Dept of Physics and Astronomy
Astronomy345M, 2008-09

Gravitation and Relativity II

Aims and Objectives

Chapter 2: **Static Models with Spherical Symmetry**

Aim: To understand the mathematical and physical principles which underpin the derivation of the Schwarzschild metric.

Objectives: By the end of the course students should be familiar with, and (where appropriate) be able to apply in the context of algebraic and numerical problems:

- The definition and properties of an orthogonal metric
- The general form of the metric for a static, spherically symmetric spacetime (S4)
- How to compute the Christoffel symbols for S4
- *Given* the components of the Ricci tensor for S4, how to derive the form of the Schwarzschild metric.

Chapter 3: **The Schwarzschild Metric and Classical Tests of GR**

Aim: To understand the classical predictions of GR – advance of pericentre, light deflection, gravitational redshift and time delay – and how they differ from Newtonian gravity.

Objectives: By the end of the course students should be familiar with, and (where appropriate) be able to apply in the context of algebraic and numerical problems:

- The observational evidence for the classical predictions of GR, including the advance of perihelion of Mercury, the binary pulsar system, the system OJ287, gravitational strong lensing and microlensing systems, the Pound-Rebka experiment etc.
- How to derive and solve the equation of motion for the Newtonian 2-body planetary orbit.
- How to derive the equivalent expression for GR.
- The approximate GR solution of a precessing ellipse, and its rate of precession
- How to derive and solve the quasi-Newtonian light deflection formula for a photon passing close to a point mass
- How to derive the equivalent expression for GR, and in particular the origin of the extra factor of two
- The equation defining the Einstein ring radius of a gravitational lens, and the numerical value of this radius for galactic and cosmological cases
- How to derive an expression for gravitational redshift within the framework of the Schwarzschild metric
- How to derive an expression for the Shapiro effect, for the gravitational time delay in terms of the Newtonian gravitational potential

Chapter 4: **Einstein's Equations for Static Spherically Symmetric Stars**

Aim: To understand the GR description of the interior structure of a star, and how it differs from the classical, Newtonian description.

Objectives: By the end of the course students should be familiar with, and (where appropriate) be able to apply in the context of algebraic and numerical problems:

- *Given* the components of the Einstein tensor and energy-momentum tensor for the interior of a spherically symmetric star, how to solve Einstein's equations to obtain the Oppenheimer-Volkoff equation
- How the OV equation reduces to the equation of hydrostatic equilibrium, for the case of a weak gravitational field
- How to solve the OV equation exactly for a star of constant density
- Limits on the radius of static stars within the framework of GR, and their extension to non-static stars via Buchdahl's Theorem.

Chapter 5: **Gravitational Radiation**

Aim: To understand the origin of gravitational waves as metric perturbations of a non-stationary spacetime, and to understand qualitatively the implications of this result for their detection and generation.

Objectives: By the end of the course students should be familiar with, and (where appropriate) be able to apply in the context of algebraic and numerical problems:

- The definition of a non-stationary spacetime and a weak gravitational field and 'nearly flat' spacetime
- The definition of background Lorentz and gauge transformations
- The approximate form of Einstein's equations in a nearly flat spacetime, expressed in the Lorentz gauge
- How the free-space solution of these equations is a wave equation propagating at the speed of light
- Qualitatively, how the adoption of the transverse traceless gauge reduces to only two the number of free parameters in the wave solution
- The effect of gravitational waves on the proper distance between test particles, for the 'plus' and 'cross' polarisation states

Chapter 6: **Black Holes**

Aim: To understand the physical nature of static black holes, within the framework of the Schwarzschild metric of GR.

Objectives: By the end of the course students should be familiar with, and (where appropriate) be able to apply in the context of algebraic and numerical problems:

- The nature of the Schwarzschild surface as a coordinate singularity, so that a test particle released from outside the Schwarzschild radius will reach it in a finite proper time (but infinite coordinate time as viewed by a distant observer)
- The behaviour of photons and material particles inside the event horizon, and in particular the geometry of null cones – meaning that photons and material particles cannot escape once they have crossed the event horizon

- The effect of gravitational redshift on black holes, and the timescale over which they ‘switch off’ as their stellar progenitor collapses inside the event horizon
- Qualitatively, the effect of frame dragging for a rotating black hole
- Qualitatively, the phenomenon of Hawking radiation and the Hawking lifetime of a black hole.

Chapter 7: **GR and Cosmology**

Aim: To understand how GR provides a natural theoretical framework for some of the key results of observational cosmology.

Objectives: By the end of the course students should be familiar with, and (where appropriate) be able to apply in the context of algebraic and numerical problems:

- How GR encapsulates the cosmological principle
- Qualitatively, how to derive the Robertson-Walker metric, as the most general form for a spherically symmetric metric consistent with the cosmological principle
- Given Einstein tensor for the Robertson-Walker metric and the energy-momentum tensor for the cosmic fluid, how to derive the Friedmann equations.
- Qualitatively, how Einstein’s cosmological constant was introduced to provide a static universe solution, and why it failed.