## 4. Inverse Compton Sources

In HEA1 we also developed mathematical machinery to describe the spectrum of radiation from an inverse Compton source.

Some revision about inverse compton scattering:
A low energy photon collides with a relativistic electron and gains energy at the expense of the electron (e.g. radio photons might be boosted to $X$-ray energies)


In most astrophysical situations it is still OK to assume that, in the rest frame of the electron, the energy of the photon before collision is much less than the rest mass energy of the electron-i.e.:-

$$
h v \ll m_{e} c^{2}
$$

This means that we don't need to use the Klein-Nishina formula, but can assume

$$
Q_{I C} \approx Q_{T}=\frac{8}{3} \pi r_{e}^{2}
$$

A head-on collision gives the maximum energy transfer to the outgoing photon. Averaging over all scattering directions gives approximately:

$$
\begin{equation*}
\left\langle\rho_{2}\right\rangle \cong \frac{4}{3} \gamma_{1}^{2} \rho_{1} \tag{4.3}
\end{equation*}
$$

And the average power emitted by a single electron is

$$
\left(\frac{d E}{d t}\right)_{I C}=L_{I C}=\frac{4}{3} Q_{I C} c \gamma^{2} U_{v_{0}}
$$

In fact eq. (4.4) is the limiting case for a highly relativistic electron with $U \sim C$

For $U<c$ the more correct result is

$$
L_{I C}=\frac{4}{3} Q_{I C} c U_{v_{0}}\left(\frac{v}{c}\right)^{2}
$$

We will return to this case later.

We will consider (possible) inverse Compton emission in three different scenarios:

1. Synchrotron self Compton radiation from $A G N$ and GRB jets
2. The diffuse cosmological X-ray background
3. The Sunyaev-Zel'dovich effect

We see this low frequency cut-off in synchrotron spectra from AGN and GRBs


What about the second peak?

This is called
Synchrotron
Self-Compton
See Section 4

### 4.1 Synchrotron Self-Compton Emission

In a strong synchrotron source, such as an AGN jet, relativistic electrons spiral around magnetic field lines and radiate photons.

Those photons may then Inverse Compton scatter with other relativistic electrons in the source, boosting the photons to much higher energies.

We will then see these IC photons as a second peak in the source spectrum.


The SSC emission may again show a lower and upper cutoff, just like the synchrotron peak.

Recall that

$$
\nu_{S}=\frac{3}{2} \gamma^{2}\left(\frac{e B}{2 \pi m}\right)=\frac{3}{2} \gamma^{2} v_{L}
$$

And if a photon of frequency $v_{S}$ undergoes an inverse Compton collision, then the inverse Compton photon has average frequency

$$
\left\langle v_{S S C}\right\rangle \cong \frac{4}{3} \gamma^{2} \nu_{S}=2 \gamma^{4} \nu_{L}
$$

For an ultra-relativistic electron (e.g. with $\gamma \sim 100-1000$ ) the increase in photon frequency compared with the Larmor frequency can be huge. ( $10^{8}$ to $10^{12}$ times).

For a spherical source of synchrotron radiation, we can relate the luminosity and radiation energy density:

$$
L_{S}=4 \pi R^{2} \frac{c}{4} U_{S}
$$

So, the luminosity of a spherical, homogeneous volume containing $N_{e}$ electrons is

$$
L_{S S C}=\frac{4}{3} Q_{I C} c N_{e} \gamma^{2} U_{S}
$$

We can rewrite this as $L_{S S C}=\frac{4}{3} Q_{I C} c N_{e} \gamma^{2} \frac{L_{S}}{\pi R^{2} c}$ and, substituting for $Q_{I C}$, we see that

$$
\frac{L_{S S C}}{L_{S}}=\frac{32}{9}\left(\frac{r_{e}}{R}\right)^{2} N_{e} \gamma^{2}
$$

so that we can have $L_{S S C} \approx L_{S}$ (see example Sheet 3)

### 4.2 The Diffuse Cosmological X-ray Background

- All-sky maps at X-ray and gamma ray wavelengths reveal strongly anisotropic emission, coming mainly from the plane of the Milky Way galaxy
- At low X-ray energies ( $\varepsilon \leq 0.5 \mathrm{keV}$ ) this background mainly originates from hot gas in the interstellar medium.
- The temperature of the gas is up to a few million degrees (recall $k T \cong 1 \mathrm{keV} \Rightarrow T \cong 10^{7} \mathrm{~K}$ ).

Source of heating: supernovae (shock heated remnants)
hot star winds (gas bubbles around hot massive stars)

- Also hot intracluster gas in nearby galaxy clusters - e.g. Virgo, Coma


All-sky ROSAT map: $0.25-1 \mathrm{keV}$

Coma cluster


### 4.2 The Diffuse Cosmological X-ray Background

- At higher energies $2 \mathrm{keV}<\varepsilon<10 \mathrm{keV}$ still some galactic point sources (e.g. XRBs) and cosmological sources (e.g. intracluster gas in galaxy clusters).
- After subtracting these sources, however we are left with a diffuse background which is nearly isotropic at energies $\varepsilon>1 \mathrm{keV}$


Differential number flux

- Integrating over photon energies above $\varepsilon>1 \mathrm{keV}$

$$
\left.\frac{d F}{d \Omega} \sim 10^{6} \int_{1}^{\infty} \varepsilon_{(\mathrm{keV})}{ }^{-2.3} \sim 10^{6}\right\} \text { photons } \mathrm{m}^{-2} \mathrm{~s}^{-1} \mathrm{Sr}^{-1}
$$

Assuming the emission at $\varepsilon>1 \mathrm{keV}$ is the result of diffuse processes occurring uniformly throughout a sphere of radius $D$

Consider the flux at the Earth, from a shell between $r$ and $r+d r$ in a cone of solid angle $d \Omega$

$d F=\frac{j d V}{4 \pi r^{2}}=\frac{j r^{2} d r d \Omega}{4 \pi r^{2}}$

$$
j=\text { Emissivity per unit volume }
$$

From the entire cone

$$
\frac{d F}{d \Omega}=\int_{0}^{D} \frac{j d r}{4 \pi}=\frac{\bar{j} D}{4 \pi}
$$

What is the origin of this emissivity?

Until quite recently there were two main contenders:

1. Inverse Compton scattering of diffuse photons by very high energy (e.g. cosmic ray) electrons.
2. Large numbers of discrete sources at cosmological distances which are (or at least were) too small to be resolved.

How large a $\bar{j}$ do we need?
Suppose we take $D$ equal to the Horizon distance (max. proper distance of a source now whose light has just reached us).
$D \sim 5 \times 10^{26} \mathrm{~m} \Rightarrow \bar{j}=\frac{4 \pi}{D} \frac{d F}{d \Omega} \sim 2 \times 10^{-20}$ photons s ${ }^{-1} \mathrm{~m}^{-3}$

Could IC scattering in intergalactic space achieve the observed photon flux?

Suppose we have a diffuse background of photons at frequency $v_{0}$, and after IC scattering by electrons of Lorentz factor $\gamma$ the photons have a typical energy $\varepsilon \sim \frac{4}{3} \gamma^{2} h \nu_{0}$

Thus

$$
\underbrace{j_{I C}=\frac{4}{3} Q_{I C} c n_{e} \gamma^{2}\left(\frac{U_{v_{0}}}{h v_{0}}\right)}_{\text {Number density of electrons with } \gamma>\left(\frac{3}{4} \frac{1 \mathrm{kvv}}{h v_{0}}\right)^{1 / 2}}
$$

$n_{e}$ is very uncertain, but diffusion of cosmic rays out of galaxies suggests

$$
n_{e} \sim 10^{-7} \gamma^{-0.5} \mathrm{~m}^{-3}
$$

$$
\left.j_{I C}=\frac{4}{3} \times 10^{-7} \times 6.65 \times 10^{-29} \times 3 \times 10^{8} \gamma^{1.5}\left(\frac{U_{v_{0}}^{2}}{h v_{0}}\right)\right) \text { photons s }{ }^{-1} \mathrm{~m}^{-3}
$$

photon energy in eV

Consider photons from the CMBR: $T=3 \mathrm{~K} \Rightarrow h v_{0} \sim k T=2.6 \times 10^{-4} \mathrm{eV}$

Also $U_{v_{0}}=a T^{4} \sim 4 \times 10^{5} \mathrm{eV} \mathrm{m}^{-3} \quad$ (Recall $a=\frac{4 \sigma}{c}=7.53 \times 10^{-16} \mathrm{Jm}^{-3} \mathrm{~K}^{-4}$ )

So $j_{I C} \sim 4 \times 10^{-18} \gamma^{1.5}$ photons $\mathrm{s}^{-1} \mathrm{~m}^{-3}$

Comparison with eq. (4.16) suggests that we can comfortably produce enough emissivity to explain the observed number flux of diffuse $X$-ray photons.

For a long time, therefore, Inverse Compton scattering of the CMBR photons was considered a plausible mechanism to explain the diffuse $X$-ray background.

## However:

- More recent observations (e.g. from XMM - better angular resolution and sensitivity) have resolved the diffuse background into discrete sources.
- Good evidence that sources are Active Galaxies: i.e. X-ray emission is coming from the accretion of matter onto a supermassive black hole (see also GAL 2).
- These AGN are obscured in the optical, UV, IR by dust, but X-rays reveal their presence all the same.
- Supports prevailing view that all galaxies contain supermassive black holes at their cores.


### 4.3 The Sunyaev-Zel'dovich Effect

- We saw in HEA1, and in Section 4.2, that rich galaxy clusters can be strong sources of X-ray thermal bremsstrahlung emission, from hot intra-cluster gas.
- The energetic electrons which produce these bremsstrahlung photons in distant clusters can have a significant effect on CMBR photons, as they pass through these clusters on their way to us.
- The electrons can inverse Compton scatter the CMBR photons to higher energies, changing the spectrum of the CMBR in the direction of the cluster.
- This phenomenon is called the Sunyaev-Zel'dovich Effect and observations of the S-Z effect in galaxy clusters can be a very useful cosmological tool.
- Exploiting the S-Z effect is a good example of multiwavelength astronomy since we combine microwave / radio and X-ray observations of a cluster.


CMBR black-body spectrum averaged over whole sky with $T=2.725 \mathrm{~K}$

CMBR spectrum in direction of the cluster, shifted to higher frequencies by energy gain from inverse Compton electrons in cluster plasma


If we observe the CMBR in the Rayleigh-Jeans part of the black-body curve, the effect of the inverse Compton scattering is to produce a small dip in the CMBR spectrum, at a given frequency, in the direction of the cluster.


Conversely, at frequencies above about 200 GHz , the IC scattering produces an increment in the observed spectrum, compared with a pure black-body CMBR



Shows $\quad I_{S-Z}-I_{C M B R}$ as a function of frequency

- The temperature of X-ray intracluster gas implies that the electrons are fast but not highly relativistic (see also Section 9, HEA1 notes).
- Thus $\gamma \sim 1$ and for a single electron $L_{I C}=\frac{4}{3} Q_{I C} c U_{v}\left(\frac{v}{c}\right)^{2}$

Luminosity of inverse Compton-produced photons of frequency $v$

- Number of interactions per unit time between the electron and the photon distribution $=Q_{I C} n_{\text {phot }} c$

[^0]
## 9. Bremsstrahlung

Bremsstrahlung radiation is produced from the interaction of electrons with a proton at rest

## electron

For X-ray production, we want an electron which is fast but still non-relativistic
e.g. consider an electron with kinetic energy $E=10 \mathrm{keV}$

Compare this with the electron's rest mass energy:-

$$
\frac{1 / 2 m v^{2}}{m c^{2}}=0.02 \Rightarrow \frac{v^{2}}{c^{2}}=0.04 \Rightarrow v=0.2 c
$$

## 7. Reaction cross section

Consider a beam of particles (e.g. electrons) with number density $n$ (particles $\mathrm{m}^{-3}$ ), and velocity $v\left(\mathrm{~ms}^{-1}\right)$ incident on a thin 'target' containing $N_{\mathrm{T}}$ particles (e.g. protons) and with area $\mathrm{A}_{\mathrm{T}}\left(\mathrm{m}^{2}\right)$ perpendicular to the incident beam.


$$
N_{\mathrm{T}}=\int_{V} n_{\mathrm{T}}(\vec{r}) d V
$$

## 7. Reaction cross section

7.1 Incident Flux = number of beam particles crossing per unit area of the target per unit time

$$
F=n v \quad \mathrm{~m}^{-2} \mathrm{~s}^{-1}
$$

7.2 Reaction Rate $=$ number of interactions per unit time

$$
R \propto F N_{\mathrm{T}}
$$

## 7. Reaction cross section

We define the constant of proportionality to be the Reaction Cross Section, $Q$, which has units of area

$$
R=F N_{\mathrm{T}} Q
$$

The reaction cross section can be thought of as defining an effective area for collisions / interactions between the beam and target particles.

Number of beam particles passing through target per unit time $=F \mathrm{~A}_{\mathrm{T}}$

Number of interactions per unit time $=F N_{\mathrm{T}} Q$
Collision probability $=\frac{F N_{\mathrm{T}} Q}{F \mathrm{~A}_{\mathrm{T}}}=\frac{N_{\mathrm{T}} Q}{\mathrm{~A}_{\mathrm{T}}}$
i.e. we can think of a disc of area $Q$ associated with each target particle.

- Also $n_{\text {phot }}=U_{v} / h v$

So, average energy gained per electron-photon interaction
$=\quad \frac{\text { Average energy per unit time from IC interactions }}{\text { Number of interactions per unit time }}$

$$
\begin{equation*}
\langle\Delta \varepsilon\rangle=\frac{L_{I C}}{Q_{I C} n_{\mathrm{phot}} c}=\frac{\frac{4}{3} Q_{I C} c U_{v}(v / c)^{2}}{Q_{I C}\left(U_{v} / h v\right) c}=\frac{4}{3}\left(\frac{v}{c}\right)^{2} h v \tag{4.23}
\end{equation*}
$$

Fractional energy gain per electron-photon interaction

$$
\begin{equation*}
\left\langle\frac{\Delta \varepsilon}{\varepsilon}\right\rangle=\frac{4}{3}\left(\frac{v}{c}\right)^{2} \tag{4.24}
\end{equation*}
$$

- Since the electrons have a thermal distribution, $\frac{1}{2} m_{e}\left\langle v^{2}\right\rangle=\frac{3}{2} k T$ Hence,

$$
\begin{equation*}
\left\langle\frac{\Delta \varepsilon}{\varepsilon}\right\rangle=\frac{4 k T}{m_{e} c^{2}} \ll 1 \tag{4.25}
\end{equation*}
$$

So, if the initial photon energy is $\varepsilon_{\text {init }}$, then after 1 IC scattering

$$
\left\langle\frac{\varepsilon_{1}}{\varepsilon_{\text {init }}}\right\rangle=1+\frac{\langle\Delta \varepsilon\rangle}{\varepsilon_{\text {init }}}=1+\frac{4 k T}{m_{e} c^{2}}
$$

After 2 IC scatterings $\left\langle\frac{\varepsilon_{2}}{\varepsilon_{\text {init }}}\right\rangle=\left[1+\frac{4 k T}{m_{e} c^{2}}\right]^{2}$

After $N$ IC scatterings $\left\langle\frac{\varepsilon_{N}}{\varepsilon_{\text {init }}}\right\rangle=\left[1+\frac{4 k T}{m_{e} c^{2}}\right]^{N} \approx 1+\frac{4 N k T}{m_{e} c^{2}}$

To compute the total S-Z effect for a cluster, we must integrate along a line-of-sight column $\ell$ of plasma through the cluster

$$
\begin{equation*}
\left\langle\frac{\Delta \varepsilon}{\varepsilon}\right\rangle_{S-Z} \propto \int_{\ell} n_{e} \frac{k T}{m_{e} c^{2}} d \ell \tag{4.27}
\end{equation*}
$$

where $n_{e}$ is the density of electrons in the cluster plasma.

The S-Z effect as a cosmological distance indicator

- For e.g. an isothermal, spherical cluster of uniform electron density $n_{e}$ and volume $V$, measurement of the S-Z effect allows estimation of $\Gamma_{S Z}=n_{e} V$
- X-ray bremsstrahlung observations of the cluster allow estimation of the emission measure, which is equal to $\xi_{0}=n_{P}{ }^{2} V$
(For a plasma of pure hydrogen then $n_{e}=n_{P}$ )

Suppose we have a plasma with uniform temperature $T=T_{0}$
Then

$$
\begin{equation*}
\xi(T)=\xi_{0} \times \delta\left(T-T_{0}\right) \tag{2.4}
\end{equation*}
$$

Independent of temperature

And

$$
\begin{equation*}
\frac{d J}{d \varepsilon}=2\left(\frac{2}{\pi m_{e}}\right)^{1 / 2} \frac{Q_{0} m_{e} c^{2}}{k^{1 / 2} \varepsilon} \xi_{0} \frac{e^{-\varepsilon / k T_{0}}}{T_{0}^{1 / 2}} \tag{2.5}
\end{equation*}
$$

Matching up with HEA1, Sect. 11, we see that

$$
\begin{equation*}
\xi_{0}=n_{P}^{2} V \tag{2.6}
\end{equation*}
$$

Or, if the proton number density is not constant $\xi_{0}=\int_{V} n_{P}(\vec{r})^{2} d V$

So, finally, the quantity $\Gamma_{S Z}{ }^{2} / \xi_{0}$ is an estimate of $V$, the cluster volume and, the cluster diameter

$$
\Delta=2\left(\frac{3 V}{4 \pi}\right)^{1 / 3}
$$

We can then compare our S-Z diameter measurement with the observed angular diameter of the cluster and redshift of the cluster to estimate its distance.

S-Z distances can be used to estimate the Hubble Constant (expansion rate of the Universe) and measure the geometry of the Universe (i.e. open closed or flat).

Pros:

- S-Z distances on much greater scale than most other methods.
- Distance errors quite large but don't grow much with distance
- Completely different physics from other distance indicators


## Cons:

- Very small effect; requires highly sensitive CMBR measurements
- Clusters may not be spherical; bias towards those clusters along line of sight?
- Clusters may not be isothermal, uniform density plasma - more parameters to fit


## A survey of the $S-Z$ effect in

 hundreds of galaxy clusters is one of the main objectives of the Planck satellite, an ESA mission to study the CMBR, due for launch in 2007.
(The arrows above denote frequencies at which Planck will observe)

## PLANCK ©esa

Planck is the European Space Agency's misslon to map the structure of the Cosmic Microwave Background in unprecedented detail. Planck will observe the Universe as it was 300000 years after the Big
ang, will constrain cosmological models and examine the birth of large-scale structure in the Universe Bang, will constrain cosmological models and examine the birth of large-scale structure in the Universe


Cestorophrsics



[^0]:    Number density of photon 'beam' particles

