1. The steady-state wind from a hot star consists of fully ionised hydrogen gas moving radially outwards with constant velocity v_0. By considering the mass per second passing through a spherical surface of radius r outside the photosphere (of radius R_*), show that $n_P(r)$, the number density of protons at radius r satisfies

$$\dot{M} = 4\pi r^2 \, v_0 n_P(r) m_P$$

where $\dot{M}$ is the mass loss rate of the star and m_P is the proton mass. (This is known as the *mass continuity equation*; see also Dr Woan's Stellar Atmospheres and Winds course).

2. Hence, show that the *source emission measure function*, $\xi(T)$, for the wind is given by

$$\xi(T) = \frac{\dot{M}^2}{4\pi r^2 \, v_0^2 \, m_P^2} \left| \frac{dr}{dT} \right|$$

3. If the temperature of the wind outside of the photosphere varies with radius according to the formula

$$T(r) = T_0 \left(\frac{r}{R_*} \right)^{-\alpha}$$

where α and T_0 are constants, derive an expression for $r(T)$, and hence show that $\xi(T) = 0$ for $T > T_0$ and

$$\xi(T) = \frac{\dot{M}^2}{4\pi\alpha \, v_0^2 \, m_P^2 \, T_0^{1/\alpha} \, R_*} T^{\frac{1}{\alpha}-1} \qquad \text{for T} \leq \text{T}_0$$

4. Determine the integrated source emission measure, Ξ, first by integrating $\xi(T)$ over temperature, i.e.

$$\Xi = \int_0^\infty \xi(T) dT$$

and then via the volume integral

$$\Xi = \int_V n_P^2 \, dV$$

showing that these two expressions are equivalent. Thus verify the relation

$$\xi(T) = \frac{d}{dT} \int_V n_P^2 \, dV$$

5. Assuming that the X-ray emission from the hot wind is thermal bremsstrahlung, show that the differential luminosity of the star is given by

$$\frac{dL}{d\epsilon} = 2 \left(\frac{2}{\pi m_e} \right)^{\frac{1}{2}} \frac{Q_0 m_e c^2 \, \dot{M}^2}{k^{1/2} \, 4\pi\alpha \, v_0^2 \, m_P^2 \, T_0^{1/\alpha} \, R_*} \int_0^{T_0} T^{\frac{1}{\alpha}-\frac{3}{2}} \, e^{-\epsilon/kT} dT$$

6. Applying the substitution $x = \epsilon/kT$, or otherwise, show that the above expression may be reduced to

$$\frac{dL}{d\epsilon} \propto \epsilon^{\frac{1}{\alpha}-\frac{1}{2}} \int_{\frac{\epsilon}{kT_0}}^{\infty} x^{-\frac{1}{2}-\frac{1}{\alpha}} e^{-x} dx$$

7. Hence, explain why – for large X-ray photon energies – the shape of the differential photon luminosity is independent of α.