$$\tilde{g}(k) = \frac{1}{2\pi}\int_{-\infty}^{\infty} g(y)\, e^{iky}\, dy = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x)\, s(y-x)\, dx\, e^{iky}\, dy$$

Let $z = y - x \Rightarrow y = x + z$

when $y = -\infty \quad z = -\infty$

$y = \infty \quad z = \infty$

$$\tilde{g}(k) = \frac{1}{2\pi}\int_{z=-\infty}^{\infty} \frac{1}{2\pi}\int_{x=-\infty}^{\infty} f(x)\, s(z)\, dx\, e^{ik(x+z)}\, dz$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x)\, e^{ikx}\, dx \;\frac{1}{2\pi}\int_{-\infty}^{\infty} s(z)\, e^{ikz}\, dz = \tilde{f}(k)\,\tilde{s}(k) \quad \text{as required.}$$

The observed galaxy spectrum is a smoothed version, or convolution, of the (average) stellar spectrum, smoothed by the LOSVD.

The CCF 'falls off' from its maximum value, which should occur at $\bar{v}_{LOS}$, because features in the observed spectra no longer 'line up'.

The smaller the spread in LOSVD $\Rightarrow$ the smaller σ_{LOS}

$\Rightarrow$ the closer the galaxy spectrum is to the modelled stellar spectrum, shifted by $\bar{v}_{LOS}$

$\Rightarrow$ the more steeply the CCF falls off for $v \neq \bar{v}_{LOS}$

So the width of the CCF peak is sensitive to σ_{LOS}

3) $I(R) = I_0 \exp\left[-R/R_D\right]$

$$L_D = \int_0^{2\pi}\int_0^{\infty} I(R)\, R\, dR\, d\theta = 2\pi I_0 \int_0^{\infty} e^{-R/R_D}\, R\, dR$$

Let $z = R/R_D \Rightarrow L_D = 2\pi I_0 R_D^2 \int_0^{\infty} z\, e^{-z}\, dz$

$$\int_0^\infty z e^{-z}\, dz = \left[-z e^{-z}\right]_0^\infty + \int_0^\infty e^{-z}\, dz$$

$$= \left[-e^{-z}\right]_0^\infty = 1$$

$$\Rightarrow \quad L_D = 2\pi I_0 R_D^2$$

Fraction within α disk scale lengths :

$$f = 2\pi I_0 R_D^2 \int_0^\alpha z e^{-z}\, dz \Big/ 2\pi I_0 R_D^2$$

$$= \left[-z e^{-z}\right]_0^\alpha + \left[-e^{-z}\right]_0^\alpha$$

$$= -\alpha e^{-\alpha} + 1 - e^{-\alpha}$$

$$= 1 - (\alpha + 1) e^{-\alpha}$$

For $\alpha = 5$, $f = 1 - 6e^{-5} = 0.96$ as required

4) $$M = k - 2.5 \log_{10} L$$

$$= k - 2.5 \log_{10} V_{max}^{\beta}$$

$$= k - 2.5\beta \log_{10} V_{max}$$ as required