What if LIGO were 40 km long?

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based on DCC T1400554
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Why bother?

Subsystems that make up Advanced LIGO:

- Suspensions
- Coatings
- Mirrors
- Facilities

have already improved leaps and bounds over initial LIGO. Can we expect this trend to continue beyond Advanced LIGO?

A more straightforward approach might be to extend the length of the arms.

University of Glasgow

Why increase the arm length?

Arm length determines the bandwidth of the detector:

$$f_{\text{pole}} = \frac{\text{FSR}}{\pi} \arcsin\left(\frac{1 - r_1 r_2}{2\sqrt{r_1 r_2}}\right) \tag{1}$$

$$\text{FSR} = \frac{c}{2L} \tag{2}$$

For Advanced LIGO, $L = 4\,\text{km}$, so $f_{\text{pole}} \approx 10\,\text{kHz}$.

Important for Advanced LIGO is the 40 Hz to 300 Hz range, where NS/NS binary coalescences take place. Other important frequencies might be even lower, such as black hole binaries, so it would be useful to increase the arm length by a factor of 10 (thus decreasing f_{pole} by 10 to $\approx 1\,\text{kHz}$)

What gets better?

Stochastic noise sources

Stochastic noise sources are decreased by a factor equivalent to the increase in arm length.

- Thermal noise
- Gravity gradients
- Oscillator noise

How? Because more 'averaging' is performed in the longer arms. Light has to travel for longer in 40 km arms, so more blending of phases will take place. Average stochastic noise tends to zero as storage time tends to infinity.

What gets better?

Coating thermal noise

Beam size ω grows with $\sqrt{L}$, and coating noise decreases linearly with beam size, so we get a $\sqrt{\frac{40}{4}}$ improvement.

What stays the same?

Technical noise sources, e.g. control loop noise

(Probably)

What gets worse?
Infrastructure Requirements

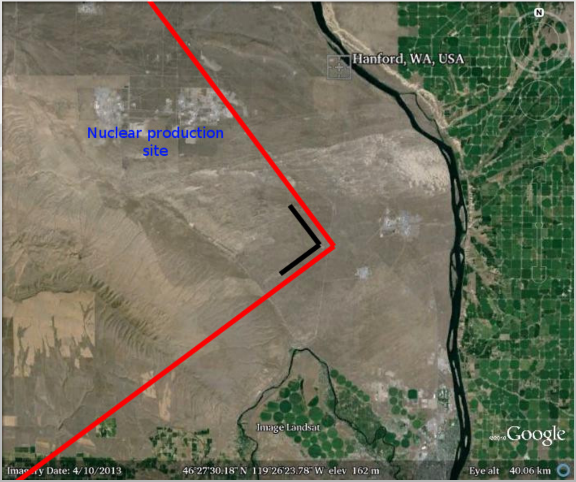
The most obvious one: we need narrow strips of land 40 km long.

What gets worse?
Infrastructure Requirements

Can we extend the arm lengths at existing sites?

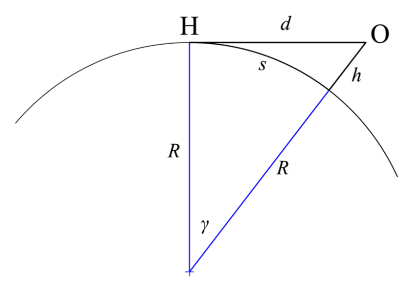
What gets worse?
Infrastructure Requirements

This won't work, because the nuclear production site gets in the way!



What gets worse?

Line of sight on Earth



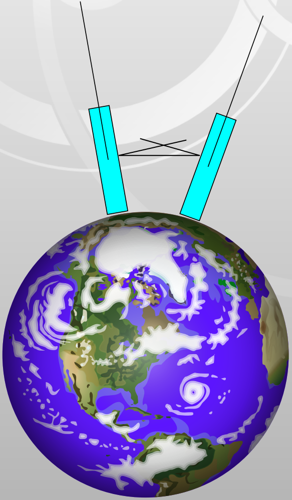
$$(R+h)^2 = R^2 + d^2$$
$$R^2 + 2Rh + h^2 = R^2 + d^2 \quad (3)$$
$$d = \sqrt{h\,(2R+h)}$$

For $R = 6400\,\mathrm{km}$ and $h = 1.0\,\mathrm{m}$, we get $d = 3.6\,\mathrm{km}$ which happens to be nice for Advanced LIGO.

For 40 km LIGO, the end station would be 100 m above the ground if it were a straight line.

What gets worse?

Line of sight on Earth



This leads to increased coupling between vertical and horizontal suspension thermal noise.

$$\theta \approx \frac{L}{R} \quad (4)$$

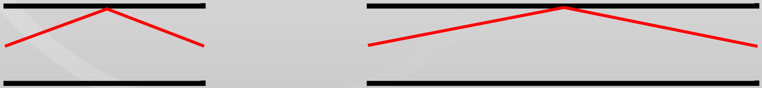
For 40 km LIGO, the angle between each mirror's normal would be $\theta \approx 0.006°$. This means that vertical displacement $y(f)$ couples into horizontal displacement $x(f)$ (i.e. the GW channel) by

$$x(f) \approx \frac{\theta y(f)}{2} \approx \frac{L y(f)}{2R} \quad (5)$$

That means that approximately 0.3% of the vertical motion will couple into horizontal motion.

What gets worse?
Scattering

The longer beam tubes would allow smaller recombination angles from scattering (increased BRDF).



Additionally, bigger optics contain larger fluctuations in the amplitude of surface distortions.

What gets worse?
Cost

Scaling the current arm design by a factor of 10 will increase costs.

Extrapolating LIGO costs from 1994 with inflation, it can be estimated that 40 km tubes would cost upwards of **$530M**. That's not taking into account any extra infrastructure for placing the tubes 50 m above or below the ground!

For reference, the total initial LIGO budget was $300M!

But...

In principle, we can overcome these technical challenges and reap the benefits of longer arm cavities.

(And as an example of this kind of thinking, look to ET)

How to reduce costs

A cheaper vacuum system design would save money, but the original LIGO vacuum system was already far cheaper than any previous large scale UHV system.

It doesn't seem feasible that a suitable 40 km vacuum system could be built.

But there's a crazy idea that avoids this cost...

The crazy idea

Don't use a vacuum system at all!

The crazy idea

Instead, could we propagate light between two mountains, 40 km apart?

Why do we use vacuum systems?

- Gas scattering changes the refractive index the light sees randomly
- Environmental shielding from temperature fluctuations, different gases, etc.

How bad would it be if we didn't use UHV?

Rayleigh scattering by gas molecules is the dominant attenuation mechanism. For cross section σ in $\frac{\text{cm}^2}{\text{molecule}}$, we get

$$\sigma(\lambda) = 64\pi^5 \frac{\alpha^2(\lambda)}{\lambda^4} \frac{\text{cm}^2}{\text{molecule}} \tag{6}$$

where $\alpha(\lambda)$ is the molecular polarisability.

For our current detectors, $\alpha(1064\,\text{nm}) = 1.5 \times 10^{-24} \frac{\text{cm}^3}{\text{molecule}}$.

However, $\alpha(10\,\mu\text{m}) = 4.4 \times 10^{-32} \frac{\text{cm}^3}{\text{molecule}}$!

Attenuation in Gas

The reduction in intensity of the cavity for a distance of z cm for a nominal intensity $I(0)$ is

$$\frac{I(z)}{I(0)} = \exp^{-\sigma z \rho_{\mathrm{air}}} \quad (7)$$

where ρ_{air} is the average density of molecules in the air in the path of the light. At atmospheric pressure, $\rho_{\mathrm{air}} = 3 \times 10^{19} \frac{\mathrm{molecules}}{\mathrm{cm}}$.

At 1064 nm, we get an attenuation of 5.3% whereas for 10 µm we get 5.3 ppm.

So 10 µm seems to win over 1064 nm in terms of scattering in air!

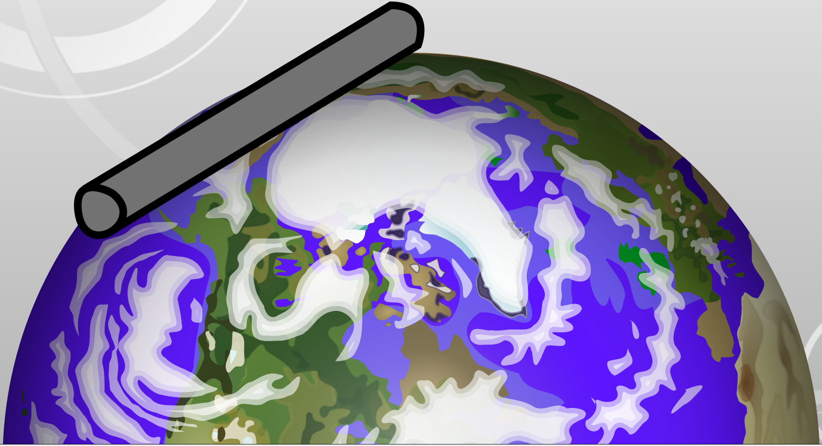
Avoiding Turbulence

Turbulence caused by high wind appears to be a major concern for propagation in air. Additionally, obtaining reliable estimates of the effect of turbulent air on the beam is difficult, and little literature appears to exist concerning this.

The paper makes an attempt to calculate the effect, but it's complicated.

Using a Tube

Instead, **we can perhaps use a beam tube at air pressure**. This allows the advantage of environmental shielding but avoids the significant costs associated with ultra high vacuum.



Using a Tube

How much could a low-fi pipe cost? **Look to the oil and gas industry.**

As of 2013, the industry managed to install pipes at a cost of approximately \$10k per kilometer per cm diameter. For 40 km LIGO that would cost **\$50M** with 1.2 m tubes, an order of magnitude less than \$530M UHV tubes.

Using a Tube

But will this tube still be susceptible to internal air currents, since it's a large enclosed volume at room temperature?

Temperature gradients between the sides of the beam tubes will distort the propagation of the beam and so present a noise source unless controlled.

Convection of air occurs if the temperature gradient between the tube walls increases above 0.04 K (for a tube diameter equal to Advanced LIGO's)*.

Thus, to avoid convection noise, the tubes will need to be placed in a very stable environment in terms of temperature.

*Using complicated atmospheric modelling - see the paper for more details.

Stable Temperature

$$\boxed{\frac{\mathrm{d}T}{\mathrm{d}z} = 0.04\,\mathrm{K}}$$

Where do we find such a stable environment? **Underground!**

How deep? We can estimate this with the diffusion equation:

$$T(z) = T_{\text{mean}} + \Delta T_{\text{surface}} \exp^{-z\sqrt{\frac{\omega}{2\kappa}}} \exp^{i\left(\omega t - z\sqrt{\frac{\omega}{\kappa}}\right)}, \tag{8}$$

where $\kappa = \frac{k_{\text{thermal}}}{\rho c}$ is the **thermal diffusivity** ($\frac{\mathrm{m}^2}{\mathrm{s}}$), k_{thermal} is the **thermal conductivity** ($\frac{\mathrm{Watts}}{\mathrm{m\ K}}$), ρ is the **density** ($\frac{\mathrm{kg}}{\mathrm{m}^3}$), c is the **specific heat capacity** ($\frac{\mathrm{Joules}}{\mathrm{kg\ K}}$) and $\omega = \frac{2\pi}{\tau}$ where τ is the **period of temperature oscillations** (s).

Stable Temperature

$$T(z) = T_{\text{mean}} + \Delta T_{\text{surface}} \exp^{-z\sqrt{\frac{\omega}{2\kappa}}} \exp^{i\left(\omega t - z\sqrt{\frac{\omega}{\kappa}}\right)},$$

Differentiating this equation allows an estimate of the temperature gradient:

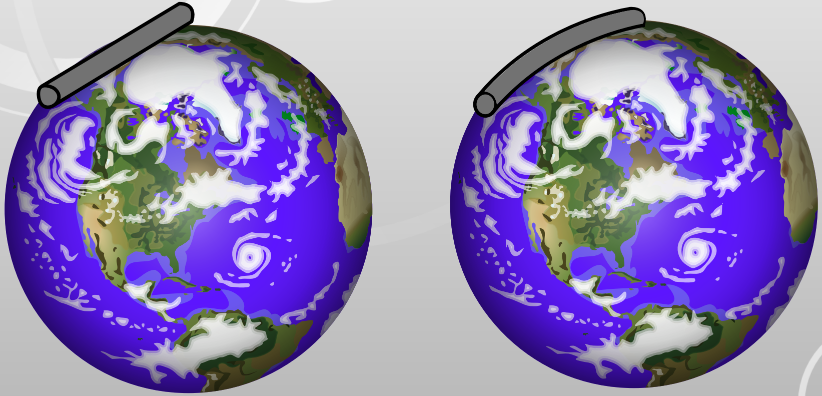
$$\frac{\mathrm{d}T}{\mathrm{d}z} = \Delta T_{\text{surface}} \sqrt{\frac{\pi}{\tau\kappa}} \exp^{-z\sqrt{\frac{\pi}{\tau\kappa}}} \qquad (9)$$

For various materials, we obtain the following required depths to reach the target of $\frac{\mathrm{d}T}{\mathrm{d}z} = 0.04\,\mathrm{K}$ (for yearly fluctuations of $8.5\,^{\circ}\mathrm{C}$):

Material	**Diffusivity** $\left(\frac{\mathrm{m}^2}{\mathrm{s}}\right)$	**Depth** (m)
Water	0.14×10^{-6}	6.14
Ice	1.10×10^{-6}	13.80
Sand	0.24×10^{-6}	7.60
Clay	0.18×10^{-6}	6.80
Rock	1.40×10^{-6}	15.10

Steering the Beam

We probably don't want to make a completely straight pipe. Instead, we can bend it across the surface of the Earth.



But how do we steer it?

Steering the Beam

Over 40 km the pipe needs to bend by about $\frac{1}{1000}$ of the Earth's circumference ($\approx 40\,000$ km), so that's $\frac{360^\circ}{1000} = 0.36^\circ$ between the cavity mirrors (w.r.t horizon).

We need to correct the beam's direction as it travels along the tube.

Steering the Beam

The index of refraction of air for 10 µm light is 2.8×10^{-4} near standard temperature and pressure (STP). The rate of change with temperature is:

$$\left(\frac{\mathrm{d}n}{\mathrm{d}T}\right)_{\mathrm{P=0.5\,bar}} = -9.3 \times 10^{-7} \left[\frac{1}{\mathrm{K}}\right] \tag{10}$$

If we integrate the deflection over distance Δx and height y, we obtain the deflection as a function of this rate of change of refractive index, α:

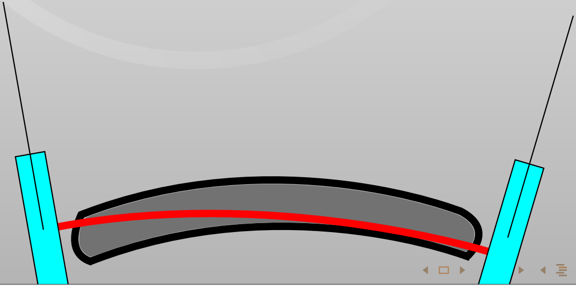
$$\alpha = \int_0^{\Delta x} \frac{\mathrm{d}n}{\mathrm{d}y}\mathrm{d}x = -9.3 \times 10^{-7} \int_0^{\Delta x} \frac{\mathrm{d}T}{\mathrm{d}y}\mathrm{d}x \tag{11}$$

Steering the Beam

$$\alpha = \int_0^{\Delta x} \frac{\mathrm{d}n}{\mathrm{d}y}\mathrm{d}x = -9.3 \times 10^{-7} \int_0^{\Delta x} \frac{\mathrm{d}T}{\mathrm{d}y}\mathrm{d}x$$

For example, maintaining a 0.1 K temperature gradient in the tube between top and bottom would, over 10 km, deflect the beam by 0.038°.

Over 40 km there is enough scope to keep the beam in the tube at both ends if it is given a biased trajectory at each end.



Steering the Beam

Alternative considered in paper: radially symmetric ring heaters on tube to focus the beam using air as the lens.

It's possible, and perhaps practical, but it can lead to additional phase noise in the TEM 00 mode and can be influenced strongly by transverse motion of the tube (e.g. from seismic activity).

Phase Compensation

It might be the case that, however hard we try to avoid it, we pick up phase noise during propagation.

Another approach considered is to use two overlapped lasers, both at 10 μm, but with one frequency shifted to an absorption resonance in the gas medium. Then the non-resonant beam sees a phase shift of:

$$\phi_{\mathrm{main}} = \phi_{\mathrm{n}} + \phi_{\mathrm{GW}}, \tag{12}$$

where ϕ_{n} is the phase accumulated by refractive index noise and ϕ_{GW} is our gravitational wave;

while the other sees a polarisation induced phase due to resonant polarizability g:

$$\phi_{\mathrm{ref}} = g\phi_{\mathrm{n}} + \phi_{\mathrm{GW}}, \tag{13}$$

Phase Compensation

Then we rearrange and get:

$$\phi_{\mathrm{GW}} = \frac{g\phi_{\mathrm{main}} - \phi_{\mathrm{ref}}}{g - 1} \quad (14)$$

So as long as we understand g well, we can compensate for phase noise during propagation. **Understanding g is not trivial, though.**

Also, standard gases in air do not contain resonance lines near 10 μm. It might require something like:

- Water
- Methane
- Carbon dioxide

to be mixed with the air in the tubes.

Other Considerations

This is clearly a challenging undertaking!

The paper goes into more detail about these topics:

- Effect of seismic displacements on tube and gas
- Effect of acoustic coupling through walls of tube to the gas
- Side-injection of light into beam tubes (due to opaque optics at 10 μm)
- Spring materials to help avoid coupling of vertical suspension noise into longitudinal

Challenges and Strengths

- Different wavelength
  - Different coatings
  - Different absorption
  - Different scattering from surfaces
  - Quantum noise/efficiency at detectors
  - Laser noise
- Much longer, do we have space?
- Different environment
  - UVH to (near-) atmospheric pressure
  - Temperature control more important
  - Seismic coupling into air
- Different limiting noise sources
  - Acoustic coupling?
  - Seismic-induced refractive index noise?

- Maximise sensitivity over important observation frequencies
- Cheaper facilities per unit length
- Better stochastic noise performance
- Less challenging cleanliness requirements
- Possible phase compensation system
- New science!