution to the expressions of the previous section:

$$f(p) = \frac{n_{e,i}}{(2\pi m T_{e,i})^{1/2}} \exp\left(-\frac{p^2}{2mT_{e,i}}\right)$$

Substituting Maxwell distribution we immediately obtain

$$\varepsilon_l(\omega,k) = 1 + \sum_j \frac{1}{k^2\lambda_{Dj}^2}\left[1 + F\left(\frac{\omega}{\sqrt{2}kv_{Tj}}\right)\right]$$

$$\varepsilon_t(\omega,k) = 1 + \sum_j \frac{\omega_{pj}^2}{\omega^2} F\left(\frac{\omega}{\sqrt{2}kv_{Tj}}\right)$$

$$\lambda_{Dj} = \sqrt{\frac{T_j}{4\pi e^2 n_j}}, \quad v_{Tj} = \sqrt{\frac{T_j}{m_j}}, \quad \omega_{pj} = \sqrt{\frac{4\pi e^2 n_j}{m_j}}$$

$$F(x) = \frac{x}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp(-z^2)dz}{z - x - i0}$$

is *plasma dispersion function*

Landau damping

Let us consider an imaginary part of dielectric tensor

$$\mathrm{Im}\varepsilon(\omega, \mathbf{k})_l = -\sum_j \frac{4\pi^2 e^2}{k^2} \int \mathbf{k}\frac{\partial f}{\partial \mathbf{p}} \delta(\omega - \mathbf{k}\mathbf{v}) d\mathbf{p}$$

It says that the waves in plasma are damped even in the collisionless plasma. For Maxwellian distribution we have

$$\gamma_k \approx -\sqrt{\frac{\pi}{8}} \sum_j \frac{\omega_{pj}}{(k\lambda_{Dj})^3} \exp\left(-\frac{1}{2(k\lambda_{Dj})^2}\right)$$

The damping is not randomization of collisions, but a transfer of wave energy into resonant oscillation of particles. Note that
For $k\lambda_{De} > 1$ the damping rate would exceed frequency of oscillations.

Langmuir waves

The dispersion relation for longitudinal electrostatic oscillations:

$$\varepsilon(\omega, k)_l = 0$$

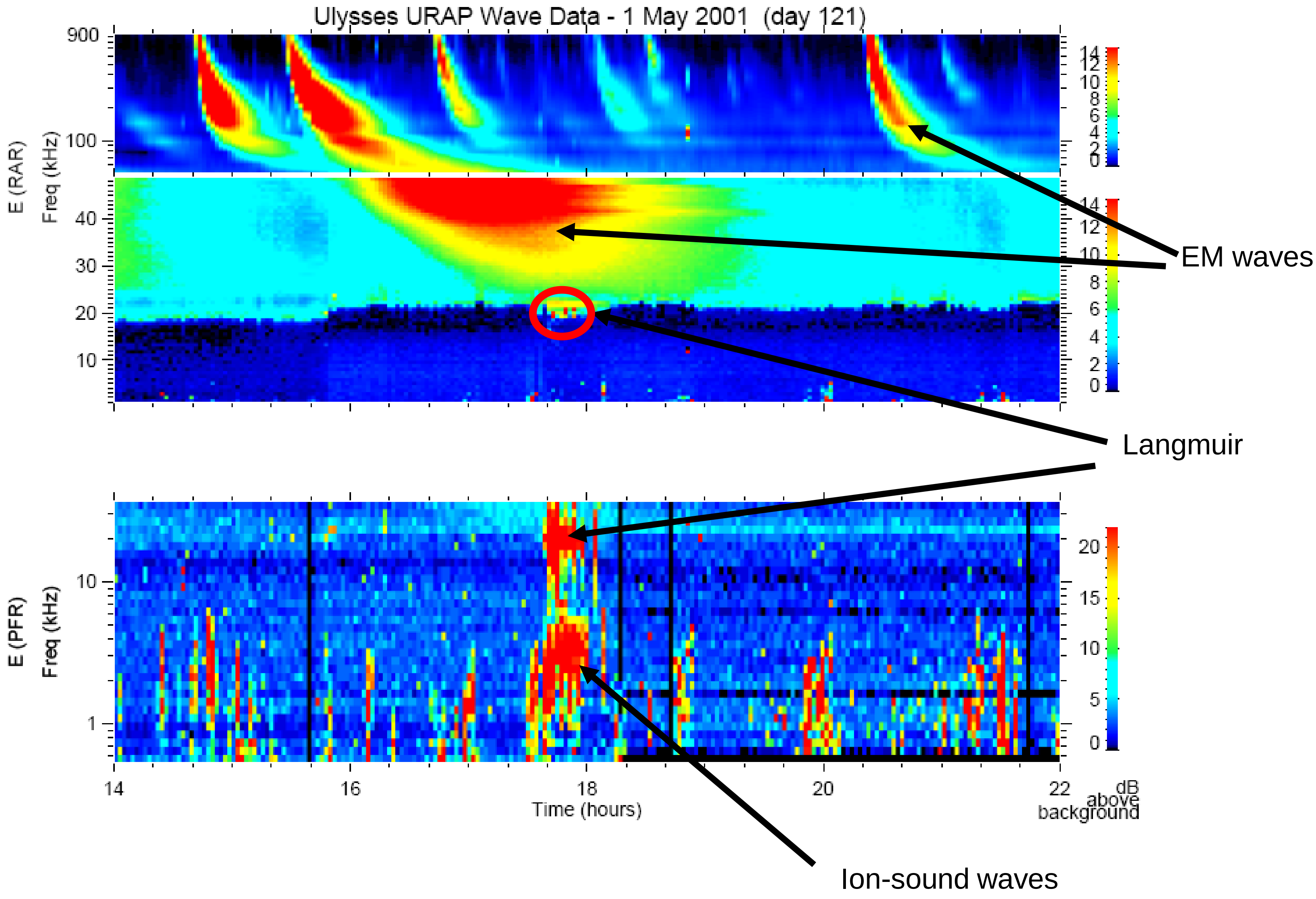
In the limit $\omega >> kv_{Te} >> kv_{Ti}$ we find for the real part

$$\omega(k)_l = \omega_{pe}(1 + \frac{3k^2\lambda_{De}^2}{2})$$

which is the dispersion relation for *Langmuir waves*. The imaginary part of the frequency is

$$\gamma(k) = -0.5\omega_{pe}\mathrm{Im}\varepsilon(\omega, k)$$

In-situ observations of Langmuir waves



Electromagnetic waves

For transverse waves the dispersion relation is given by

$$\omega^2 = c^2 k^2 / \varepsilon_t$$

High frequency waves $\omega \gg k v_{Te}$ orresponds to ordinary electromagnetic waves. We find

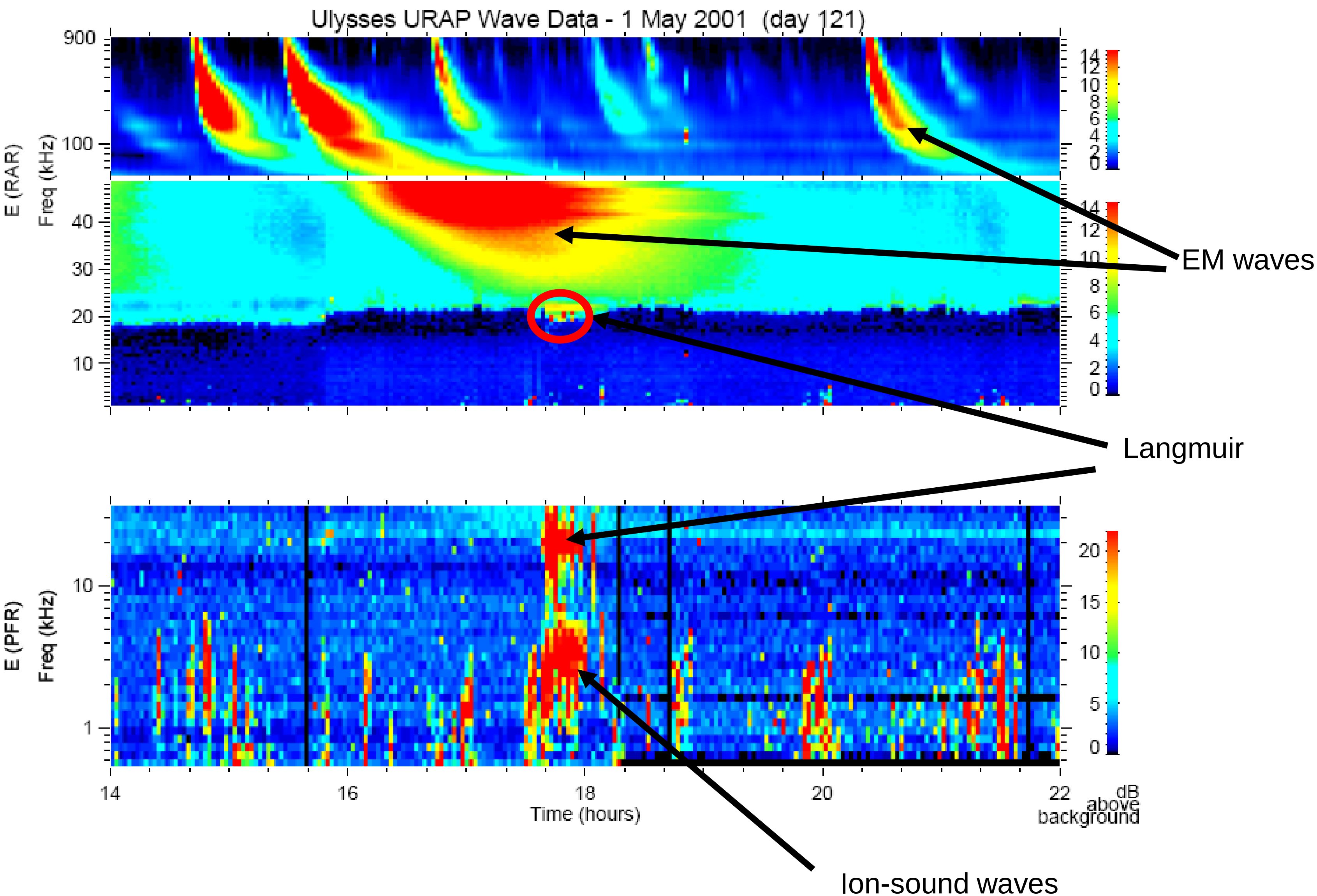
$$\omega(k)_t^2 = \omega_{pe}^2 + k^2 c^2$$

This relation is correct for all values of **k**. Note Landau damping is does not exist since phase velocities are greater than speed of light.

For low frequency waves ions again are not important and the solution has only imaginary part and no waves can propagate

$$\omega(k)_t = -i\sqrt{\frac{2}{\pi}} \frac{k^3 c^2 v_{Te}}{\omega_{pe}^2}$$

Radio waves


Ulysses URAP Wave Data - 1 May 2001 (day 121)
900
100
40
30
20
10
E (RAR)
Freq (kHz)
14
12
10
8
6
4
2
0
EM waves
Langmuir
10
1
E (PFR)
Freq (kHz)
20
15
10
5
0
14
16
18
20
22
Time (hours)
dB above background
Ion-sound waves

Let us consider the range $kv_{Ti} << \omega << kv_{Te}$: have the solution of dispersion equation in the form

$$\omega(k)_s^2 = \omega_{pi}^2 \frac{k^2\lambda_{De}^2}{1+k^2\lambda_{De}^2}$$

These waves are called *ion-sound waves*. For $k\lambda_{De} << 1$ have

$$\omega(k)_s = k\sqrt{\frac{T_e}{M}}$$

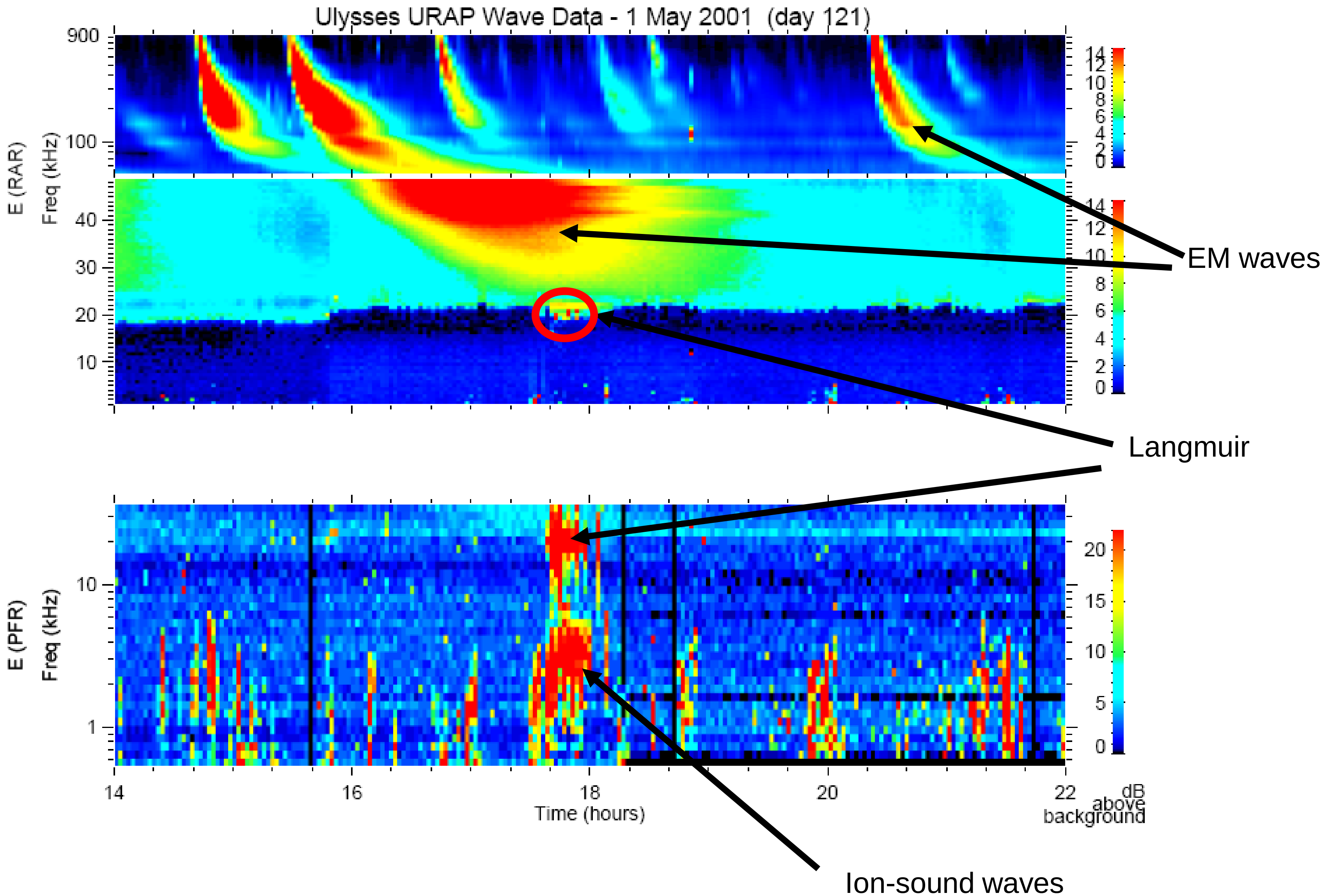
with the dispersion as for ordinary sound waves.

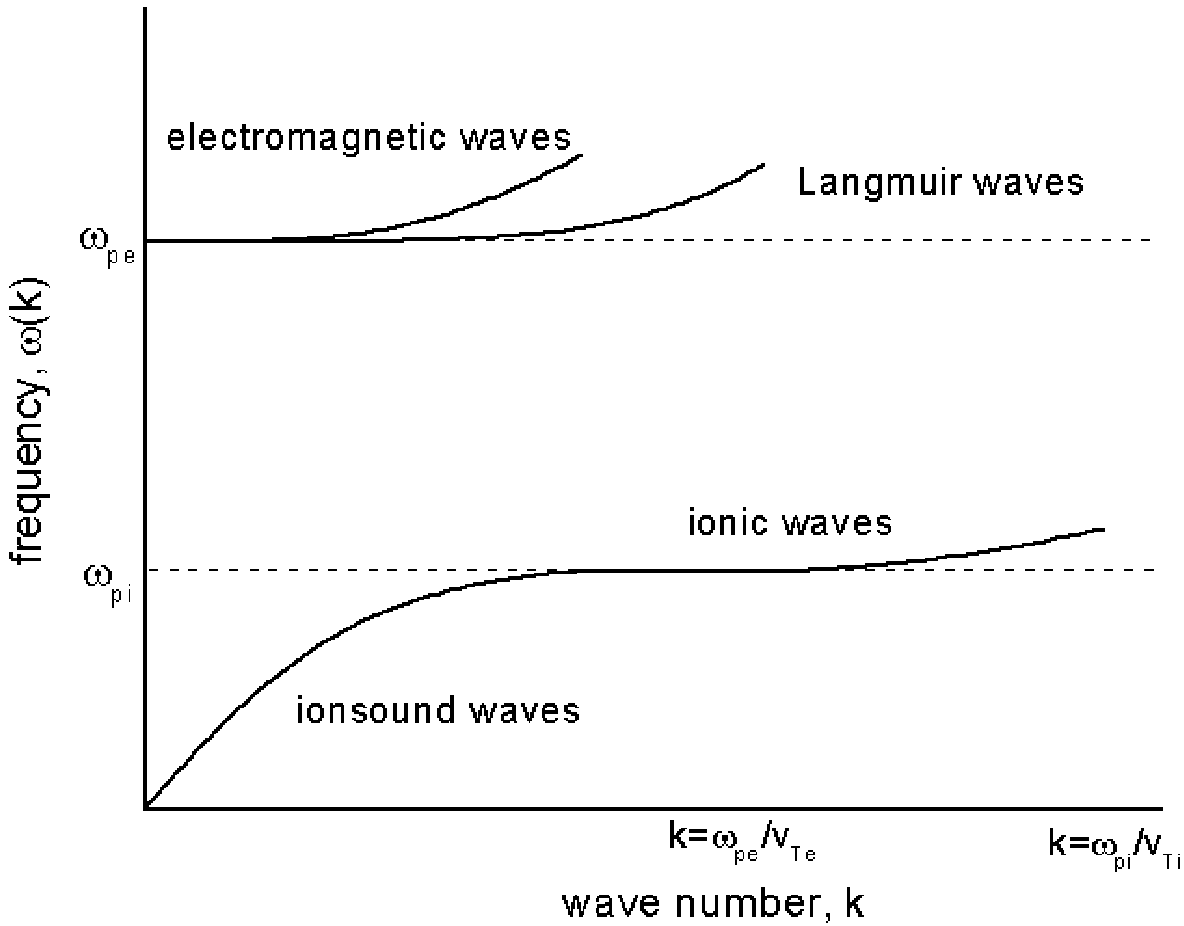
This waves are subject to strong damping. The rate accounting only electrons is

$$\gamma(k)_s = \omega\sqrt{\frac{\pi m}{8M}}$$

Note that these waves require $T_e >> T_i$

Ion-sound waves




electromagnetic waves
Langmuir waves
ω_pe
frequency, ω(k)
ionic waves
ω_pi
ionsound waves
k=ω_pe/v_Te
k=ω_pi/v_Ti
wave number, k

The resonant condition is when *the wave has zero frequency in the rest frame of particle:*

Recall Landau damping: $$\mathrm{Im}\varepsilon(\omega, \mathbf{k})_l = -\sum_j \frac{4\pi^2 e^2}{k^2} \int \mathbf{k} \frac{\partial f}{\partial \mathbf{p}} \delta(\omega - \mathbf{kv}) d\mathbf{p}$$

For unmagnetised plasma – Cherenkov resonance:

$$\omega - \boldsymbol{k} \cdot \boldsymbol{v} = 0$$

In magnetised plasma –cyclotron resonance possible:

$$\omega - s\Omega - k_{||} v_{||} = 0,$$

Cherenkov resonance condition