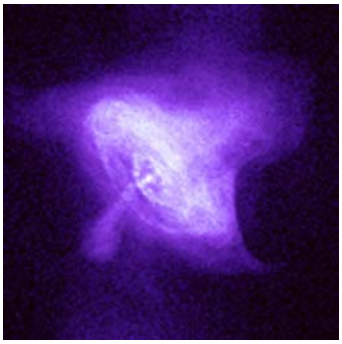
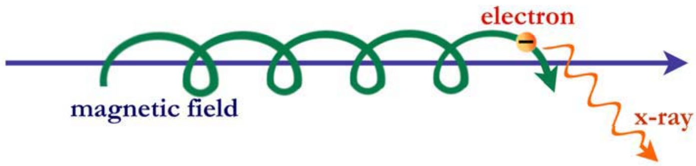
3. Cyclo/Synchrotron Sources

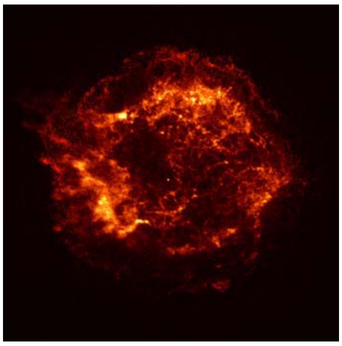
In HEA1 we developed some mathematical machinery to describe the spectrum of radiation from a synchrotron source (e.g. a supernova remnant).

Crab Nebula





Cassiopeia A



In this section we discuss how we can estimate the **magnetic field strength** of a synchrotron source.

Consider, for simplicity, a homogeneous source of volume V, magnetic field B, and with N electrons each of the same energy $E = \gamma m_e c^2$

[we could consider a *spectrum* of electron energies, but the algebra would be messier]

3.1 Estimating B for a synchrotron source: equipartition

Suppose we can measure the source volume, its synchrotron luminosity and frequency. Then, from HEA1 Sec 15

$$\boxed{L_S = \frac{4}{3} Q_T N c \gamma^2 U_B} \qquad (3.1)$$

and

$$\boxed{\nu_S = \frac{3}{2}\gamma^2\left(\frac{eB}{2\pi m_e}\right) = \frac{3}{2}\gamma^2 \nu_L} \qquad (3.2)$$

Also $U_B = \dfrac{B^2}{2\mu_0}$ so

$$\boxed{L_S = \frac{4}{3} Q_T N c \gamma^2 \frac{B^2}{2\mu_0}} \qquad (3.3)$$

Magnetic energy density

Equations (3.2) and (3.3) are 2 equations in 3 unknown parameters: $B \quad N \quad \gamma$

These two equations alone, then, cannot fix the values of these 3 parameters.

We can have:

A. Models with low B field and high Lorentz factor

B. Models with high B field and low Lorentz factor

Provided that $\gamma^2 B$ is consistent with the observed ν_S we are then free to adjust N so that $N\gamma$ matches the observed L_S

Models A and B differ, however, in their **total energy**

Provided the synchrotron source is not in a rapid state of change, but is near equilibrium, then we expect the source to be in *neither* state A or state B.

Can show from thermodynamic arguments that systems tend to **virialise** by moving to a minimum energy state.

Consider total energy

Magnetic energy = energy density x volume

$$\boxed{E_{\text{total}} = E_B + E_e \quad = \quad \frac{B^2}{2\mu_0}V + N\gamma m_e c^2} \qquad (3.4)$$

From (3.2)

$$\gamma = \left(\frac{4\pi m_e \nu_s}{3eB}\right)^{1/2} \qquad (3.5)$$

and from (3.1)

$$N\gamma = \frac{3}{4Q_T c}\frac{2\mu_0}{B^2}\frac{L_S}{\gamma} \qquad (3.6)$$

Combining (3.5) and (3.6)

$$\boxed{N\gamma = \frac{3}{4Q_T c}\frac{2\mu_0}{B^2}L_S\left(\frac{3eB}{4\pi m_e \nu_s}\right)^{1/2}} \qquad (3.7)$$

Substituting into (3.4)

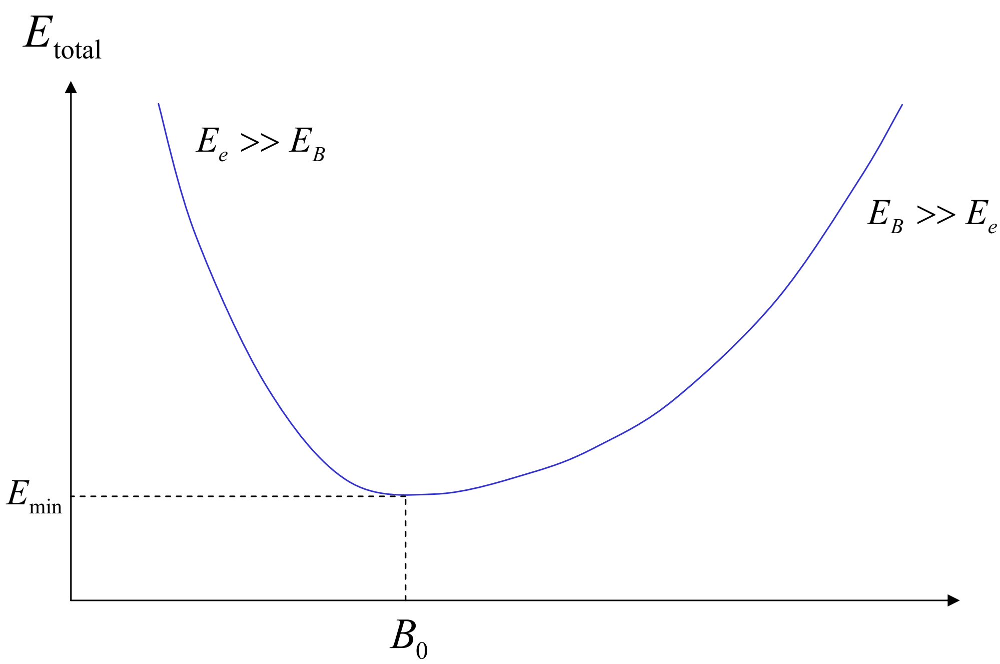
$$\boxed{E_{\text{total}} = E_B + E_e \quad = \quad \frac{V}{2\mu_0}\left(B^2 + \frac{\beta}{B^{3/2}}\right)} \qquad (3.8)$$

Where

$$\beta = \underbrace{\frac{3\mu_0^2 c}{Q_T}\left(\frac{3em_e}{4\pi}\right)^{1/2}}_{\text{Fundamental constants}} \underbrace{\left(\frac{L_S}{\nu^{1/2}\,V}\right)}_{\text{Observable from the data}}$$

We can find the total minimum energy as a function of B field by differentiating eq. (3.8).

This gives us an estimate of the B field for our synchrotron source



We can show (see Examples sheet 2) that

$$E = E_{\min} \quad \text{when} \quad \boxed{B = B_0 = \left(\frac{3\beta}{4}\right)^{2/7}} \qquad (3.9)$$

Note: we can also show that

$$\boxed{\frac{E}{E_{\min}} = \frac{3}{7}\left(\frac{B}{B_0}\right)^2 + \frac{4}{7}\left(\frac{B_0}{B}\right)^{3/2}} \qquad (3.10)$$

and when $B = B_0$ then $E_B / E_e = \frac{3}{4}$

i.e. magnetic energy ~ particle energy

⇨ **Energy equipartition**

Note also that when $B = \frac{1}{10} B_0$ then $E \sim 20\, E_{\min}$

and when $B = 10 B_0$ then $E \sim 40\, E_{\min}$

If we plug in some numbers for, e.g., the Crab Nebula (see Examples 2) then

$$\boxed{B_0 \sim 10^{-8}\ \text{Tesla}} \qquad (3.11)$$

This corresponds to a total energy of

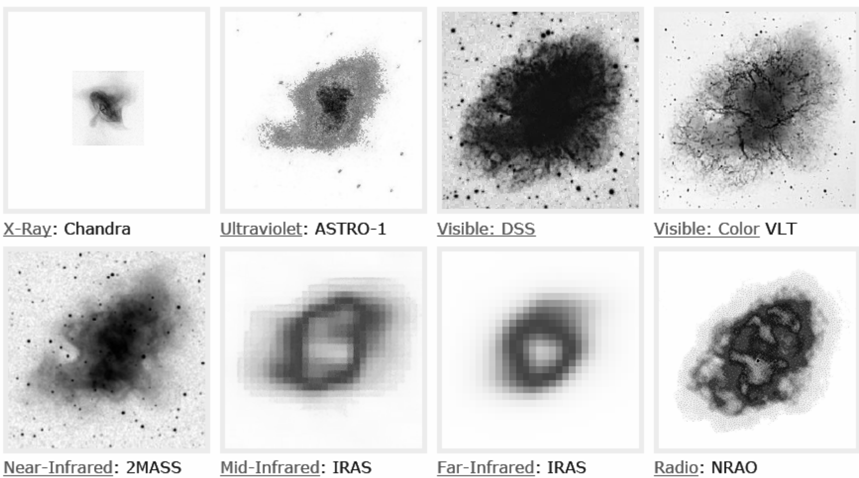
$$\boxed{E_{\min} \sim 10^{40}\ \text{J}} \qquad (3.12)$$

This corresponds to the **bolometric luminosity** of the Sun, for about 10^6 years.

3.2 Decay of synchrotron electrons

Q. **Were the high energy electrons that produce the Crab synchrotron emission accelerated in the original SN explosion, in 1054AD?**

What we observe is that the Crab is about the same size in X-rays, optical and radio. We can use this observation and an argument based on the lifetime of high-energy electrons to help answer the above question.



From NASA Goddard's Multiwavelength Astronomy site

Consider the lifetime of an electron emitting synchrotron radiation. (Similar argument in HEA1 for Inverse Compton lifetime).

We know that, for a single electron $$L_S = \left(\frac{dE}{dt}\right)_S = \frac{4}{3} Q_T \, c \gamma^2 U_B$$

and $$E = \gamma m_e c^2$$

Also, recall from HEA1 that $$Q_T = \frac{8\pi}{3}\left(\frac{e^2}{4\pi\varepsilon_0 mc^2}\right)^2 = \frac{8\pi}{3} r_e^2$$

So, a rough estimate of the lifetime of the electron is

$$\boxed{\tau_S \approx \frac{E}{\left(\frac{dE}{dt}\right)_S} = \frac{\gamma m_e c^2}{\frac{32}{9}\pi r_e^2 c\gamma^2 U_B} = \frac{9 m_e c}{32\pi r_e^2} \frac{2\mu_0}{\gamma B^2}} \qquad (3.13)$$

We can simplify this further by substituting from eq. (3.5) for the Lorentz factor in terms of the magnetic field and synchrotron frequency, i.e.

$$\gamma = \left(\frac{4\pi m_e \nu_s}{3eB}\right)^{1/2}$$

so that

$$\boxed{\begin{aligned}\tau_S &\approx \frac{E}{\left(\frac{dE}{dt}\right)_S} = \frac{9 m_e c}{32\pi r_e^2} \frac{2\mu_0}{B^2}\left(\frac{4\pi m_e \nu_s}{3eB}\right)^{-1/2} \\ &= \text{constant} \times B^{-3/2} \, \nu_S^{-1/2}\end{aligned}} \qquad (3.14)$$

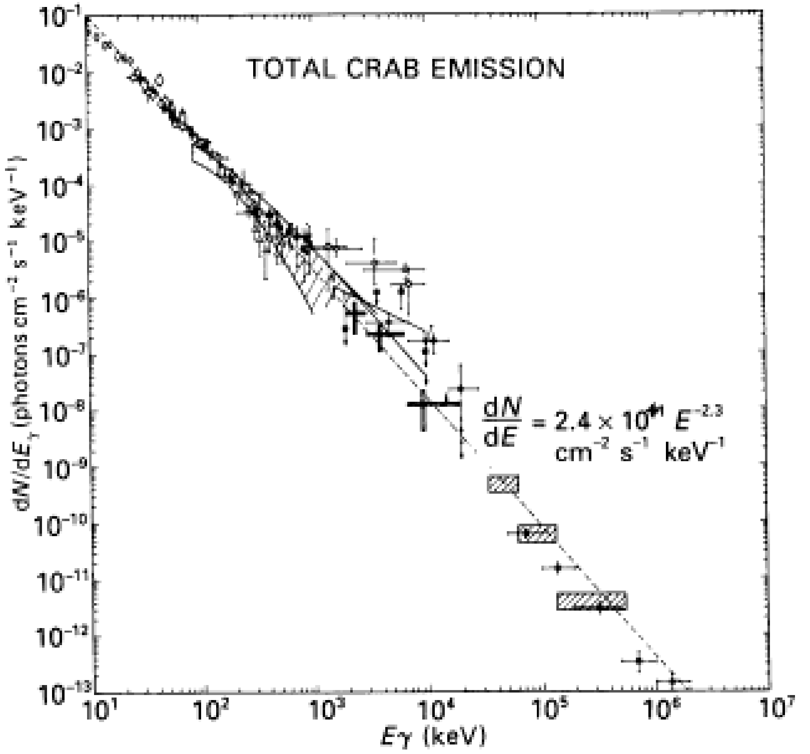
If the magnetic field is constant throughout the synchrotron emitting volume, then

$$\boxed{\tau_S \propto \frac{1}{\sqrt{\nu_S}}} \qquad (3.15)$$

The synchrotron spectrum should decay first at high frequencies (energies).

The observed emission from the Crab Nebula is well fitted by a **power law** in the X-ray and gamma ray region of the E-M spectrum

Equivalent to $dJ/d\varepsilon$



Adapted from "Handbook of Astronomy and Space Physics"

Equivalent to $dL/d\varepsilon$

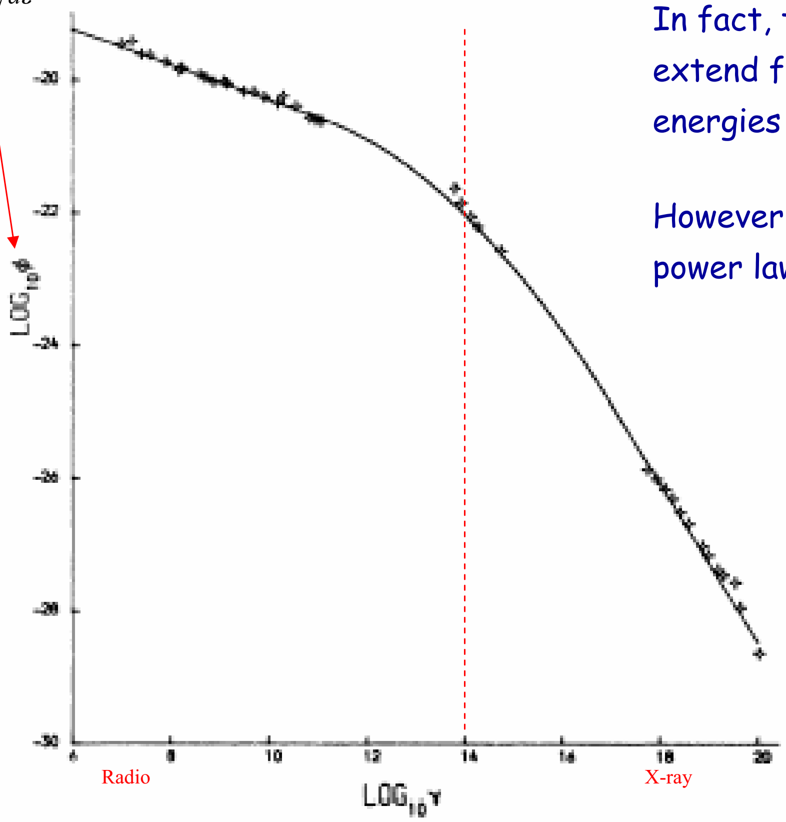


Fig. 1a. The curve shows the smooth approximation to the Crab spectrum calculated by the finite difference method described. The points shown correspond to the measured points in the Crab spectrum. (ϕ is measured in erg cm^{-2}s^{-1}Hz^{-1} and ν is measured in Hz)

Adapted from Brown et al. (1985)

In fact, this power law behaviour is seen to extend from radio to X-ray and gamma ray energies

However, there is good evidence that the power law changes *slope* at $\nu_s \sim 10^{14}$ Hz

Could the lower emission at higher energies be due to the decay of those high frequency photons since 1054 ?

Suppose we take $\nu_s \sim 10^{14}$ Hz in (3.14).

If we take $\tau_s \sim 1000$ yr then we can show that

$$\boxed{B \sim 3 \times 10^{-8}\ \mathrm{T}} \qquad (3.16)$$

This is consistent with our earlier, minimum energy estimate

So the idea that the break in the power law at high frequency could be due to synchrotron decay since the SN explosion hangs together pretty well.

Except for one major flaw…

If high (and low) energy electrons formed in the original SN explosion, and spread out radially, and if the the hig energy electrons have indeed decayed first, then we would expect the Crab to look **smaller** *at high frequency than at low frequency.*

Recall, however, that the size of the Crab is approximately the same at radio, optical and x-ray wavelengths.

Conclusion

The synchrotron-emitting electrons in the Crab were not accelerated in the SN explosion, but in the turbulent plasma *throughout* the nebula

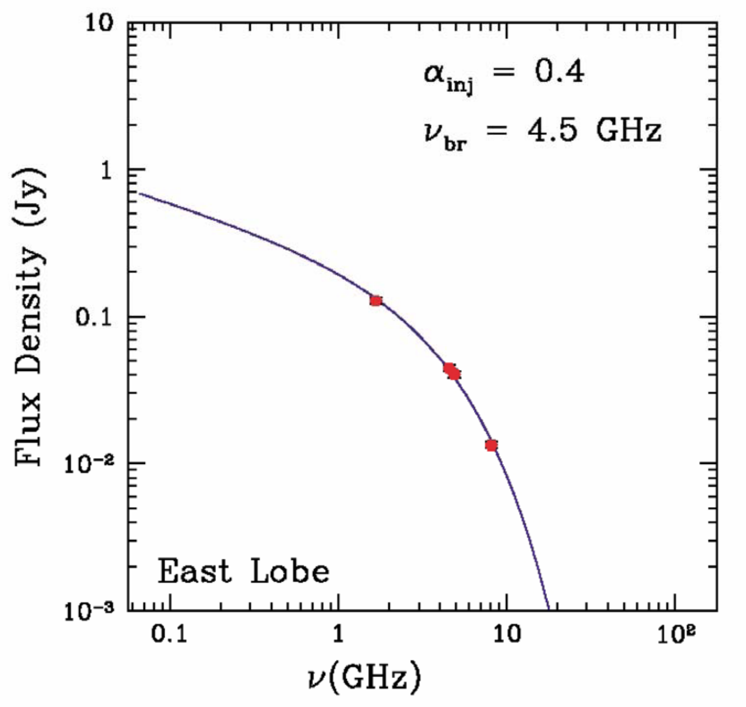
3.3 Synchrotron emission from AGN

The jets of Active Galactic Nuclei are often strong synchrotron sources, although now the emission is generally in the **radio** part of the spectrum: **radio galaxies**. (c.f. HEA1, Sect 15b and GAL2 course)

In AGN synchrotron spectra we again see evidence of a break at high frequencies, and this is again believed to be due to the radiative decay of high energy synchrotron electrons.

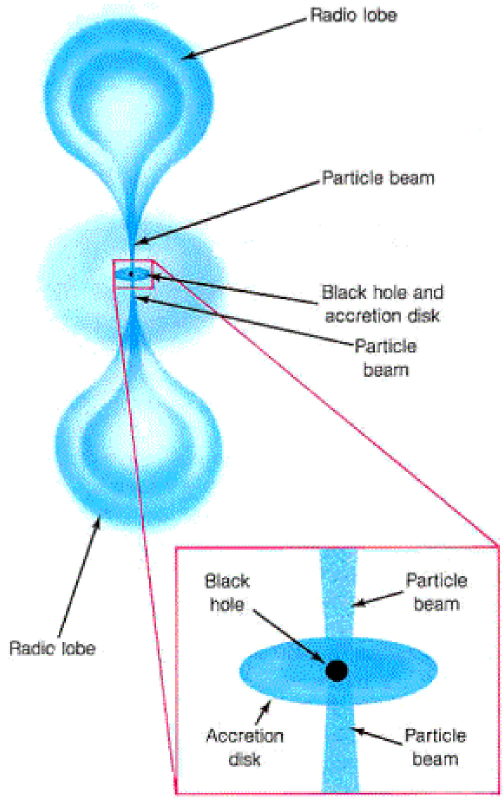
Recall eq. (3.15) above

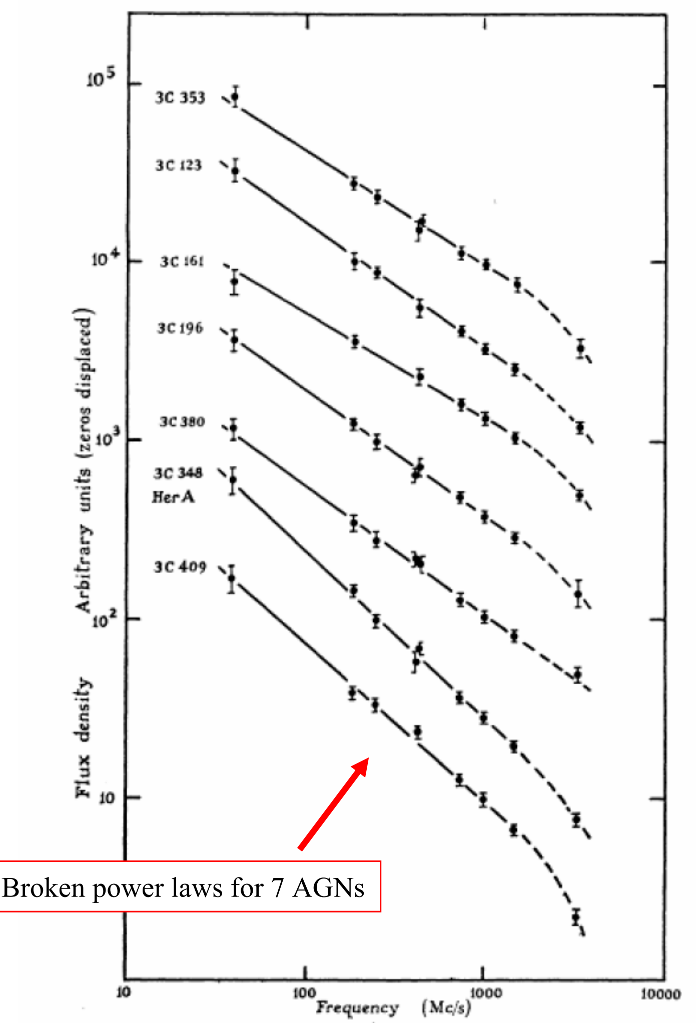
$$\boxed{\tau_S \propto \frac{1}{\sqrt{\nu_S}}}$$



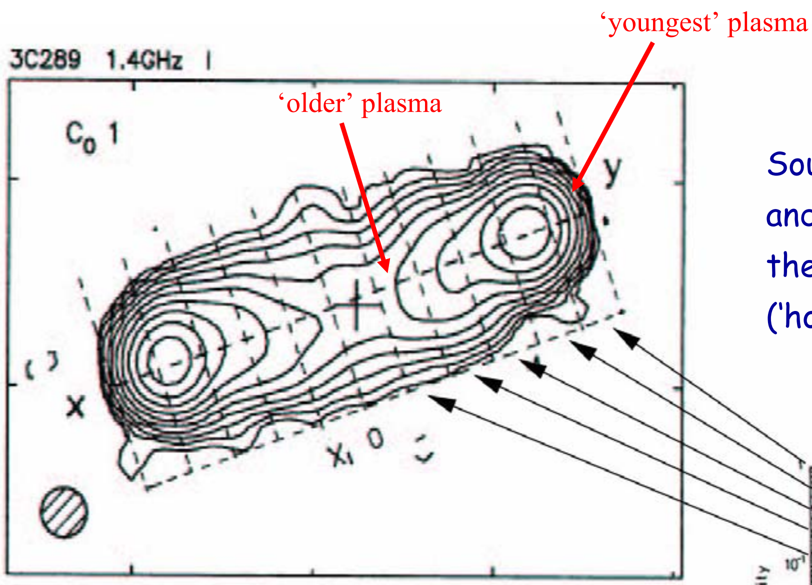
(From Cotter, Univ. of Oxford)

By measuring the **break frequency** we can estimate the age of the AGN.



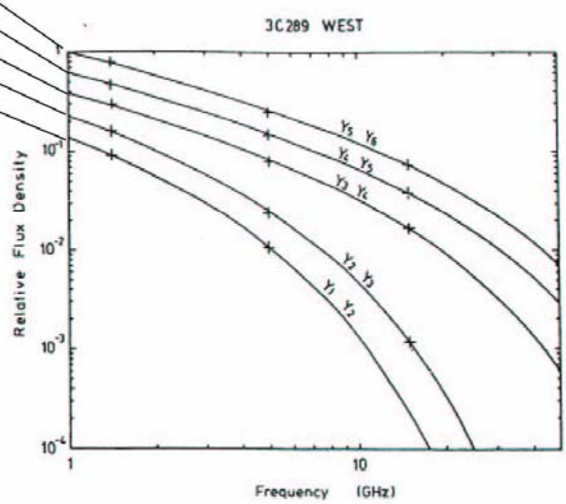


If we can map the spectrum from an extended source, we can estimate the age of different parts of the source.



Map of radio intensity for quasar 3C289

Sources grow over time (up to $0.1c$) and jets end in a strong 'shock' where they hit the intergalactic medium ('hotspot').



Synchrotron emitting plasma left behind as the source continues to grow.

Steps:

- Measure break frequency ν_{break} and total synchrotron luminosity L_S
- Use **equipartition argument** to estimate B and U_B
- Determine Lorentz factor $\gamma = \left(\dfrac{4\pi m_e \nu_{\text{break}}}{3eB}\right)^{1/2}$ of electrons at the break frequency
- Compute timescale for synchrotron decay of these electrons:

$$\tau_S \approx \frac{E}{\left(\frac{dE}{dt}\right)_S} = \frac{\gamma m_e c^2}{\frac{32}{9}\pi r_e^2 c \gamma^2 U_B}$$

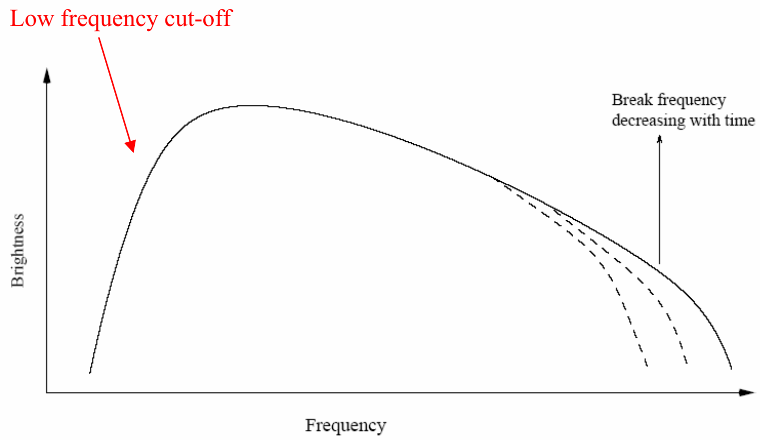
By these arguments we find lifetimes of AGN jets up to $\boxed{\tau_S \approx 10^8 \text{ years}}$

(See also simpler argument in Q.10 of Example Sheet 2, and GAL2 course)

3.4 Synchrotron self-absorption

As well as the break at high frequency due to radiative losses, synchrotron spectra also show a low frequency cut-off

In HEA1 when we derived the predicted power law photon spectrum for a power law electron energy spectrum, we implicitly assumed that **all** of the synchrotron photons emitted by each electron reach us.



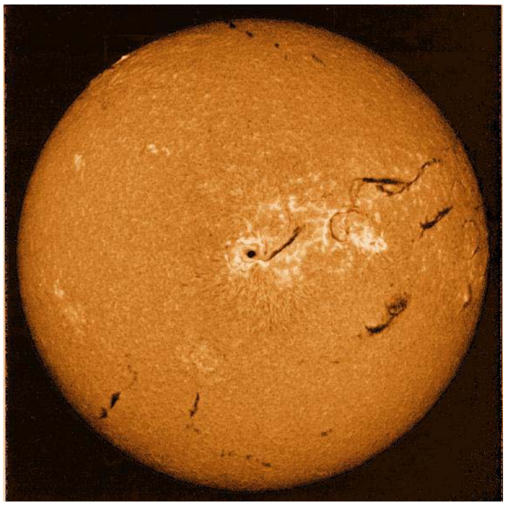
However, this will not in general be the case. As a photon propagates towards us through the plasma, there is a probability that it will interact with *another* synchrotron electron. We call this process **synchrotron self-absorption**

If the mean free path between interactions is very short, the result is that an outside observer only 'sees' synchrotron photons which are emitted from a thin layer below the surface of the source.

Below this surface layer, photons never really escape; instead we can think of the synchrotron electrons just continually exchanging photons in quasi-equilibrium.

The observed flux of synchrotron photons is, therefore, much smaller than if all the synchrotron photons escaped from the source.

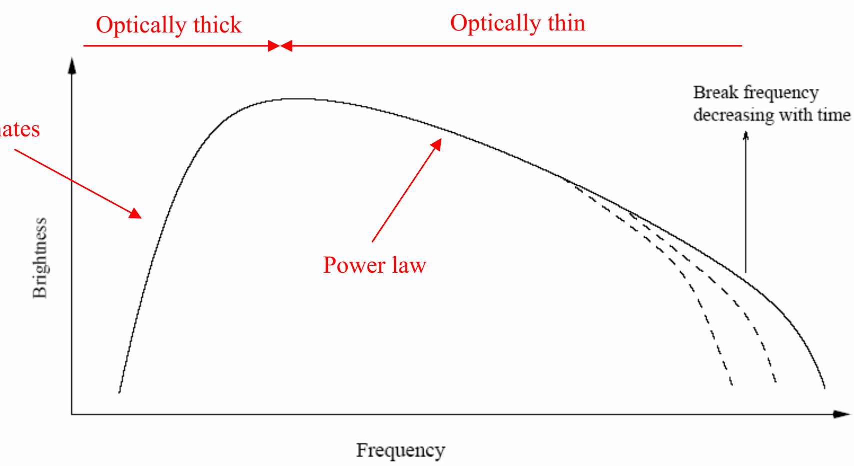
This is closely analogous to viewing the surface of a dense medium in thermal equilibrium – e.g. the **photosphere** of the Sun.



Continuing the analogy, we use the terminology:

photon mfp << source size	**Optically thick**
photon mfp > source size	**Optically thin**

- The absorption cross-section for a synchrotron electron *decreases* with frequency (in HEA1 Sect 15b we ignored this dependence, for simplicity)
- This means that the mean free path between interactions is shorter at lower frequencies.
- For a source of a given size, we 'see' deeper into the source as our observing frequency increases.
- Once the mean free path ~ the source size, we can 'see' all the way through the source. At higher frequencies we then recover the power law behaviour derived in HEA1



- The precise form of the synchrotron spectrum, including self-absorption, is very complicated (see e.g. Longair vol.2 Ch. 18)

- We can at least understand what is the *slope* of the self-absorption dominated portion of the spectrum, however.

We assign an **effective temperature** to the synchrotron electrons

(Can do this even though electrons aren't thermal)

$$\boxed{k\,T_{\text{eff}} \sim \gamma\, m_e c^2} \qquad (3.17)$$

Also, from eq. (3.5) $\quad \gamma = \left(\dfrac{4\pi m_e \nu_s}{3eB}\right)^{1/2}$ $\quad$ Hence $\quad \boxed{T_{\text{eff}} \sim \nu_S^{\,1/2}}$

(3.18)

For a self-absorbed source, effective temperature = brightness temperature

Adapted from Astronomy 2 Observational Astrophysics notes

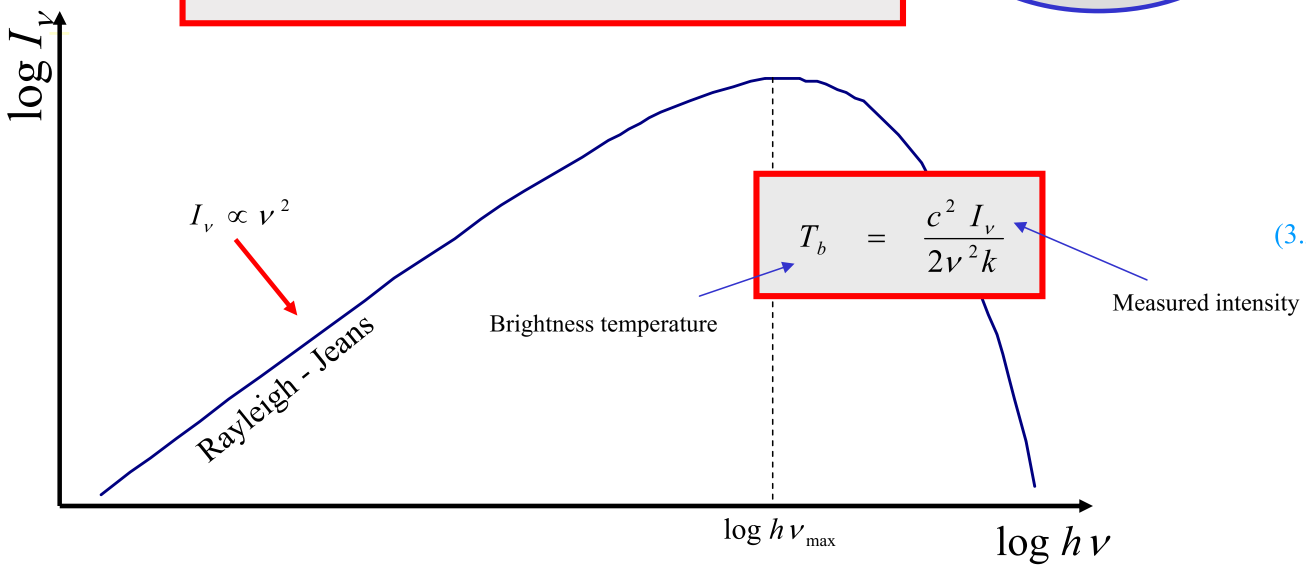
Brightness temperature

At typical radio frequencies and temperatures $\quad h\nu << kT \quad \Rightarrow \quad \exp\left(\frac{h\nu}{kT}\right) - 1 \approx \frac{h\nu}{kT}$

Hence
$$\boxed{I_\nu = \frac{2h\nu^3}{c^2\left[\exp\left(\frac{h\nu}{kT}\right)-1\right]} \cong \frac{2\nu^2 kT}{c^2}}$$

Rayleigh – Jeans approximation

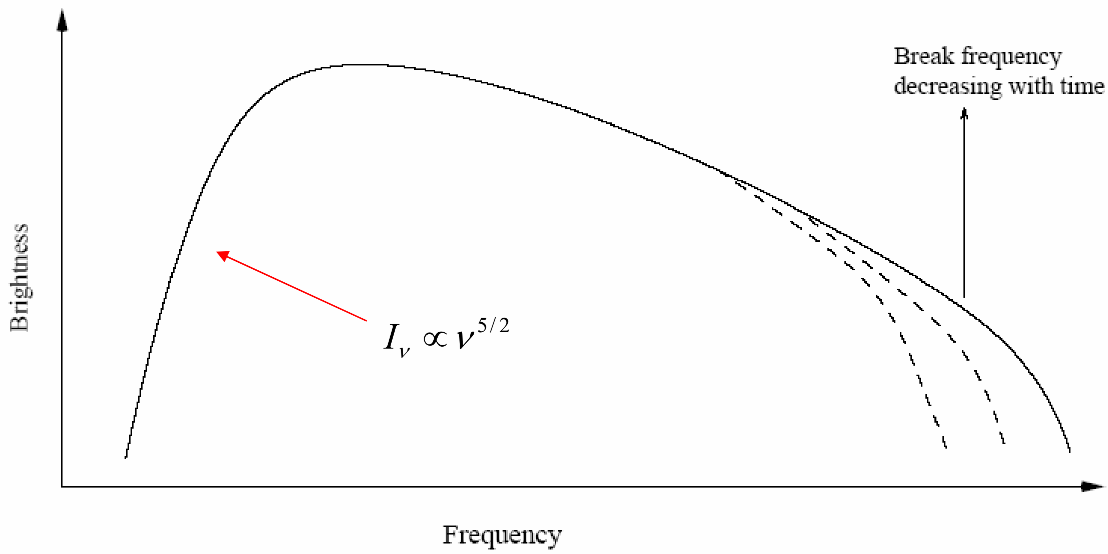
(3.19)



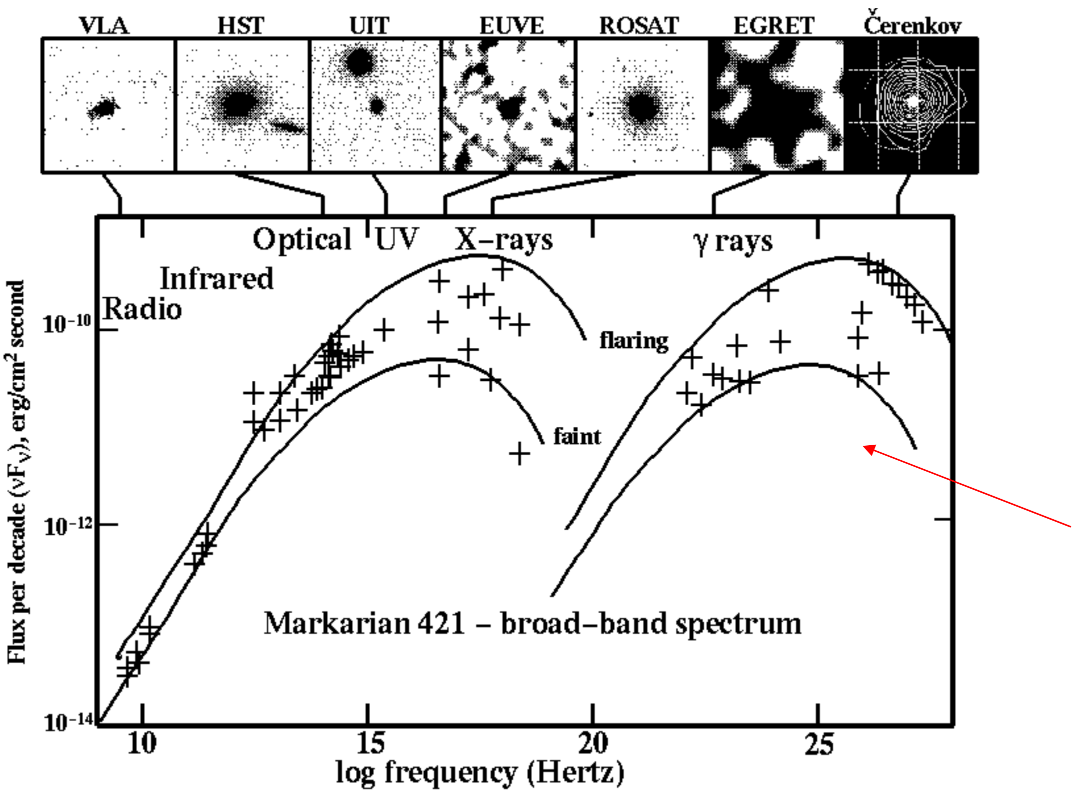
(3.20)

So the measured intensity, or brightness, in the self-absorbed domain is:

$$I_\nu \propto \nu^2 T_b \quad \text{i.e.} \quad \boxed{I_\nu \propto \nu^{5/2}} \qquad (3.21)$$



We see this low frequency cut-off in synchrotron spectra from AGN and GRBs



What about the second peak?

This is called Synchrotron Self-Compton

See Section 4

Credit: Bill Keel, Univ of Alabama)