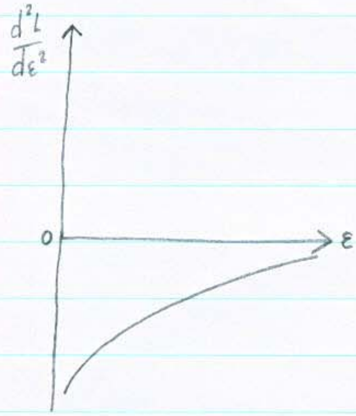
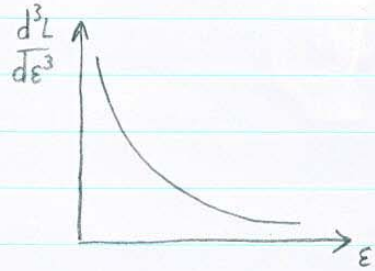
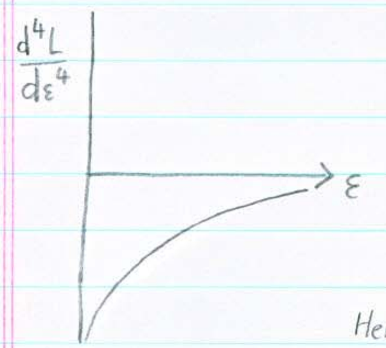
HEA II Ex Sheet 2 Model Answers

1) 





etc.

Thus $H(\varepsilon) \equiv dL/d\varepsilon$

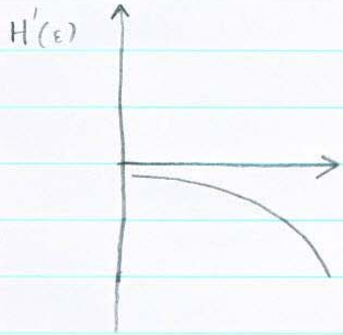
has $H'(\varepsilon) \leq 0$

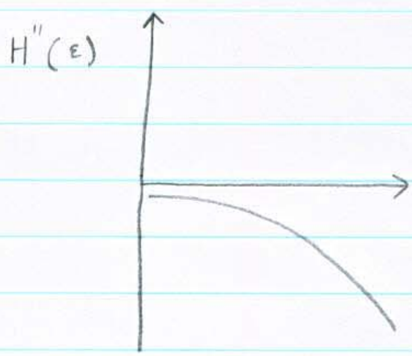
$H''(\varepsilon) \geq 0$

$H'''(\varepsilon) \leq 0$ etc

Hence $H(\varepsilon)$ satisfies the derivative constraints for thermal bremsstrahlung

2) The source cannot be 100% thermal bremsstrahlung because $H''(\varepsilon) < 0 \;\; \forall \varepsilon$, which violates the derivative constraint





For non-thermal bremsstrahlung, we require only $H'(\varepsilon) \leq 0$ which is satisfied by the data. => source could be NTB

3) $\frac{\partial E_{total}}{\partial B} = \frac{V}{2\mu_0}\left[2B - \frac{3}{2}\beta/B^{5/2}\right] = 0 \iff B^{7/2} = \frac{3}{4}\beta$

Hence $B_0 = \left(\frac{3\beta}{4}\right)^{2/7}$

4) $$E_{min} = \frac{V}{2\mu_0}\left[\left(\frac{3\beta}{4}\right)^{\frac{4}{7}} + \beta\left(\frac{4}{3\beta}\right)^{3/7}\right]$$

$$= \frac{V}{2\mu_0}\left[\left(\frac{3}{4}\right)^{4/7} + \left(\frac{4}{3}\right)^{3/7}\right]\beta^{4/7}$$

$$= \frac{V}{2\mu_0} B_0^2 \left[1 + \frac{4}{3}\right] = \frac{V}{2\mu_0} \times \frac{7}{3} B_0^2 \quad \text{as required}$$

$$\frac{E}{E_{min}} = \left(B^2 + \frac{\beta}{B^{3/2}}\right) \Big/ \frac{7}{3}B_0^2$$

$$= \frac{3}{7}\left(\frac{B}{B_0}\right)^2 + \left(\frac{4}{3}\frac{B_0^{7/2}}{B^{3/2}}\right) \Big/ \frac{7}{3}B_0^2$$

$$= \frac{3}{7}\left(\frac{B}{B_0}\right)^2 + \frac{4}{7}\left(\frac{B_0}{B}\right)^{3/2}$$

5) When $B = B_0$, $E_B = \frac{V}{2\mu_0} B_0^2 = \frac{3}{7} E_{min}$

Similarly $E_e = \frac{4}{7} E_{min}$ $\Rightarrow$ $E_B = \frac{3}{4} E_e$

6) When $B = \frac{1}{10} B_0$, $E/E_{min} = \frac{3}{7}\left(\frac{1}{10}\right)^2 + \frac{4}{7}\left(10\right)^{3/2}$

$= 18.1$

When $B = 10 B_0$, $E/E_{min} = \frac{3}{7}(10)^2 + \frac{4}{7}\left(\frac{1}{10}\right)^{3/2} = 42.9$

7) From the given data: $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left(1.5 \times 3.09 \times 10^{16}\right)^3$
$= 4.2 \times 10^{50}\ m^3$

If $E = 1\ keV = 1.6 \times 10^{-16} J$
$\Rightarrow \nu = E/h = 1.6 \times 10^{-16} / 6.63 \times 10^{-34} = 2.4 \times 10^{17}\ Hz$

Hence $\beta = \frac{3 \times \left(1.26 \times 10^{-6}\right)^2 \times 3 \times 10^8}{6.65 \times 10^{-29}} \left(\frac{3 \times 1.6 \times 10^{-19} \times 9.1 \times 10^{-31}}{4\pi}\right)^{1/2} \frac{4.9 \times 10^{30}}{\sqrt{2.4 \times 10^{17}}\ 4.2 \times 10^{50}}$

$= 9.53 \times 10^{-29}$ (in SI units)

8) So $B_0 = \left(\frac{3 \times 9.53 \times 10^{-29}}{4}\right)^{2/7} = 9.1 \times 10^{-9}\ T$

$E_{min} \sim \frac{4.2 \times 10^{50}}{2 \times 1.26 \times 10^{-6}} \times \frac{7}{3} \times \left(9.1 \times 10^{-9}\right)^2 = 3.2 \times 10^{40}\ J$

9) constant $= \frac{9\, m_e c\ 2\mu_0}{32\pi r_e^2}\left(\frac{4\pi m_e}{3e}\right)^{-1/2}$

$= \frac{9 \times 9.1 \times 10^{-31} \times 3 \times 10^8 \times 2 \times 1.26 \times 10^{-6}}{32 \times \pi \times (2.82 \times 10^{-15})^2}\left(\frac{4\pi \times 9.1 \times 10^{-31}}{3 \times 1.6 \times 10^{-19}}\right)^{-1/2} \simeq 1.59 \times 10^6$

$$\tau_s = 1000\,\text{yr} = 3.16\times10^{10}\,\text{s} = 1.6\times10^{6}\,B^{-3/2}\times\left(10^{14}\right)^{-1/2}$$

$$B^{-3/2} = \frac{3.16\times10^{10}}{1.6\times10^{-1}} = 1.98\times10^{11}$$

$$\Rightarrow \quad B \sim 3\times10^{-8}\,\text{T} \quad \text{as required}$$

10) See galaxies2, example sheet 3, Q.2