Astronomy 2 – Special Relativity – Part 1
Introduction

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Structure

The structure of the Special Relativity course is as follows.

Part 1: Introduction Central ideas and definitions. Inertial frames. How to measure lengths and times, and how to synchronise clocks. [One lecture]

Part 2: The postulates Introducing and justifying the two postulates which SR rests on. [One lecture]

Part 3: Spacetime and the Lorentz Transformation The central mathematical tool. The Minkowski diagram and the idea of spacetime. The invariant interval. [Four lectures]

Part 4: Relativistic kinematics Momentum, energy and force. [Four lectures]

Web pages:

- `http://physci.moodle.gla.ac.uk/course/view.php?id=15`
  the A2 moodle.
- `http://www.astro.gla.ac.uk/users/norman/lectures/A2SR/`
  the main web pages for this SR lecture course.

I'm going to assume that you've at least looked at these notes before the lecture – not that you've necessarily understood them, but that you at least have an idea what I'm going to be talking about, so that there will be a structure to the way to listen to the next lecture.

I've also taken pains to include a number of exercises, which are generally, but not always, keyed to particular objectives. You should make an effort to attempt these exercises: more so than with other subjects, special relativity can seem to be intelligible right up to the point where you're required to solve a problem, and you don't want to discover this for the first time in the week before the exam.

Aims and objectives for Part 1

The point of Aims and Objectives is twofold. They help me keep on track by reminding me what things it's important I cover; and they help you follow the course, by reminding you of the motivation for the material I'm covering. The distinction between the two, as far as I'm concerned, is simple.

- The *aims* are the point of the course – why you're doing the course, and why I'm teaching it. These are the insights you'll have, and the ideas you'll understand, long after the point where you've forgotten most of the details. Unfortunately, it's easy to claim, but difficult to show, you have this understanding. So. . . .

- The *objectives* are the detailed skills, mastery of which demonstrates that you have in fact achieved the aims of the course. Hint: it is a short step from objectives to exam questions.

Aims: You should

1. understand the primacy of events within Special Relativity, and the distinction between events and their coordinates in a particular frame.

2. appreciate why we have to define very carefully the process of measuring distances and times, and how we go about this.

Objectives: You should be able to demonstrate that you can

1. provide concise descriptions of the terms 'event', 'reference frame', 'inertial frame' and 'standard configuration'.

2. describe how an observer ascribes a position and time to an event.

3. describe how we might use a network of observers to measure lengths and times.

4. cast a described problem into standard configuration.

1 The basic ideas

Relativity is simple. Essentially the only *new physics* which will be introduced in this course boils down to just:

1. All inertial reference frames are equivalent for the performance of all physical experiments (the Equivalence Principle);

2. The speed of light has the same constant value when measured in any inertial frame.

The work you will do in this course consists of (a) understanding what these two axioms really mean, and (b) examining both their direct consequences, and the way that we have to adjust the physics we already know.

We will examine these axioms in part 2, study their direct consequences in part 3, and their consequences for dynamics in part 4, but before we can start on any of this, we have to understand what the axioms actually mean. For example, what is a 'reference frame', and what is special about 'inertial' frames? It turns out that we also have to be particularly careful about how we use terms like 'measurement': we have to ask precisely what we mean when we talk of measuring distances, in space or time, or how we would go about synchronising two clocks. It is this that makes SR challenging: the maths isn't particularly hard, but we have to put a lot of effort into understanding ideas we thought were already clear, and try to think precisely about processes we thought were intuitive, such as measuring the length of a stick.

1.1 Events

An 'event' in SR is something that happens at a particular place, at a particular instant of time. The standard examples of events are a flashbulb going off, or an explosion, or two things colliding.

Note that it is *events*, and not the reference frames that we are about to mention, that are primary. Events are real things that happen in the real world (we omit a mass of philosophical detail, here); the separations between events are also real (we return to this in more detail later); reference frames are a construct *we* add to events to allow us to give them numbers, and to allow us to manipulate and understand them. That is, events are not relative – everyone agrees that an event happens. SR is about how we reconcile the different measurements of an event, that different, relatively moving, observers make.

1.2 Inertial reference frames

We need to understand first what a *reference frame* is, and then what is special about an *inertial* (reference) frame.

A *reference frame* is simply a method of assigning a position, as a set of numbers, to events. Whenever you have a coordinate system, you have a reference frame. The coordinate systems that spring first to mind are possibly the (x, y, z) or (r, θ, ϕ) of physics problems. Reference frames need not be fixed to a stationary body, though. A train driver most naturally sees the world in terms of distances in front of the train. An approaching station can quite legitimately be said to be moving – speeding up and slowing down – in the driver's reference frame.

You can generate an indefinite number of reference frames, fixed to various things moving in various ways. However, we can pick out some frames as special, namely those frames which are *not accelerating*.

Imagine placing a ball at rest on a table: you'd expect it to stay in place. Similarly, if you roll a ball across a table, you'd expect it to move in a straight line. This is merely the expression of Newton's first law: 'bodies move in straight lines at constant velocity, unless acted on by an external force'. In what circumstances will this *not* be true?

Suppose you're sitting in a train which is accelerating out of a station. A ball placed on a table in front of you will start to roll towards the back of the train, rather than staying put, as Newton's first law says it should. This observation only makes sense from the point of view of someone on the station platform, who sees the ball as stationary, and the train being pulled from under it. The station is an inertial frame, and the accelerating train carriage, where Newton's law appears not to hold, is not. Similarly, if you are perched on a spinning children's roundabout, and toss a ball to someone on the opposite side, it'll veer off to one side (interpreting this as either 'it'll appear to veer off to one side, from your point of view' or, more formally, 'it'll be measured to veer off, as observed by someone using the rotating reference frame which is fixed to the roundabout'). This motion, again, is only immediately intelligible from the point of view of someone standing watching all this go on, who sees the ball go exactly where it should, but the person it's aimed at turn out of the way. The playpark is an inertial frame, the spinning roundabout is not. In both cases, you can tell if you're the one in the non-inertial frame: in the first case you feel yourself pushed back into the train seat, and in the second case, your perch on the roundabout stops you flying off, and you feel yourself thrown towards the outside by centrifugal 'force', and held in your place only by the force exerted on you by your seat.

Acceleration and force are intimately connected with the notion of inertial frames – an inertial frame is one which isn't accelerated in any way. From that, you would be correct to conclude that once the train has stopped accelerating, and is speeding smoothly on its way (we imagine a perfectly smooth track), it becomes an inertial frame again; if you closed your eyes, you wouldn't be able to tell if you were on a moving train or at rest in the station. Anything you can do whilst standing on a station platform (such as juggling, perhaps), you can also do whilst racing through that station on a train, irrespective of the fact that, to the person watching the performance from the platform, the balls you're juggling with are moving at a hundred miles an hour, or so.

Newton's second law is more quantitative, since it relates the amount of force applied to an object, the amount it is accelerated, and the body's inertial mass, through the well-known relation $F = ma$.

We can therefore define an 'inertial frame' as follows:

> *Definition of Inertial Frames:* An inertial reference frame is a reference frame, with respect to which Newton's second law holds, to an adequate approximation.

That is, an intertial frame is one which is not accelerating. It follows that inertial frames are, in the context of SR, infinite in extent; also, that all inertial frames move with constant velocity, so that no pair of such frames mutually accelerate.

Of course, there is rather more to it than that. This definition suffices for Special Relativity, but once we consider General Relativity (GR) we have both the need, and the mathematical tools, for a more fundamental definition. In brief, in GR the definition of an inertial frame is one which is in free fall, meaning moving freely in a gravitational field, or freely floating, unaccelerated in interstellar space. This definition is locally consistent with the definition in SR, but allows us to start to discuss frames which are mutually accelerating.

To be precise, I shouldn't really talk of train carriages and station platforms as inertial frames. Firstly, there are tiny corrections due to the fact that we are on a rotating, curved, planet; we can ignore these. Secondly, we should be careful when talking about throwing balls or juggling (as I do repeatedly) within an inertial frame, since, because of the presence of the force of gravity, a frame sitting on earth is not inertial according to GR's stricter definition. However, as long as we are talking about SR rather than GR, as long as all the relevant motion (of inertial frames) is horizontal, and as long as no-one throws the ball further than a hundred miles or so (!), denying ourselves any mention of projectile motion would achieve nothing beyond removing a vivid and natural example to focus on. If you really want to, you can remove gravity from the examples by imagining the events taking place not in train carriages going through stations, but in space capsules flying past asteroids, with some suitably baroque arrangement of air jets or rockets, to supply the forces when necessary.

The mass which is the constant of proportionality in Newton's second law is what defines *inertial mass*. It is distinct from gravitational mass, which describes how closely bound the object is to the gravitational field, and is the proportionality constant in Newton's law of gravitation – it is this that you measure when you weigh something. In fact, these two quantities, though logically completely distinct, are always measured to be equal, and it is the examination of this astonishing observation that leads us to General Relativity.

Ex.1

1.3 Measuring lengths and times: simultaneity

How do we measure times? In SR, we repeatedly wish to talk about the time at which an event happens. If the event happens in front of our nose, we can just look at our own watch. This is an important point. One of the things we can hold onto in the rest of this course, is that if two events at the same spatial position happen at the same time, they are simultaneous for everybody. This is why a time measurement by a local observer is always reliable. Observers at that same spatial position but moving in different frames may produce different numbers, but their measurement is, by definition, the measurement of the time of that event in that frame. Einstein made this particularly clear:

> We must take into account that all our judgments in which time plays a part are always judgments of *simultaneous events*. If, for instance, I say, "That train arrives here at 7 o'clock," I mean something like this: "The pointing of the small hand of my watch to 7 and the arrival of the train are simultaneous events." [1]

If the event happens some distance away (answering a question such as "what time does the train pass the next signal box?"), however, or if we want to know what time was measured by someone in a moving frame (answering, for example, "what is the time on the train-driver's watch as the train passes through the station?"), things are not so simple, as most of the rest of this lecture course makes clear. SR is very clear about what we mean by 'the time of an event': when we talk about the time of an event, we *always* mean the time of the event *as measured on a clock carried by a local observer*, that is, an observer at the same spatial position as the event (which is rather unfortunate if the event in question is an explosion of some type – but what are friends for?), who is stationary with respect to the frame they represent. We will typically imagine more than one observer at an event; indeed we imagine one local observer per frame of interest, stationary in that frame, and responsible for reporting the space and time coordinates of the event 'as measured in that frame'.

We suppose that a frame has a limitless supply of observers, scattered throughout space. These 'same-frame' observers have two properties of importance: as well as being at rest in their frame, the clocks they carry are synchronised with all others in their frame. This implies a specific procedure for synchronising clocks: this isn't too difficult to define, within the context of the two axioms mentioned at the start of Sect. 1 and discussed in greater detail in the next part, and both Rindler [2, §2.5] and Taylor and Wheeler [3, §2.6], for example, give details. When an observer is asked for the coordinates of an event which happened near them (in their frame, only ever in their frame), they can respond with the spatial coordinates of their fixed station, and the time the event happened as noted from their local clock.

If we find a number of observers located immediately adjacent to an event, the above doesn't imply that all observers report the *same* time. The observers may have watches set to different time zones, their watches may be designed to run at different rates (perhaps someone has an exotic watch that ticks out 1000ths of a day), someone's computer might be ticking out seconds since the start of 1970, or the watches may be running at different rates for relativistic reasons that we'll come to later. We assume that despite these complications, all of the clocks are *good clocks*: whatever they tick out, they do so in a linear way.

The key *mathematical* observation is that there is a set of *linear* transformations between the times being shown on the different observers' clocks and watches.

For example, imagine a trainful of world leaders speeding through a busy station when a protestor throws a paint-bomb at the train. Questioned afterwards, a white-with-a-hint-of-puce commuter explains that it happened exactly when the 7.29 express was due, and that he is the 117th commuter along from the coffee-stall. Describing the same incident, the member of the entourage nearest the impact describes it happening three hours before the conference starts (the only timescale which matters to the jet-lagged passengers on the train), and two carriages up from the president. The two local observers have different origins of space and of time, and the two sets of observers, on the train and on the platform, have clocks which are synchronised with respect to the others in their own frame, but have little relation to the clocks in the other.

So much for coordinates; how should we measure the length of a moving object? The obvious methods – some variant of laying a ruler along the object to be measured and examining the marks on the ruler – make too many assumptions about how the world is. To measure something accurately this way, you have to correct for the time it takes for light to travel from the ruler and the (moving) object to you. You obviously have to measure both ends of the object simultaneously: does that 'simultaneously' depend on where you're standing relative to the object? The obvious approaches to this problem beg too many questions to which SR provides surprising answers.

The way we measure lengths and times in SR is therefore as follows. We station observers at strategic points in the reference frames of interest (in principle, we have at our disposal an infinite array of observers scattered throughout space). We can know these observers' coordinates in one frame or another. The observers make measurements of events which happen at their location, and afterwards compare notes and draw conclusions. For example, you would measure the 'length of a rod' by subtracting the coordinates of the two observers who observed opposite ends of the rod at a prearranged time.

We can summarise what we have discovered so far. This approach relies on three things.

1. It requires a specific procedure for synchronising clocks.

2. It assumes that there is no ambiguity about two events at the same position and time being regarded as simultaneous. This has to be true: the fact that two cars crash – because they were in the same position at the same time, and so are attempting to occupy the same space simultaneously – cannot depend in any way on your point of view.

3. We assume that moving clocks measure the passage of time accurately. We do not assume anything about accelerating clocks, because in SR we largely avoid discussion of acceleration, but we do assume that there is nothing magical about motion which causes clocks to go wrong. This is known as the *clock hypothesis*, and is related to the Equivalence Principle which is one of the axioms of SR.

1.4 Standard configuration

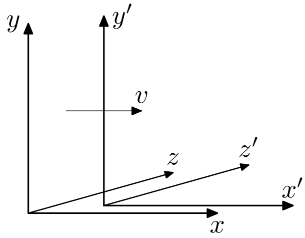

Figure 1: Standard config.

Finally, a bit of terminology to do with reference frames. Two frames S and S', with spatial coordinates (x, y, z) and (x', y', z') and time coordinates t and t' are said to be in *standard configuration* if:

1. they are aligned so that the (x, y, z) and (x', y', z') axes are parallel;
2. the frame S' is moving along the x axis with velocity V;
3. we set the zero of the time coordinates so that the origins coincide at $t = t' = 0$ (which means that the origin of the S' frame is always at position $x = Vt$).

This arrangement, shown in Fig. 1, makes a lot of example problems somewhat easier, and is the arrangement assumed by the 'Lorentz Transformation' which we will meet later.

When we refer to 'frame S' and 'frame S'', we will interchangably be referring either to the frames themselves, or to the sets of coordinates (t, x, y, z) or (t', x', y', z'). Ex.1

2 Further reading

When learning relativity, even more than with other subjects, you benefit from hearing or reading things multiple times, from different authors, and from different points of view. I mention a couple of good introductions below, but there is really no substitute for going to section ‘Physics C25’ in the library, looking through the SR books there, and finding one which makes sense to *you*.

Other textbooks you might want to look at are:

- Carroll & Ostlie [4] is the set book for Astronomy 2, and has a chapter on relativity.
- Taylor & Wheeler [3] is an excellent account of Special Relativity, written in a style which is simultaneously conversational and rigourous. It introduces the subject from a geometrical perspective from the very beginning, which makes it quite natural to make links to General Relativity.
- Rindler [2] contains a clear account of the subject, using a rather similar tack to my own in this course. Rindler takes great pains to confront and explain the subtleties involved in the subject, and explains things sparsely but precisely. I think that both Rindler and Taylor & Wheeler see SR essentially as a prologomenon to GR, and are consequently admirably careful in their treatment of SR. Only the first half or so of this book covers SR: the remainder is about GR, and goes substantially beyond the scope of this course.
- French [5] is a justly well-known account. It takes a rather traditional approach to the subject: it’s good on the experiments, such as the Michelson-Morley experiment, but lacks excitement for me.
- Barton [6] is another book with a rather traditional approach. It goes into a great deal of detail about experimental corroboration of the results of SR (a little more modern than French), and explains its results with great care. These are both valuable features, but it

might be best to treat this book as a supporting resource, as it would be easy to find the quantity of detail overwhelming.

- Moore [7] is another book which takes a fairly geometrical approach. It takes pains to explain the ideas of inertial frames clearly.

- Schutz [8] is a textbook on *General* Relativity (in fact it's the set book for the A3 course on that subject), and is therefore generally beyond the scope of this course. However the first chapter gives a breakneck account of SR, and the second a similarly abstract account of vector analysis, which were influential on parts 3 and 4 of this course.

- Einstein's own popular account of relativity [9] is very readable, though it's naturally a little old-fashioned in places. You can see the influence of this book, and its examples, in many later SR textbooks.

- *The Principle of Relativity* [10] is a collection of (translations of) original papers on the Special and General theories, including Einstein's paper of 1905 [9], but also some earlier papers by Lorentz suggesting interpretations of the Michelson-Morley experiment. The first few sections, at least, of the 1905 paper are worth reading as a current introduction to SR.

All of these books, except Carroll & Ostlie, which you are supposed to have yourself, and Barton, which is not in the Library, are on restricted loan in the Reading Room.

Other books which have interesting notes on SR include: David Mermin's book [11] is a (very good) collection of essays, covering a wide variety of topics from the practice of physics, through quantum mechanics, to relativity. Chapters 19 to 21 are about unusual approaches to the teaching of relativity. John Bell's book [12] is another collection of (sometimes rather high-level) essays; they're mostly illuminating essays on quantum mechanics, but chapter 9 is about a rather classically-oriented approach to relativity, which would be rather nicely partnered with Lorentz's papers in [10]. Another book to mention, just because I like it, is

Malcolm Longair's collection of lecture notes and case studies, covering several areas in theoretical physics [13]: it pulls no mathematical punches, but is full of insights, including a chapter on SR.

There are several popular science books which are about, or which mention, relativity – these aren't to be despised just because you're now doing the subject 'properly'. These books tend to ignore any maths, and skip more pedantic detail (so they won't get you through an exam), but in exchange they spend their effots on the underlying ideas. Those underlying ideas, and developing your intuition about relativity, are things that can sometimes be forgotten in more formal courses. I've always liked [14], which is a cartoon book but very clear, and I've heard good things about Brian Cox's *Why does* $E = mc^2$*?*.

Questions

These questions are from session 2001–2.

Will there be worked examples? Sometimes it's useful to explain something by working through it, and there are one or two places where I do that. However, most of the 'worked examples' comprise working you do yourself, in the examples at the end of each part.

If you're in a free-falling lift and you release a ball, surely it would fly to the top of the lift. Possibly surprisingly, no. Remember that, neglecting air-resistance, a light object falls just as fast as a heavy one (recall Galileo's supposed experiment with the Tower of Pisa). This will also be true if the light object is *inside* the heavier one, as will be the case with the lift. Therefore, the ball will accelerate at the same rate as the lift, and so will *not* more relative to it. This is discussed at much greater length in the 'General Relativity' part of this course. This is linked to the last part of Example 1.1.

When a falling lift reached its terminal velocity, would it still be an inertial frame? At that point it would be moving at a constant velocity so yes, in the slightly loose sense of Special Relativity, but no, not in the more fundamental sense of General Relativity. See the dangerous-bend paragraphs at the end of Sect. 1.2 for further discussion of the distinction here.

References

[1] Albert Einstein. Zur Elektrodynamik bewegter Körper (on the electrodynamics of moving bodies). *Annalen der Physik*, 17:891, 1905. Reprinted in [10].

[2] Wolfgang Rindler. *Essential Relativity: Special, General and Cosmological*. Springer-Verlag, 2nd edition, 1977.

[3] Edwin F Taylor and John Archibald Wheeler. *Spacetime Physics*. W H Freeman, 2nd edition, 1992.

[4] Bradley W Carroll and Dale A Ostlie. *Modern Astrophysics*. Addison Wesley, 1996.

[5] A P French. *Special Relativity*. Chapman and Hall, 1971.

[6] Gabriel Barton. *Introduction to the Relativity Principle*. John Wiley and Sons, 1999.

[7] Thomas A Moore. *A Traveler's Guide to Spacetime*. McGraw-Hill, 1995.

[8] Bernard F Schutz. *A First Course in General Relativity*. Cambridge University Press, second edition, 2009.

[9] Albert Einstein. *Relativity: The Special and the General Theory*. Routledge Classics, 2001.

[10] H A Lorentz, A Einstein, H Minkowski, and H Weyl. *The Principle of Relativity*. Dover, 1952.

[11] N David Mermin. *Boojums All the Way Through*. Cambridge University Press, 1990.

[12] J S Bell. *Speakable and Unspeakable in Quantum Mechanics*. Cambridge University Press, 1987.

[13] M S Longair. *Theoretical Concepts in Physics*. Cambridge University Press, 1984.

[14] Joseph Schwartz and Michael McGuinness. *Introducing Einstein*. Icon Books, 1999. Used to be known as 'Einstein for Beginners'.

Examples

Example 1.1 (section 1.2)

Which of these are inertial frames? (i) A motorway bridge; (ii) a stationary car; (iii) a car moving at a straight line at a constant speed; (iv) a car cornering at a constant speed; (v) a stationary lift; (vi) a free-falling lift (the last one is rather subtle and relates to the dangerous-bend paragraphs at the end of Sect. 1.2). (Objective 1)

Example 1.2 (section 1.4)

A train carriage moves through a station at a speed of $50\,\mathrm{ms}^{-1}$. The carriage is $10\,\mathrm{m}$ long, and the station platform is $100\,\mathrm{m}$ long. Identify two reference frames, in standard configuration, suitable for analysing this problem, including their origins and their orientation, and taking the point when the back of the carriage is at one end of the station as the zero of time. At time $t = 1\,\mathrm{s}$, someone lets off a banger at the forward end of the carriage. Draw a diagram at this time, and mark the ends of the carriage and the platform in the appropriate frames. (Objective 4)