Kinetics of waves and particles in plasmas

II. Interaction of waves and particles

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0.1 Two level system

Let us consider an elementary process of emission/absorption of a wave by a particle. Due to conservation of energy and momentum, a particle with energy ε and momentum $\mathbf{p}$ emitting /absorbing a wave change its energy and momentum following

$$\varepsilon = \varepsilon' \pm \hbar\omega, \quad \mathbf{p} = \mathbf{p}' \pm \hbar\mathbf{k} \tag{1}$$

Therefore the number of particles with ε, $\mathbf{p}$ has reduced by one $f(\mathbf{p}) - 1$, while the number of particles with $f(\mathbf{p} \pm \hbar\mathbf{k}) + 1$ increased by one.

The number of waves can be conveniently defined as

$$N(\mathbf{k}^\sigma) = \frac{W(\mathbf{k})^\sigma}{\hbar\omega^\sigma} \tag{2}$$

If we introduce the number of waves $N(\mathbf{k})^\sigma$ of the sort σ as above, in each transaction the number of waves is either reduced by one (absorption) or increased by one (emission).

0.2 Kinetic equations for waves and particles

Independently on the nature of the particles and the media, we can build kinetic equations for particles and waves introducing the probability of the spontaneous emission $w_{\mathbf{p}}^{\mathbf{k}}$ of a wave $\hbar\omega(\mathbf{k})$ by a particle with momentum $\mathbf{p}$. The increase of waves due to emission is

$$\frac{\mathrm{d}N_{\mathbf{k}}^{+}}{\mathrm{d}t} = \int (N_{\mathbf{k}} + 1) w_{\mathbf{p}}^{\mathbf{k}} f_{\mathbf{p}} \mathrm{d}\mathbf{p} \tag{3}$$

while the decrease due to absorption is

$$\frac{\mathrm{d}N_{\mathbf{k}}^{-}}{\mathrm{d}t} = \int N_{\mathbf{k}} w_{\mathbf{p}}^{\mathbf{k}} f_{\mathbf{p}-\hbar\mathbf{k}} \mathrm{d}\mathbf{p} \tag{4}$$

The total change of the number of waves is described by the equation subtracting lower equation from the upper

$$\frac{\mathrm{d}N_{\mathbf{k}}}{\mathrm{d}t} = \int \left[(N_{\mathbf{k}} + 1) w_{\mathbf{p}}^{\mathbf{k}} f_{\mathbf{p}} - N_{\mathbf{k}} w_{\mathbf{p}}^{\mathbf{k}} f_{\mathbf{p}-\hbar\mathbf{k}}\right] \mathrm{d}\mathbf{p} \tag{5}$$

In the same approximation, $N_{\mathbf{k}} \gg 1$, we have the second equation

$$\frac{\mathrm{d}f_{\mathbf{p}}}{\mathrm{d}t} = \int \left[w^{\mathbf{k}}_{\mathbf{p}+\hbar\mathbf{k}} (f_{\mathbf{p}+\hbar\mathbf{k}} - f_p) - w^{\mathbf{k}}_{\mathbf{p}} (f_{\mathbf{p}} - f_{\mathbf{p}-\hbar\mathbf{k}}) \right] N_{\mathbf{k}} \mathrm{d}\mathbf{k} \tag{6}$$

0.3 Fokker-Plank approximation

Since $\hbar\mathbf{k} \ll \mathbf{p}$ (*Fokker-Plank approximation*) we can expand in $\hbar\mathbf{k}$ the right-hand side terms of the kinetic equations (5,6).

$$f(\mathbf{p} \pm \hbar\mathbf{k}) = f(\mathbf{p}) \pm \hbar\mathbf{k}\frac{\partial f(\mathbf{p})}{\partial\mathbf{p}} + \frac{1}{2}\left(\hbar\mathbf{k}\frac{\partial}{\partial\mathbf{p}}\right)^2 f(\mathbf{p}) + \ldots \tag{7}$$

Using the fact that Cherenkov emission has a resonant character $w_{\mathbf{p}}^{\mathbf{k}} = w(\mathbf{v},\mathbf{k})\delta(\omega(\mathbf{k}) - \mathbf{kv})$ one obtains in simplified one-dimensional case, which takes place in many physical situations

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = \frac{\hbar^2}{m^2}\frac{\partial}{\partial v}\left[\frac{k^2}{v}N_k w(v,k)\frac{\partial f}{\partial v}\right] \tag{8}$$

$$\frac{\partial N_k}{\partial t} = \frac{\hbar}{m}w(v,k)N_k\frac{\partial f}{\partial v}, \quad \omega(k) = kv \tag{9}$$

0.4 Quasi-linear relaxation

This effect results in relaxation of the system and lead to the formation of a steady state - plateau at the distribution function $f = \mathrm{const}$.

$$f(t \to \infty, v) = \frac{n'}{v_0}, \quad v < v_0, \tag{10}$$

and generate high level of oscillations

$$N_k(t \to \infty, v) = \frac{m}{\hbar} \frac{n' \omega(k)}{k^3} \int\limits_0^{\omega(k)/k} \left[\frac{n'}{v_0} - g_0(v) \right] \mathrm{d}v, \; v < v_0, \tag{11}$$

where $g_0(v)$ is normalized to beam density n', and v_0 is the maximum velocity of the beam.