Galaxies - II Examples 3 Model Answers

1) (a) $M_{bol} = m_{bol} - 5\log d - 25$

$d = cz/H_0 = (3\times10^5 \times 0.19)/70 = 814\,\text{Mpc}$

$\Rightarrow M_{bol} = -24.65$

(b) $L_{bol} = L_{bol,\,sun} \cdot 10^{-0.4(M_{bol} - 4.75)}$

$= 3.86\times10^{26} \times 10^{0.4(29.40)} = 2.23\times10^{38}\,\text{W}$

c) $L_{Edd} \simeq \dfrac{4\pi G M m_p c}{\sigma_T}$ so $L \le L_{Edd} \Rightarrow M \ge \dfrac{2.23\times10^{38} \times 6.65\times10^{-29}}{4\pi \times 6.673\times10^{-11} \times 1.673\times10^{-27} \times 3\times10^{8}}$

$M \ge 3.52\times10^{37}\,\text{kg}$

$= 1.77\times10^{7}\,M_\odot$

d) $L = \eta \dot{M} c^2$

$\Rightarrow \dot{M} = \dfrac{2.23\times10^{38}}{0.1 \times 9\times10^{16}}\,\text{kg s}^{-1} = 2.48\times10^{22}\,\text{kg s}^{-1}$

$= 7.8\times10^{29}\,\text{kg yr}^{-1}$

$= 0.39\,M_\odot\,\text{yr}^{-1}$

e) From notes, assuming a black body,

$$T = \left(\frac{G M \dot{M}}{8\pi\sigma r^3}\right)^{1/4}$$

$$R_s = \frac{2GM}{c^2} = \frac{2\times6.673\times10^{-11} \times 3.52\times10^{37}}{9\times10^{16}} = 5.2\times10^{10}\,\text{m}$$

$r = 50R_s = 2.6\times10^{12}\,\text{m}$

$$T = \left[\frac{6.673\times10^{-11} \times 3.52\times10^{37} \times 2.48\times10^{22}}{8\pi \times 5.671\times10^{-8} \times (2.6\times10^{12})^3}\right]^{1/4}$$

$= 3.9\times10^{4}\,\text{K}$

2) (a) At 1400 MHz $S_\nu = 9.12 \times 10^{-24}\ Wm^{-2}\ Hz^{-1}$

$$\Rightarrow S_\nu = 9.12 \times 10^{-24} \left(\frac{\nu}{1.4\times10^9}\right)^{-0.6}$$

$$\text{Flux} = \int_{10^7}^{3\times10^9} 9.12 \times 10^{-24} \left(\frac{1.4\times10^9}{\nu}\right)^{0.6}$$

$$= 2.80 \times 10^{-18} \left[\frac{\nu^{0.4}}{0.4}\right]_{10^7}^{3\times10^9}$$

$$= \frac{2.80\times10^{-18}}{0.4}\left[6.178\times10^3 - 6.309\times10^2\right]$$

$$= 3.88 \times 10^{-14}\ Wm^{-2}$$

b) $d = cz/H_0 = 6.73\ Mpc = 2.076\times10^{23}\ m$

$\therefore L = 4\pi d^2 F = 2.102 \times 10^{34}\ W \simeq 5.45\times10^9\ L_{bol,\odot}$

$L_{MW,\,radio} \simeq 2500\ L_{bol,\odot} \Rightarrow L_{CenA,\,radio} = 2\times10^6\ L_{MW,\,radio}$

$L_{CenA,\,radio} \simeq 0.05\ L_{MW,\,opt}$

c) $\tau \simeq E/L = 10^{50}/(0.5\times 2.102\times10^{34})$

$= 9.5 \times 10^{15}$ s

$= 3\times10^8$ years

d) $E_{tot} = 2E_{mag}$

$= 2 \times V \times U_{mag}$

$= 2 \times \frac{4}{3} \times \pi \times (3.086\times10^{20})^3\ U_{mag}$

$\Rightarrow U_{mag} = 10^{50}/2.46\times10^{62} = \frac{B^2}{2\mu_0}$

$\therefore B = (2\times4\times10^{-13}\times4\pi\times10^{-7})^{1/2}$

$= 10^{-9}$ T

e) $$\nu_s = \frac{3}{2}\gamma^2 \frac{eB}{2\pi m_e}$$

$$\Rightarrow \gamma = \left[\frac{4\pi m_e \nu_s}{3eB}\right]^{1/2}$$

$$= \left[\frac{4\pi \times 9.1\times10^{-31} \times 1.4\times10^{9}}{3\times 1.6\times10^{-19} \times 10^{-9}}\right]^{1/2}$$

$$= 5.78\times10^{3}$$

3) $$L = 10^{12} L_{sun} = 3.86\times10^{38}\ W$$

$$= 1.218\times10^{46}\ J\,yr^{-1}$$

$$= \left(\frac{1.218\times10^{46}}{9\times10^{16}}\right) kg\,yr^{-1} \simeq 0.07\ M_\odot\ yr^{-1}$$

Take $L \simeq 10^{11} L_\odot$

$\Rightarrow 10\ M_\odot\ yr^{-1}$ of gas ejected

$\Rightarrow 1\ M_\odot\ yr^{-1}$ released as energy

$$\therefore L_{MW} \simeq \frac{10^{12}}{0.07} L_\odot = 1.43\times10^{13} L_\odot$$

4) a) $$\frac{dL}{d\lambda} = \frac{dL}{d\nu}\cdot\frac{d\nu}{d\lambda} \propto \nu^{-\alpha}\cdot\left(\frac{-c}{\lambda^2}\right) \qquad \nu = \frac{c}{\lambda}$$

$$\propto \lambda^{\alpha-2} = K\lambda^{\alpha-2} \quad \text{(say)}$$

b) $$\int_{\lambda_1}^{\lambda_2} L_\lambda\left(\lambda/_{1+z}, t_0\right) d\lambda = K\int_{\lambda_1}^{\lambda_2}\left(\frac{\lambda}{1+z}\right)^{\alpha-2} d\lambda$$

$$= (1+z)^{2-\alpha} K\int_{\lambda_1}^{\lambda_2}\lambda^{\alpha-2}\,d\lambda$$

$$\therefore k = 2.5\log_{10}(1+z) - 2.5\log_{10}\left[(1+z)^{2-\alpha}\right]$$

$$\therefore \quad k = \left[2.5\,(\alpha - 2) + 2.5\right] \log_{10}(1+z)$$

$$= (\alpha - 1) \times 2.5 \log_{10}(1+z)$$

c) Since for many quasars $L_\nu \propto \nu^{-1}$ (i.e. $\nu L_\nu \simeq$ const.) it follows that $k \simeq 0$