HEA II Ex Sheet 1 — Model Answers

1) High mass XRB : Mass of optical companion $\gg M_\odot$
Low mass XRB : " " " " $\sim M_\odot$

X-ray power source believed to be accretion of mass from optical companion onto compact object $\Rightarrow$ release of large amount of P.E.

HMXBs – concentrated along gal. plane, but at all gal. longitudes
LMXBs – peaked towards gal. centre, but over wider range of latitudes.

This is consistent with our SF understanding : HMXBs require recent SF, which needs gas-rich environment (i.e. gal. disk). LMXBs can be much older systems - i.e. in galactic bulge, Globular clusters.

2) Energy yield / unit mass $\simeq \dfrac{GM_{NS}}{R_{NS}} = \dfrac{6.673\times10^{-11}\times2\times2\times10^{30}}{10^4}$

$$= 2.67\times10^{16}\ m^2 s^{-2}$$
$$= 0.3c^2\ J\,kg^{-1}$$
$$= 30\%\ \text{of rest mass energy}$$

XRB luminosity comes from disk as well as NS, reducing energy yield. Some energy lost to friction, KE of disk rotation

(2)

3) $$F_{grav} = \frac{GMm_p}{r^2}\left[1 - \frac{L\sigma_T}{4\pi GMm_pc}\right]$$

$$= \frac{GMm_p}{r^2}\left[1 - L/L_{crit}\right] \quad \text{as required}$$

$$L_{crit} = \frac{4\pi GMm_pc}{\sigma_T} = 4\pi G\left(\frac{M}{M_\odot}\right)\frac{M_\odot m_p c}{\sigma_T}$$

$$= \frac{4\pi \times 6.673\times10^{-11} \times 2\times10^{30} \times 1.67\times10^{-27} \times 3\times10^{8}}{6.67\times10^{-29}}\left(\frac{M}{M_\odot}\right) \text{ W}$$

$$= 1.27\times10^{31}\left(\frac{M}{M_\odot}\right) \text{ W} = 3.2\times10^{4} L_\odot \left(\frac{M}{M_\odot}\right)$$

4) $$L_{acc} = \frac{GM}{r}\left[1 - \frac{L_{acc}}{L_{crit}}\right]\dot{M} = \frac{GM\dot{M}}{r} - L_{acc}\frac{GM\dot{M}}{rL_{crit}}$$

$$L_{acc}\left[1 + \frac{GM\dot{M}}{rL_{crit}}\right] = \frac{GM\dot{M}}{r}$$

$$\Leftrightarrow \quad L_{acc}\left[\frac{rL_{crit} + GM\dot{M}}{rL_{crit}}\right] = \frac{GM\dot{M}}{r}$$

$$\Leftrightarrow \quad L_{acc} = \frac{GM\dot{M}\,L_{crit}}{rL_{crit} + GM\dot{M}} = \frac{L_{crit}}{1 + L_{crit}/GM\dot{M}/r}$$

(3)

For small $\dot{M}$ $\quad L_{acc} \ll L_{crit} \Rightarrow L_x \sim \frac{GM\dot{M}}{r}$

$$\Leftrightarrow \dot{M} \sim \frac{L_x}{GM/r}$$

taking $M = 2M_\odot$ $\quad \dot{M} \sim \left(\frac{L_x}{10^{31}W}\right) \frac{\times 10^{31} \times 10^4}{6.67\times10^{-11}\times 4\times10^{30}} \ kg\,s^{-1}$

$$= 3.75 \times 10^{14} \ kg\,s^{-1} \left(L_x/10^{31}W \right)$$

$$= 1.18 \times 10^{22} \ kg\,yr^{-1} \left(L_x/10^{31}W \right)$$

$$\simeq 6 \times 10^{-9} \left(\frac{L_x}{10^{31}W}\right) M_\odot\,yr^{-1}$$

5) $\quad m_p \frac{dv_w}{dt} = m_p v_w \frac{dv_w}{ds} = \frac{K}{s^2}$

$$\Leftrightarrow v_w\,dv_w = \frac{K}{m_p}\frac{ds}{s^2}$$

$$\Leftrightarrow \frac{1}{2}v_w^2 = -\frac{K}{m_p}\frac{1}{s} + C$$

Take $v_w = v_\infty$ when $s \to \infty$ $\Rightarrow C = \frac{1}{2}v_\infty^2$

Also, take $v_w \to 0$ as $s \to R_{opt}$

$$\Leftrightarrow 0 = \frac{1}{2}v_\infty^2 - \frac{K}{m_p}\frac{1}{R_{opt}} \quad \Leftrightarrow \frac{1}{2}R_{opt}v_\infty^2 = \frac{K}{m_p}$$

$$\therefore \frac{1}{2}v_w^2 = \frac{1}{2}v_\infty^2 - \frac{1}{2}\frac{R_{opt}}{s}v_\infty^2$$

$$\Leftrightarrow v_w = v_\infty\left(1 - \frac{R_{opt}}{s}\right)^{1/2}$$

(4)

$$V_{rel}^2 = v_w^2 + v_{orbit}^2$$

$$= v_\infty^2\left(1 - \frac{R_{opt}}{s}\right) + \frac{GM_{opt}}{s} = v_\infty^2\left(1 - \frac{1}{x}\right) + \frac{GM_{opt}}{R_{opt}}\frac{R_{opt}}{s}$$

$$= v_\infty^2\left(1 - \frac{1}{x}\right) + v_\infty^2 \frac{y}{x}$$

$$= v_\infty^2\left(\frac{x + y - 1}{x}\right)$$

$$\Longleftrightarrow \quad V_{rel} = V_\infty\left(\frac{x + y - 1}{x}\right)^{1/2}$$

6) Assume source emits isotropically $\Rightarrow$ $L = 4\pi d^2 F$

a) $L_x = 4 \times \pi \times \left(4.6 \times 10^6 \times 3.09 \times 10^{16}\right)^2 \times 7.86 \times 10^{-15}$ W
$= 2 \times 10^{33}$ W

b) $$L_{Edd} = \frac{4\pi \times 6.67 \times 10^{-11} \times 5 \times 2 \times 10^{30} \times 1.67 \times 10^{-27} \times 3 \times 10^8}{6.65 \times 10^{-29}} \text{ W}$$

$$= 6.3 \times 10^{31} \text{ W} \ll L_x$$

This suggests the emitting region has a larger mass in order to be sub-Eddington.

We require $L_{Edd} \simeq 2 \times 10^{33}$ W

$$\Rightarrow M_x \simeq \frac{6.65 \times 10^{-29} \times 2 \times 10^{33}}{4\pi \times 6.67 \times 10^{-11} \times 1.67 \times 10^{-27} \times 3 \times 10^8} = 3.2 \times 10^{32} \text{ kg} \simeq 160 \, M_\odot$$

(5)

c) Radius of X-ray emitting region (assuming black-body)

$$R = \left(\frac{L_x}{4\pi\sigma T^4}\right)^{1/2} = 1.3 \times 10^{7}\ m \gg \text{e.g. } R_{NS}$$

This suggests the quasi-soft is unlikely to be a 'normal' compact object.

d) If the X-ray flux were emitted anisotropically this might be explicable from a source of $5 M_\odot$