Gravitation & Relativity I: A3/A4 Lectures, October 2000

AIMS AND OBJECTIVES

This course provides the first part of an introduction to the theory of general relativity. Elementary tensor calculus on riemannian manifolds is introduced as a means of describing curved spacetimes and arriving at Einstein's field equations. Astrophysical applications will be discussed in Gravitation and Relativity II.

COURSE CONTENT (10 lectures)

1. **The equivalence principle and its physical consequences.** Gravitational redshift and bending of light. Gravitation as curvature of spacetime. Illustration of curvature at the surface of the Earth.

2. **Covariance.** Component versus coordinate free notation.

3. **Review of special relativity** Invariant distance $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$. Lorentz transformation. Spacelike, timelike, and null intervals. Proper time. Notion of covariance in special relativity. Four vectors, four velocity, four momentum and four acceleration. Tensors in special relativity.

4. **Spacetime as a manifold.** Manifolds and coordinate systems. Transformation of dx^i as prototype contravariant vector. Covariant vector components with gradient as example. Tangent vector and one forms. The metric tensor $g_{\mu\nu}$. $g^{\mu\nu}$ as inverse. $g^{\mu\alpha}g_{\alpha\nu} = \delta^\mu_\nu$. Tensors of higher rank. Raising and lowering indices. Contraction.

5. **Covariant differentiation :** Parallel displacement of scalars, vectors and tensors. Christoffel symbols: $\Gamma^\mu_{\alpha\beta}$. Christoffel symbols expressed in terms of $g_{\mu\nu,\alpha}$. Covariant differentiation of vectors and of tensors of higher rank.

6. **Geodesics:** Geodesic equation in Riemannian space as world line of test particle. Null geodesics and photons.

7. **Energy momentum tensor:** Physical interpretation, conservation of energy momentum

8. **Riemann-Christoffel tensor, and Ricci tensor.** Derivation of Riemann Christoffel tensor from equation of geodesic deviation. Bianchi identities.

9. **Einstein's equations:** $G_{\mu\nu} = kT_{\mu\nu}$. $T^{\mu\nu}_{\ ;\nu} = 0$, following from Bianchi identities.

10. **Weak field limit and correspondence with Newtonian gravitation.**

RECOMMENDED BOOKS

Schutz, B. "A First Course in General Relativity", CUP (1985)
(This book is not essential for purchase, but is highly recommended)

Books for consultation:

- Misner, Thorne and Wheeler, "Gravitation", Freeman(1973)
- Wald, R.M. "General Relativity", University of Chicago (1984)
- Rindler, W. "Essential Relativity", Springer (1977).
- Berry, M. "Principles of Relativity and Cosmology", CUP(1976)
- Papapetrou, A. "Lectures in General Relativity", Reidel (1974)
- Ohanion, H.C. "Gravitation and Spacetime", Norton (1976)
- d'Inverno R. "Introducing Einstein's Relativity", Oxford University Press (1992)

(None of the above books is essential, or indeed worthwhile, for purchase but may be useful for occasional consultation and general background reading)

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Martin Hendry, October 2000