Escape of atmospheres

NAIVE ARGUMENT

If the speed of a particular atom or molecule exceeds the Escape speed, then it will escape the atmosphere.

i.e. if $v > \sqrt{\frac{2GM_p}{R_p}} = v_{esc} \Rightarrow$ ESCAPE

BUT atoms have a DISTRIBUTION of speeds (Maxwell Boltzmann)

Rule of thumb A planet will have lost a particular component of its atmosphere if

$$v_{rms} > \frac{1}{6} v_{esc}$$

"root mean square" speed $= \sqrt{\overline{v^2}}$

Recall :- $\frac{1}{2} m \overline{v^2} = \frac{3}{2} kT \Rightarrow$ $T = \frac{m v_{rms}^2}{3k}$

So a particle of mass, m, will have escaped if:-

$$T > \frac{m v_{esc}^2}{3k \cdot 36} = \frac{1}{54} \frac{GM_p m}{k R_p} = T_{esc}$$