HEA II Ex Sheet 3 Model Answers

1) Assuming the source is isotropic, $L_x = 4\pi d^2 F$

$$d = 50 \times 3.09 \times 10^{19}\ \mathrm{m} \qquad 5\ \mathrm{keV} = 8 \times 10^{-16}\ \mathrm{J}$$

$$F = \left(\frac{50 \times 8 \times 10^{-16}}{0.04}\right)\ \mathrm{Wm^{-2}} = 10^{-12}\ \mathrm{Wm^{-2}}$$

$$\Rightarrow \quad L_x = 4\pi \times \left(1.545 \times 10^{21}\right)^2 \times 10^{-12}\ \mathrm{W}$$

$$= 3 \times 10^{31}\ \mathrm{W}$$

2) $$L_x = \int_{0.5}^{10} \frac{dL}{d\varepsilon}\, d\varepsilon = \int_{0.5}^{10} 2\left(\frac{2}{\pi m_e}\right)^{\frac{1}{2}} \frac{n_p^2\, V Q_0\, m_e c^2}{(kT)^{\frac{1}{2}}}\, e^{-\varepsilon/kT}\, d\varepsilon$$

Our goal is to determine n_p

$kT = 1.38 \times 10^{-15}\ \mathrm{J} = 8.6\ \mathrm{keV}$, so putting $y = \varepsilon/kT$

when $\varepsilon = 0.5, \quad y = 0.058 = y_1$

$\varepsilon = 10, \quad y = 1.163 = y_2$

$$L_x = 2\left(\frac{2}{\pi m_e}\right)^{\frac{1}{2}} (kT)^{\frac{1}{2}}\, V Q_0\, m_e c^2 \int_{y_1}^{y_2} e^{-y}\, dy \; n_p^2$$

$$= \text{const.} \times \left[-e^{-y}\right]_{y_1}^{y_2} n_p^2$$

$$\text{const.} = 2 \times \left(\frac{2}{\pi \times 9.1 \times 10^{-31}}\right)^{\frac{1}{2}} \times \left(1.38 \times 10^{-15}\right)^{\frac{1}{2}} \times 2.67 \times 10^{67} \times 1.54 \times 10^{-31} \times 9.1 \times 10^{-31} \times \left(3 \times 10^{8}\right)^2$$

$$= 2 \times 10^{31} \quad \text{(in SI units)}$$

(2)

So, $5 \times 10^{37} = 2 \times 10^{31} \times \left[e^{-0.058} - e^{-1.163} \right] n_p^2$

$\Leftrightarrow \quad n_p = 2 \times 10^3$ protons / m^3

Total mass of intracluster gas $= m_p n_p V$

$= 8.9 \times 10^{43}$ kg

Thermal energy $= 3 N_p k T$

$= 3 n_p V k T$

$= 3 \times 2 \times 10^3 \times 2.67 \times 10^{67} \times 1.38 \times 10^{-15}$ J

$= 2.2 \times 10^{56}$ J

Cooling time $\tau \sim E/L = \dfrac{2.2 \times 10^{56}}{5 \times 10^{37}} = 4.4 \times 10^{18}$ s

$= 1.4 \times 10^{11}$ yrs

3) Assuming $U_B \sim U_{opt} = 10^6 \, eV m^{-3}$

$= 1.6 \times 10^{-13} \, J m^{-3}$

$= \dfrac{B^2}{2\mu_0}$

$\Rightarrow \quad B^2 = 1.6 \times 10^{-13} \times 2 \times 1.26 \times 10^{-6}$

$B = 6.4 \times 10^{-10}$ T

A photon of $\lambda = 600$nm has $\nu = 5 \times 10^{14}$ Hz $\Rightarrow E_\nu = 3.3 \times 10^{-19}$ J

$\therefore \quad U_{opt}$ corresponds to $\left(1.6 \times 10^{-13} / 3.3 \times 10^{-19} \right) = 4.8 \times 10^5$ photons m^{-3}

Taking $\nu_L = \dfrac{eB}{2\pi m_e} = \dfrac{1.6 \times 10^{-19} \times 6.4 \times 10^{-10}}{2\pi \times 9.1 \times 10^{-31}} = 17.9$ Hz

We want $\nu_s = 10^{18}$ Hz $\Rightarrow \frac{3}{2}\gamma^2 = \frac{10^{18}}{17.9}$

$$\Longleftrightarrow \quad \gamma = 1.9 \times 10^{8}$$

4) Number density of CMBR photons $= \frac{5 \times 10^{-14}}{3 \times 10^{-4} \times 1.6 \times 10^{-19}}$ m^{-3} $\simeq 10^9$ photons m^{-3}

$\Rightarrow n_\gamma / n_H \simeq 10^3$

5) Column density $= 2 \times 10^{26}$ H atoms m^{-2}

We take this as 2×10^{26} H m^{-2} $= n_H \times d$

$$= \left(\frac{3 \times 10^{-21}}{1.67 \times 10^{-27}}\right) d$$

$$\Rightarrow \quad d = \left(\frac{1.67 \times 10^{-27} \times 2 \times 10^{26}}{3 \times 10^{-21}}\right) \text{ m}$$

$$= 1.1 \times 10^{20} \text{ m} \simeq 3.6 \text{ kpc}$$

If the column density were 2×10^{28}, this would imply a distance of 360 kpc which is too large for our galaxy.

6) Using the thermal bremsstrahlung formula:

$$L_X = 2\left(\frac{2}{\pi m_e}\right)^{1/2} (kT)^{1/2}\, V Q_0 m_e c^2 \left[e^{-y_1} - e^{-y_2}\right] n_p^2$$

where $y_i = \varepsilon_i / kT$

If we assume that we integrate over X-ray energies from 0.1 keV to 100 keV (i.e. soft to hard)

(4)

$kT = 1.38 \times 10^{-16} J = 0.86 \text{ keV} \Rightarrow y_1 = 0.116, \; y_2 = 116$

Integrated flux $= 10^{-4} \, Wm^{-2}$

$\Rightarrow L_x = 4\pi d^2 F$ (assuming isotropic emission)

$= 2.8 \times 10^{19} \, W$

Putting all this together, with $V = \pi r^2 h = 9.8 \times 10^{22} \, m^3$

$$L_x = 2 \times \left(\frac{2}{\pi \times 9.1 \times 10^{-31}}\right)^{1/2} \left(1.38 \times 10^{-16}\right)^{1/2} \times 9.8 \times 10^{22} \times 1.54 \times 10^{-31} \times 9.1 \times 10^{-31} \times 9 \times 10^{16} \times 0.89 \times n_p^2$$

$$\Leftrightarrow \quad n_p^2 = \frac{2.8 \times 10^{19}}{2.2 \times 10^{-14}} = 3.4 \times 10^{16} \, m^{-3}$$

Assuming hydrogen plasma, $n_p = n_e \Rightarrow n_e = 3.4 \times 10^{16} \, m^{-3}$

Thermal energy density $= 3 n_e kT = 14 \, Jm^{-3}$

mag. energy density $U_B = \frac{B^2}{2\mu_0} = \frac{0.2^2}{2 \times 1.26 \times 10^{-6}} = 1.6 \times 10^4 \, Jm^{-3}$

7) Applying the same formulae as in Q.6; $y_1 = 0.06$, $y_2 = 58.8$

$$L_x = 2 \times \left(\frac{2}{\pi \times 9.1 \times 10^{-31}}\right)^{1/2} \left(2.76 \times 10^{-16}\right)^{1/2} \times \frac{4}{3}\pi \times (10^8)^3 \times 1.54 \times 10^{-31} \times 9.1 \times 10^{-31} \times c^2 \times 0.94 \times 10^{28}$$

$= 1.4 \times 10^{16} \, W$

Mass of plasma $= 10^{14} \times 1.67 \times 10^{-27} \times \frac{4}{3}\pi (10^8)^3 = 7 \times 10^{11} \, kg$

Mass of each sphere $= 1.75 \times 10^{11}$ kg

no. of protons in each sphere $= 1.05 \times 10^{38}$

number density $= 1.05 \times 10^{38} / (\frac{4}{3}\pi\, 10^{21}) = 2.5 \times 10^{16}\ m^{-3}$

$$\Rightarrow L_x = 4 \times 2 \times \left(\frac{2}{\pi \times 9.1 \times 10^{-31}}\right)^{1/2} \left(2.76 \times 10^{-16}\right)^{1/2} \times \frac{4}{3}\pi \left(10^{7}\right)^{3} \times 1.54 \times 10^{-31} \times 9.1 \times 10^{-31} \times 9 \times 10^{16} \times 0.94 \times \left(2.5 \times 10^{16}\right)^{2}$$

So the volume $\downarrow$ 250, no. density $\uparrow (250)^2$

$\Rightarrow$ luminosity increased by a factor of 250

$\Rightarrow L_x \simeq 3.5 \times 10^{18}$ W

9) Number of neutrinos released $= \left(\frac{10^{46} \times (1.6 \times 10^{-19})^{-1}}{10^{6}}\right) = 6.25 \times 10^{58}$

How many neutrinos passed through each m^2

$$N_\nu = \frac{6.25 \times 10^{58}}{4\pi \times (50 \times 3.09 \times 10^{19})^2} = 2 \times 10^{15}$$

taking the projected area of each human as $\sim 1\ m^2$

$\Rightarrow 2 \times 10^{15}$ passed through each person

10) Scale height, $H \sim \frac{2kT}{m_{Fe}\, g}$

$$g = \frac{GM}{R^2} = \frac{6.67 \times 10^{-11} \times 1.4 \times 2 \times 10^{30}}{10^{8}} = 1.9 \times 10^{12}\ kg\,ms^{-2}$$

$m_{Fe} = 56 \times 1.67 \times 10^{-27} = 9.35 \times 10^{-26}$ kg $\Rightarrow H = 1.6 \times 10^{-4}$ m

$= 0.16$ mm