

1. A bright solar mass galactic X-Ray source has a luminosity of 10^{31}W , emitting a black body spectrum peaked at 2.5 keV. Calculate the temperature and radius of the source.
Assuming the origin of the energy source is gravitational, due to the accretion of matter onto the surface of the source from a companion star, estimate the mass transfer rate in units of the solar mass per year.
2. Consider the Planck function, which describes the energy spectrum of black body radiation:-

$$B_\nu(T) = \frac{2h\nu^3/c^2}{e^{h\nu/kT} - 1}$$

Verify that, expressed in terms of wavelength, λ , the Planck function takes the form:-

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

Prove the *Stefan Boltzmann law*, which states that:-

$$B \propto T^4$$

3. Verify the expression given in the notes for the cooling time, τ , of a spherical mass, M , of ionised hydrogen of radius, R , and temperature, T , emitting black body radiation, viz:-

$$\tau \simeq \frac{140 \times (M/M_\odot)}{(R/R_\odot)^2 \times (T/10^7 \text{K})^3} \text{ seconds}$$

4. Consider a fully ionised plasma of hydrogen at a distance, R , from a compact object of mass, M and luminosity, L . Write down expressions for the forces acting on an electron-proton pair in the plasma due to:-
 - (a) the gravitational attraction by the mass, M
 - (b) repulsion due to 'radiation pressure', i.e. the absorption of (a fraction of) the photon momentum due to Thomson scattering by the electron of photons emitted by the source.

Hence show that the condition for these two forces to balance leads to the expression:-

$$L = \frac{4\pi GMm_p c}{Q_T}$$

where m_p is the proton mass and Q_T is the Thomson cross section. (This luminosity is known as the *Eddington limit*).

5. Consider a fully ionised, homogenous plasma of hydrogen, occupying volume, V . Show that the differential emission rate due to bremsstrahlung is given by:-

$$\frac{dJ}{d\epsilon} = \frac{Q_0 m c^2 V n_P}{\epsilon} \int_\epsilon^\infty \frac{F(E)}{E} dE$$

where $F(E)$ is the differential electron energy flux spectrum, Q_0 is the bremsstrahlung cross section, m is the electron mass and n_P is the plasma density.

Hence, verify the solution given in the lecture notes for $F(E)$ in terms of $dJ/d\epsilon$, viz:-

$$F(E) = \frac{E}{n_P V Q_0 m c^2} \left[-\frac{dJ}{d\epsilon} - \epsilon \frac{d^2 J}{d\epsilon^2} \right]_{\epsilon=E}$$