Galaxies II   Examples 5   Model Answers

1) Units of force = $kg\,m\,s^{-2}$ units of ρ = $kg\,m^{-3}$

Units of G = $m^3 s^{-2} kg^{-1}$ units of v = $m s^{-1}$

$$F \approx K (GM)^a v^b \rho^c$$

$\Rightarrow$ We require $[kg][m][s]^{-2} = \left([m]^3[s]^{-2}\right)^a \left([m][s]^{-1}\right)^b \left([kg][m]^{-3}\right)^c$

$$\Rightarrow \quad c = 1$$
$$3a + b - 3c = 1$$
$$-2a - b = -2$$

Substituting $3a + b = 4$
$2a + b = 2$

Subtracting $a = 2 \Rightarrow b = -2$

$$\therefore \quad F \approx \frac{K (GM)^2 \rho}{v^2} \quad \text{as required}$$

2) (a) From Sect. 1 notes, in outer halo $\frac{mv^2}{r} = \frac{GM_r m}{r^2}$

$$\Rightarrow \quad M_r = \frac{v^2}{G} r \quad \Rightarrow \quad \frac{dM_r}{dr} = \frac{v^2}{G} = 4\pi r^2 \rho(r)$$

$$\therefore \quad \rho = \frac{v^2}{4\pi G r^2}$$

$$\Rightarrow \quad F \approx \frac{K (GM)^2 v^2}{4\pi G v^2 r^2} = \frac{K G M^2}{4\pi r^2} \quad \text{as required}$$

(b) $L = Mvr$ $\tau = \frac{KGM^2}{4\pi r}$

(c) $\frac{dL}{dt} = Mv\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{\tau}{Mv} = -\frac{KGM}{4\pi r v}$

(need minus sign since drag force acts in opposite sense to L)

(d) Re-arranging $dt = -\frac{4\pi v}{KGM} r\,dr$

$$\int_0^{t_{GC}} dt = -\frac{4\pi v}{KGM}\int_{r_{init}}^{0} r\,dr \qquad (\text{assuming } v \text{ const to } r \simeq 0)$$

$$t_{GC} = \frac{2\pi v\, r_{init}^2}{KGM}$$

e) $r_{init} = r_{max}$ for $t_{GC} = t_{Gal}$

$$\Rightarrow \quad r_{max} = \left(\frac{KGM\, t_{Gal}}{2\pi v}\right)^{\frac{1}{2}}$$

$M = 5\times10^6 M_\odot = 10^{37}$ kg
$v = 2.5\times10^5$ ms^{-1} $\Rightarrow$ $r_{max} \simeq 10^{20}$ m $= 3.2$ kpc
$t_{Gal} = 10$ Gyr $= 3.16\times10^{17}$ s

f If $M \sim 10^8$ then r_{max} increases by a factor of $\sqrt{20}$

$$\Rightarrow \quad r_{max} \simeq 14 \text{ kpc}$$

This is a bit smaller than the radius of M31, so our simple model isn't adequate to explain completely the absence of $10^8 M_\odot$ globular clusters

3) Applying formula from (2) $t_{LMC} \sim \frac{2\pi v r^2}{KGM}$

$M = 2\times10^{10} M_\odot = 4\times10^{40}$ kg
$v = 2.2\times10^5$ ms^{-1} $\Rightarrow$ $t_{LMC} = 5\times10^{16}$ s $= 1.6\times10^9$ yrs
$r = 50$ kpc $= 1.55\times10^{21}$ m

4) (a) Strong UV features are Balmer emission lines due to hot, massive stars formed in the initial burst of star formation.

These features decline sharply as the hot stars leave the Main Sequence (and are not replaced by new SF)

b) Red 'bump' is due to the formation of post-MS red giant stars

c) far-UV emission comes from hot post-MS remnants, i.e. white dwarfs

d) From the table, stars leaving MS between 4 and 13 Gyr will have masses of about 1 – 1.25 $M_\odot$

From the C-M diagram, post-MS evolutionary tracks are almost vertical => colours show little change during this time.

Hence, as stars leave the MS between t = 4 Gyr and t = 13 Gyr, the shape of the spectrum shows very little evolution over this time.