1. In a simple model for galaxy formation, a newly formed galaxy is assumed to be surrounded by a spherical halo of hot fully ionised hydrogen gas, with density $n_P \propto r^{-a}$ and temperature $T \propto r^{-b}$. Show that the spatially integrated differential luminosity of the halo, due to X-ray bremsstrahlung emission, has the form

$$\frac{dL}{d\epsilon} \propto \epsilon^{-\left(\frac{6+b-4a}{2b}\right)}$$

2. The galaxy cluster Abell 3570 emits X-rays in the energy range 0.1 – 2.4 keV, predominantly produced by thermal bremsstrahlung from the intracluster gas, which is at a mean temperature of $3 \times 10^7$ K. The energy flux of X-rays in the 0.1 – 2.4 keV range detected at the Earth from this cluster is $1.3 \times 10^{-15}$ Wm$^{-2}$.

   Assuming that Abell 3570 is a spherical cluster of radius 25 kpc at a redshift of $z = 0.037$, and the intracluster gas is an isothermal plasma of fully ionised hydrogen, estimate the number density of protons in the intracluster gas. (Take $H_0 = 70$ kms$^{-1}$Mpc$^{-1}$).

3. The differential flux of 1 keV X-ray photons, emitted as thermal bremsstrahlung from a solar flare, is observed at the Earth to be $10^{12}$ photons m$^{-2}$ s$^{-1}$ keV$^{-1}$. Treating the source as a sphere of radius $1.33 \times 10^7$ m, containing a homogeneous plasma of ionised hydrogen gas at a temperature of $2.5 \times 10^7$ K, calculate the number density of protons in the source. What is the luminosity of the flare for X-rays in the energy range 1 – 10 keV?

4. Consider the Inverse Compton collision of ultra-relativistic cosmic ray electrons, of energy $\gamma m_e c^2$, with low ebergy photons. Assuming that the photons gain, on average, a factor of $\frac{4}{3}\gamma^2$ in energy, show that the Inverse Compton luminosity of a source containing $N$ electrons of energy $\gamma m_e c^2$ is

$$L_{IC} = \frac{4}{3} Q_T N \gamma^2 U_{rad}$$

   where $U_{rad}$ is the ambient radiation energy density. Similarly, show that the synchrotron luminosity from the same electrons in a magnetic field is given by

$$L_S = \frac{4}{3} c Q_T N \gamma^2 U_{mag}$$

   where $U_{mag}$ is the magnetic energy density.

   Calculate the galactic magnetic field strength needed to give a synchrotron luminosity radiated by cosmic ray electrons equal to the Inverse Compton luminosity from scattering of the same electrons off the 2.7 K cosmic microwave background radiation.

5. Show that the Lorentz factor, $\gamma$, of an ultra-relativistic electron emitting synchrotron radiation in a constant magnetic field, $B$, decays according to the equation

$$\gamma(t) = \gamma_0 \left(1 + C\gamma_0 t\right)^{-1}$$

   where $\gamma_0$ is the initial Lorentz factor of the electron and $C$ is a constant given by

$$C = \frac{2 Q_T c B^2}{3 m_e c^2 \mu_0}$$