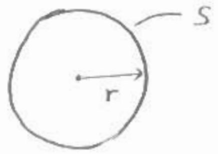
X-Ray Astrophysics I — Example Sheet 2 — Model Answers

(1) $\dot{M}$ = mass loss rate from star



= mass per second passing through surface, S

= (volume × density) through S in 1 second

= $4\pi r^2 v_0\, n_p(r)\, m_p$ as required

(2) $$\xi(T) = \int_{S_T} \frac{n_p^2(\underline{r})\, dS}{\left|\frac{dT}{dr}\right|}$$

S_T denotes a spherical surface of constant temperature, T, radius, r

$\Rightarrow$ we can take $n_p^2(\underline{r})$ and $\left|\frac{dT}{dr}\right|$ outside integral

$$\xi(T) = \frac{n_p^2}{\left|\frac{dT}{dr}\right|} \times 4\pi r^2$$

$$= \frac{\dot{M}^2\, 4\pi r^2}{(4\pi r^2)^2 v_0^2 m_p^2}\left|\frac{dr}{dT}\right| \qquad = \frac{\dot{M}^2}{4\pi r^2 v_0^2 m_p^2}\left|\frac{dr}{dT}\right|$$

(3) $$T(r) = T_0\left(\frac{r}{R_*}\right)^{-\alpha} \qquad r \geq R_*$$

$$\Leftrightarrow \quad T = T_0\left(\frac{R_*}{r}\right)^{\alpha}$$

$$\Leftrightarrow \quad r^{\alpha} = R_*^{\alpha}\,\frac{T_0}{T}$$

$$\Leftrightarrow \quad r = R_*\left(\frac{T_0}{T}\right)^{\frac{1}{\alpha}} = R_*\, T_0^{1/\alpha}\, T^{-1/\alpha}$$

$$\Rightarrow \quad \left|\frac{dr}{dT}\right| = \frac{1}{\alpha} R_*\, T_0^{1/\alpha}\, T^{-\frac{1}{\alpha}-1}$$

Since $r \geq R_* \Rightarrow T \leq T_0$

$\therefore \quad \xi(T) = 0 \quad \forall\, T > T_0$

for $0 \leq T \leq T_0$,

$$\xi(T) = \frac{\dot{M}^2}{4\pi R_*^2 T_0^{2/\alpha} T^{-2/\alpha} v_0^2 m_p^2} \times \frac{1}{\alpha} R_* T_0^{1/\alpha} T^{-1/\alpha - 1}$$

$$= \frac{\dot{M}^2}{4\pi\alpha v_0^2 m_p^2 T_0^{1/\alpha} R_*} T^{\frac{1}{\alpha} - 1}$$

(4) $$\Xi = \int_0^\infty \xi(T)\, dT = \frac{\dot{M}^2}{4\pi\alpha v_0^2 m_p^2 T_0^{1/\alpha} R_*} \int_0^{T_0} T^{\frac{1}{\alpha}-1}\, dT$$

$$= \frac{\dot{M}^2}{4\pi\alpha v_0^2 m_p^2 T_0^{1/\alpha} R_*} \left[\alpha T^{1/\alpha} \right]_0^{T_0}$$

$$= \frac{\dot{M}^2}{4\pi v_0^2 m_p^2 R_*}$$

Also $$4\pi \int_{R_*}^\infty n_p^2(r)\, dr = \frac{\dot{M}^2 \, 4\pi}{m_p^2 v_0^2} \int_{R_*}^\infty \frac{r^2 dr}{(4\pi r^2)^2}$$

$$= \frac{\dot{M}^2}{4\pi m_p^2 v_0^2} \left[-\frac{1}{r} \right]_{R_*}^\infty$$

$$= \frac{\dot{M}^2}{4\pi v_0^2 m_p^2 R_*}$$

Thus $\int n_p^2\, dV = \int \xi(T)\, dT$, confirming that $\xi(T) = \frac{d}{dT} \int_V n_p^2\, dV$

(5) From notes, $\frac{dL}{d\varepsilon} = 2\left(\frac{2}{\pi m_e}\right)^{\frac{1}{2}} \frac{Q_o m_e c^2}{k^{\frac{1}{2}}} \int_0^\infty \xi(T) \frac{e^{-\varepsilon/kT}}{T^{\frac{1}{2}}} dT$

$$= 2\left(\frac{2}{\pi m_e}\right)^{\frac{1}{2}} \frac{Q_o m_e c^2 \dot{M}^2}{k^{\frac{1}{2}} 4\pi\alpha R_* v_o^2 m_p^2 T_o^{1/\alpha}} \int_0^{T_o} T^{\frac{1}{\alpha}-\frac{3}{2}} e^{-\varepsilon/kT} dT$$

(6) Put $x = \frac{\varepsilon}{kT} \Rightarrow dx = -\frac{\varepsilon}{kT^2} dT = -\frac{\varepsilon}{kT}\frac{dT}{T} = -x\frac{dT}{T}$

$$\text{integral} = \int_0^{T_o} T^{\frac{1}{\alpha}-\frac{1}{2}} e^{-\varepsilon/kT} \frac{dT}{T}$$

$$= \int_{\frac{\varepsilon}{kT_o}}^{\infty} \left(\frac{\varepsilon}{kx}\right)^{\frac{1}{\alpha}-\frac{1}{2}} e^{-x} \frac{dx}{x}$$

$$= \int_{\frac{\varepsilon}{kT_o}}^{\infty} \frac{\varepsilon^{\frac{1}{\alpha}-\frac{1}{2}} e^{-x}}{k^{\frac{1}{\alpha}-\frac{1}{2}}} x^{\frac{1}{2}-\frac{1}{\alpha}-1} dx$$

$$\propto \varepsilon^{\frac{1}{\alpha}-\frac{1}{2}} \int_{\frac{\varepsilon}{kT_o}}^{\infty} x^{-\frac{1}{\alpha}-\frac{1}{2}} e^{-x} dx$$

(7) For $\varepsilon/kT_o \gg 1$, $\int_{\varepsilon/kT_o}^{\infty} x^{-\frac{1}{\alpha}-\frac{1}{2}} e^{-x} dx \simeq \left(\frac{\varepsilon}{kT_o}\right)^{-\frac{1}{\alpha}-\frac{1}{2}} e^{-\varepsilon/kT_o}$

$$\Rightarrow \frac{dL}{d\varepsilon} \propto \varepsilon^{\frac{1}{\alpha}-\frac{1}{2}-\frac{1}{\alpha}-\frac{1}{2}} e^{-\varepsilon/kT_o} = \varepsilon^{-1} e^{-\varepsilon/kT_o}$$

which is independent of α