## Apparent magnitudes of familiar objects

## Apparent and Absolute Magnitude

Introducing the parsec as a unit of distance also helps to define a convenient relationship, used by astronomers, between the apparent brightness of a source and its intrinsic brightness, or luminosity.

Recall from A1X that astronomers use the magnitude system to express ratios of observed flux to differences in apparent magnitude, via the equation:

$$
\begin{equation*}
m_{1}-m_{2}=-2.5 \log _{10} \frac{F_{1}}{F_{2}} \tag{5}
\end{equation*}
$$

Remember: the minus sign in front of 2.5 means that brighter objects have smaller (i.e. more negative) apparent magnitudes - see Figure 4.

We can use Equation (1) to express the apparent magnitudes of the stars in terms of their distance and luminosity, instead of their flux:

$$
m_{1}-m_{2}=-2.5 \log _{10} \frac{4 \pi D_{2}^{2} L_{1}}{4 \pi D_{1}^{2} L_{2}}
$$

Using the properties of logarithms, we can rewrite this as:

$$
\begin{equation*}
m_{1}-m_{2}=5 \log _{10} D_{1}-5 \log _{10} D_{2}+2.5 \log _{10} L_{2}-2.5 \log _{10} L_{1} \tag{6}
\end{equation*}
$$

Now suppose the two stars have the same luminosity (or, equivalently, suppose that the same star is being observed from two different places, which are a distance $D_{1}$ and $D_{2}$ respectively from the star). Equation (6) simplifies to:

$$
\begin{equation*}
m_{1}=m_{2}+5 \log _{10} D_{1}-5 \log _{10} D_{2} \tag{7}
\end{equation*}
$$



Figure 4: Illustration of the apparent magnitude scale (handout given out in A1X)

We can use Equation (7) to introduce the absolute magnitude of a star, defined as the apparent magnitude which the star would have if it were at a distance of $\mathbf{1 0}$ parsecs. Absolute magnitude is usually written as $M$ (not to be confused with mass!). Thus, in Equation (7), if we measure distance in parsecs, and set $D_{2}=10$, then $m_{2}=M$ and

$$
\begin{equation*}
m=M+5 \log _{10} D-5 \tag{8}
\end{equation*}
$$

Equation (8) is analogous to Equation (1), in that it relates the apparent magnitude, absolute magnitude and distance of a star, just as Equation (1) relates the flux, luminosity and distance of a star. Where apparent magnitudes define a logarithmic scale measuring fluxes, absolute magnitudes define a logarithmic scale measuring luminosities. In particular:

$$
\begin{equation*}
M_{1}-M_{2}=-2.5 \log _{10} \frac{L_{1}}{L_{2}} \tag{9}
\end{equation*}
$$

The quantity $\quad m-M$ is known as the distance modulus and is often written as $\mu$. Inverting Equation (8):

$$
\begin{equation*}
D=10^{0.2(m-M+5)}=10^{0.2(\mu+5)} \tag{10}
\end{equation*}
$$

where $D$ is measured in pc. In A1Y Cosmology, we will also measure distances in megaparsecs (Mpc), where $1 \mathrm{Mpc}=10^{6}$ pc. Hence, in cosmology we replace Equation (8) by:

$$
\begin{equation*}
m=M+5 \log _{10}\left(D \times 10^{6}\right)-5=M+5 \log _{10} D+25 \tag{11}
\end{equation*}
$$

where $D$ is now measured in Mpc.

