X-Ray Astrophysics — Solutions to Example Sheet 1

Black body source => Wien's displacement law holds

i.e. $\lambda_{max} T = 0.24 \text{cm}$

$$E = \frac{1.24 \times 10^{-6}}{\lambda(m)} \text{eV} = \frac{1.24 \times 10^{-7}}{\lambda(cm)} \text{keV}$$

$$E_{max} = 2.5 \text{ keV} \Rightarrow \lambda_{max} = 5 \times 10^{-8} \text{ cm}$$

$$\therefore \quad T = \frac{0.24}{5 \times 10^{-8}} \text{ K} = 5.8 \times 10^{6} \text{ K}$$

$$L = 10^{31} \text{ W} = 4\pi R^2 \sigma T^4 \Rightarrow R = \sqrt{\frac{L}{4\pi\sigma T^4}}$$

$$= \left(\frac{10^{31}}{4\pi \times 5.67 \times 10^{-8} \times (5.8 \times 10^{6})^4} \right)$$

$$= 1.1 \times 10^{5} \text{ m}$$

If luminosity arises from mass transfer, equate energy released with rest mass energy of transferred mass. (Not quite so efficient, but approx correct)

In one year energy released $E = L \times 3600 \times 24 \times 365.25$

$$= 3.16 \times 10^{38} \text{ J}$$

Equivalent rest mass $= \frac{E}{c^2} = 3.5 \times 10^{21} \text{ kg}$

$$= \frac{3.5 \times 10^{21}}{2 \times 10^{30}} M_\odot$$

Mass loss rate, $\underline{\dot{M} = 1.3 \times 10^{-9} M_\odot / \text{year}}$

2) $B_\lambda \, d\lambda = B_\nu \, d\nu$

$$\Leftrightarrow \quad B_\lambda = B_\nu(\lambda) \left|\frac{d\nu}{d\lambda}\right|$$

$c = \nu\lambda$

$\Rightarrow \nu = \frac{c}{\lambda}$

$\Leftrightarrow d\nu = \frac{-c}{\lambda^2} d\lambda$

$$= \frac{2h\left(\frac{c}{\lambda}\right)^3 \cdot \frac{1}{c^2}}{e^{hc/\lambda kT} - 1} \cdot \frac{c}{\lambda^2}$$

$$= \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

$$B = \int_0^\infty B_\lambda \, d\lambda = \int_0^\infty \frac{2hc^2/\lambda^5 \, d\lambda}{e^{hc/\lambda kT} - 1}$$

Put $s = \frac{hc}{\lambda kT}$ $\quad \lambda = \frac{hc}{ksT}$

$\Rightarrow ds = \frac{-hc}{\lambda^2 kT} d\lambda$

$d\lambda = \frac{-\lambda^2 kT}{hc}$

$d\lambda = \frac{-hc\,kT}{(ksT)^2} ds$

$$= \int_0^\infty \frac{2\,hc^2 (ks\,T)^5}{e^s - 1} \left(\frac{1}{hc}\right)^5 \frac{hc\,kT\,ds}{(kT)^2 s^2}$$

$$= \frac{2\,hc^2 k^5 T^5 hc\,kT}{(hc)^5 k^2 T^2} \int_0^\infty \frac{s^3 \, ds}{e^s - 1}$$

$$= C \int_0^\infty \frac{s^3 \, ds}{e^s - 1} T^4 \quad \propto \quad T^4$$

(3) $$\tau = \frac{U}{L} = \frac{3k}{4\pi\sigma m_p}\frac{M}{R^2 T^3}$$

$$= \frac{3k M_\odot \left(M/M_\odot\right)}{4\pi\sigma m_p R_\odot^2 \left(\frac{R}{R_\odot}\right)^2 10^{21}\left(T/10^7\right)^3}$$

$$= \frac{3 \times 1.38\times 10^{-23} \times 2\times 10^{30} \left(M/M_\odot\right)}{4\times\pi\times 5.67\times 10^{-8} \times 1.67\times 10^{-27} \times (7\times 10^8)^2 \times 10^{21}\left(T/10^7\right)^3}$$

$$= \frac{(142 \text{ seconds})\left(M/M_\odot\right)}{\left(R/R_\odot\right)^2 \left(T/10^7 K\right)^3}$$

(4) (a) $$F_G = \frac{GMm_p}{R^2} \quad (\text{ignoring } m_e)$$

(b) For photons, momentum $= \frac{E}{c}$

=> rate of change of momentum

= momentum radiated per second $= \frac{L}{c}$

Probability of a photon being scattered by a particular electron $p = \frac{Q_T}{4\pi R^2}$

=> radiation pressure transfers $$F_R = \frac{L Q_T}{4\pi R^2 c}$$

$$F_R = F_G \implies \frac{L Q_T}{4\pi R^2 c} = \frac{G M m_p}{R^2}$$

$$\iff L = \frac{4\pi G M m_p c}{Q_T}$$

(5) By energy conservation we need $E > \varepsilon$

$$\frac{dJ}{d\varepsilon} = \int_V \frac{dj}{d\varepsilon}\, dV$$

$$= \int_V n_p \int_\varepsilon^\infty F(E) \frac{dQ_B}{d\varepsilon}(\varepsilon, E)\, dE\, dV$$

Homogeneous plasma so we can take out $\int n_p\, dV = n_p V$

Also $\dfrac{dQ_B}{d\varepsilon} \simeq \dfrac{Q_0 mc^2}{\varepsilon E}$

$$\implies \frac{dJ}{d\varepsilon} = \frac{n_p V Q_0 mc^2}{\varepsilon} \int_\varepsilon^\infty \frac{F(E)}{E}\, dE \quad \text{as required}$$

Re-arranging,

$$\varepsilon \frac{dJ}{d\varepsilon} = n_p V Q_0 mc^2 \int_\varepsilon^\infty \frac{F(E)}{E}\, dE$$

(5) contd

Differentiating both sides w.r.t. ε

$$\frac{d}{d\varepsilon}\left(\varepsilon \frac{dJ}{d\varepsilon}\right)_{\varepsilon = E} = -\frac{F(E)}{E} \cdot \frac{1}{n_p V Q_o mc^2} \quad (\text{using Leibniz' rule})$$

i.e. $$\left[\frac{dJ}{d\varepsilon} + \varepsilon \frac{d^2 J}{d\varepsilon^2}\right]_{\varepsilon = E} = \frac{-F(E)}{E\, n_p V Q_o mc^2}$$

$$\Leftrightarrow \quad F(E) = \frac{E}{n_p V Q_o mc^2}\left[-\frac{dJ}{d\varepsilon} - \varepsilon \frac{d^2 J}{d\varepsilon^2}\right]_{\varepsilon = E}$$