CHAPTER 1

SPACETIME: OVERVIEW

Our imagination is stretched to the utmost, not, as in fiction, to imagine things which are not really there, but just to comprehend those things which are there.

Richard P. Feynman

1.1 PARABLE OF THE SURVEYORS

disagree on northward and eastward separations; agree on *distance*

Once upon a time there was a Daytime surveyor who measured off the king's lands. He took his directions of north and east from a magnetic compass needle. Eastward separations from the center of the town square he measured in meters. The northward direction was sacred. He measured northward separations from the town square in a different unit, in miles. His records were complete and accurate and were often consulted by other Daytimers.

Daytime surveyor uses magnetic north

A second group, the Nighttimers, used the services of another surveyor. Her north and east directions were based on a different standard of north: the direction of the North Star. She too measured separations eastward from the center of the town square in meters and sacred separations northward in miles. The records of the Nighttime surveyor were complete and accurate. Marked by a steel stake, every corner of a plot appeared in her book, along with its eastward and northward separations from the town square.

Nighttime surveyor uses North-Star north

Daytimers and Nighttimers did not mix but lived mostly in peace with one another. However, the two groups often disputed the location of property boundaries. Why? Because a given corner of the typical plot of land showed up with different numbers in the two record books for its eastward separation from the town center, measured in meters (Figure 1-1). Northward measurements in miles also did not agree between the two record books. The differences were small, but the most careful surveying did not succeed in eliminating them. No one knew what to do about this single source of friction between Daytimers and Nighttimers.

One fall a student of surveying turned up with novel open-mindedness. Unlike all previous students at the rival schools, he attended both. At Day School he learned

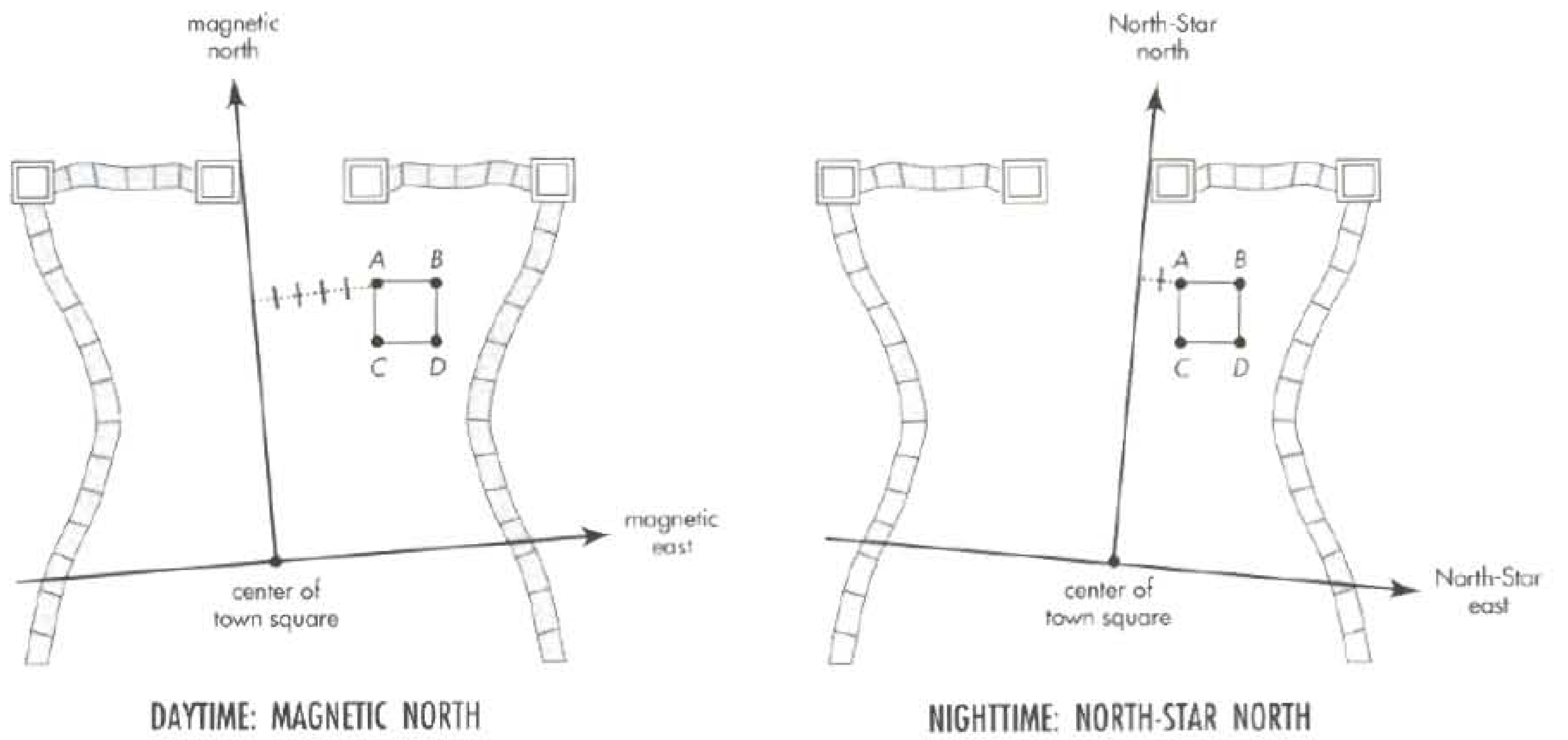

FIGURE 1-1. *The town as plotted by Daytime and Nighttime surveyors. Notice that the line of Daytime magnetic north just grazes the* left *side of the north gate, while the line of Nighttime North-Star north just grazes the* right *side of the same gate. Steel stakes* A, B, C, D *driven into the ground mark the corners of a disputed plot of land. As shown, the eastward separation of stake* A *from the north–south line measured by the Daytime surveyor is different from that measured by the Nighttime surveyor.*

from one expert his method of recording locations of gates of the town and corners of plots of land based on magnetic north. At Night School he learned the other method, based on North-Star north.

As days and nights passed, the student puzzled more and more in an attempt to find some harmonious relationship between rival ways of recording location. His attention was attracted to a particular plot of land, the subject of dispute between Daytimers and Nighttimers, and to the steel stakes driven into the ground to mark corners of this disputed plot. He carefully compared records of the two surveyors (Figure 1-1, Table 1-1).

Student converts miles to meters

In defiance of tradition, the student took the daring and heretical step of converting northward measurements, previously expressed always in miles, into meters by multiplying with a constant conversion factor k. He found the value of this conversion factor to be $k = 1609.344$ meters/mile. So, for example, a northward separation of 3 miles could be converted to $k \times 3$ miles $= 1609.344$ meters/mile $\times$ 3 miles $= 4828.032$ meters. "At last we are treating both directions the same!" he exclaimed.

Next the student compared Daytime and Nighttime measurements by trying various combinations of eastward and northward separation between a given stake and the center of the town square. Somewhere the student heard of the Pythagorean Theorem, that the sum of squares of the lengths of two perpendicular legs of a right triangle equals the square of the length of the hypotenuse. Applying this theorem, he discovered that the expression

$$\overset{\textbf{Daytime}}{\left[k \times \begin{pmatrix}\text{northward}\\ \text{separation}\\ \text{(miles)}\end{pmatrix}\right]^2} + \overset{\textbf{Daytime}}{\begin{bmatrix}\text{eastward}\\ \text{separation}\\ \text{(meters)}\end{bmatrix}^2} \qquad (1\text{-}1)$$

TABLE 1-1

TWO DIFFERENT SETS OF RECORDS; SAME PLOT OF LAND

	Daytime surveyor's axes oriented to magnetic north		Nighttime surveyor's axes oriented to North-Star north	
	Eastward (meters)	Northward (miles)	Eastward (meters)	Northward (miles)
Town square	0	0	0	0
Corner stakes:				
Stake *A*	4010.1	1.8330	3950.0	1.8827
Stake *B*	5010.0	1.8268	4950.0	1.8890
Stake *C*	4000.0	1.2117	3960.0	1.2614
Stake *D*	5000.0	1.2054	4960.0	1.2676

based on Daytime measurements of the position of steel stake *C* had exactly the same numerical value as the quantity

$$\left[k \times \overset{\textbf{Nighttime}}{\begin{pmatrix}\text{northward}\\ \text{separation}\\ \text{(miles)}\end{pmatrix}}\right]^2 + \overset{\textbf{Nighttime}}{\begin{bmatrix}\text{eastward}\\ \text{separation}\\ \text{(meters)}\end{bmatrix}}^2 \qquad (1\text{-}2)$$

computed from the readings of the Nighttime surveyor for stake *C* (Table 1-2). He

TABLE 1-2

"INVARIANT DISTANCE" FROM CENTER OF TOWN SQUARE TO STAKE C

(Data from Table 1-1)

Daytime measurements		Nighttime measurements	
Northward separation 1.2117 miles		Northward separation 1.2614 miles	
Multiply by $k = 1609.344$ meters/mile to convert to meters: 1950.0 meters		Multiply by $k = 1609.344$ meters/mile to convert to meters: 2030.0 meters	
Square the value	3,802,500 (meters)^2	Square the value	4,120,900 (meters)^2
Eastward separation 4000.0 meters		Eastward separation 3960.0 meters	
Square the value and add	+ 16,000,000 (meters)^2	Square the value and add	+ 15,681,600 (meters)^2
Sum of squares	= 19,802,500 (meters)^2	Sum of squares	= 19,802,500 (meters)^2
Expressed as a number squared	= (4450 meters)^2	Expressed as a number squared	= (4450 meters)^2
This is the square of what measurement?	**4450 meters**	This is the square of what measurement?	**4450 meters**

SAME
DISTANCE
from center of Town Square

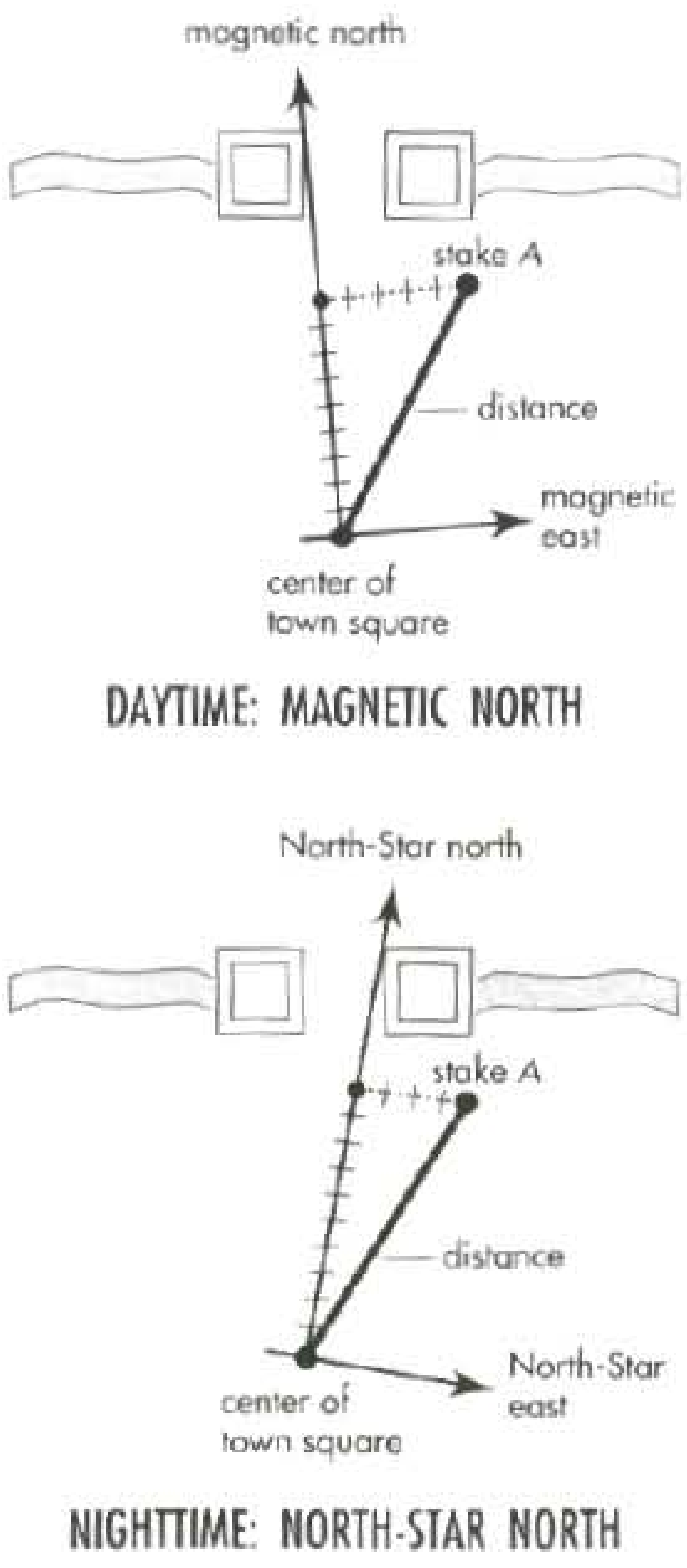

FIGURE 1-2. *The distance between stake* A *and the center of the town square has the same value for Daytime and Nighttime surveyors, even though the northward and eastward separations, respectively, are not the same for the two surveyors.*

tried the same comparison on recorded positions of stakes A, B, and D and found agreement here too. The student's excitement grew as he checked his scheme of comparison for all stakes at the corners of disputed plots — and found everywhere agreement.

Discovery: Invariance of distance

Flushed with success, the student methodically converted all northward measurements to units of meters. Then the student realized that the quantity he had calculated, the numerical value of the above expressions, was not only the same for Daytime and Nighttime measurements. It was also the square of a length: (meters)^2. He decided to give this length a name. He called it the **distance** from the center of town.

$$(\text{distance})^2 = \begin{bmatrix} \text{northward} \\ \text{separation} \\ \text{(meters)} \end{bmatrix}^2 + \begin{bmatrix} \text{eastward} \\ \text{separation} \\ \text{(meters)} \end{bmatrix}^2 \qquad (1\text{-}3)$$

He said he had discovered the **principle of invariance of distance**; he reckoned exactly the same value for distance from Daytime measurements as from Nighttime measurements, despite the fact that the two sets of surveyors' numbers differed significantly (Figure 1-2).

After some initial confusion and resistance, Daytimers and Nighttimers welcomed the student's new idea. The invariance of distance, along with further results, made it possible to harmonize Daytime and Nighttime surveys, so everyone could agree on the location of each plot of land. In this way the last source of friction between Daytimers and Nighttimers was removed.

1.2 SURVEYING SPACETIME

disagree on separations in space and time; agree on spacetime *interval*

The Parable of the Surveyors illustrates the naive state of physics before the discovery of **special relativity** by Einstein of Bern, Lorentz of Leiden, and Poincaré of Paris. Naive in what way? Three central points compare physics at the turn of the twentieth century with surveying before the student arrived to help Daytimers and Nighttimers.

The second: A sacred unit

First, surveyors in the mythical kingdom measured northward separations in a sacred unit, the mile, different from the unit used in measuring eastward separations. Similarly, people studying physics measured time in a sacred unit, called the second, different from the unit used to measure space. No one suspected the powerful results of using the same unit for both, or of squaring and combining space and time separations when both were measured in meters. Time in meters is just the time it takes a light flash to go that number of meters. The conversion factor between seconds and meters is the speed of light, $c = 299{,}792{,}458$ meters/second. The velocity of light c (in meters/second) multiplied by time t (in seconds) yields ct (in meters).

Speed of light converts seconds to meters

The speed of light is the only natural constant that has the necessary units to convert a time to a length. Historically the value of the speed of light was regarded as a sacred number. It was not recognized as a mere conversion factor, like the factor of conversion between miles and meters — a factor that arose out of historical accident in humankind's choice of units for space and time, with no deeper physical significance.

Second, in the parable northward readings as recorded by two surveyors did not differ much because the two directions of north were inclined to one another by only the small angle of 1.15 degrees. At first our mythical student thought that small differences between Daytime and Nighttime northward measurements were due to surveying error alone. Analogously, we used to think of the separation in time between two electric sparks as the same, regardless of the motion of the observer. Only with the publication of Einstein's relativity paper in 1905 did we learn that the separation in time between two sparks really has different values for observers in different states of motion — in different **frames**.

Time between events: Different for different frames

Think of John standing quietly in the front doorway of his laboratory building. Suddenly a rocket carrying Mary flashes through the front door past John, zooms down the middle of the long corridor, and shoots out the back door. An antenna projects from the side of Mary's rocket. As the rocket passes John, a spark jumps across the 1-millimeter gap between the antenna and a pen in John's shirt pocket. The rocket continues down the corridor. A second spark jumps 1 millimeter between the antenna and the fire extinguisher mounted on the wall 2 meters farther down the corridor. Still later other metal objects nearer the rear receive additional sparks from the passing rocket before it finally exits through the rear door.

One observer uses laboratory frame

Another observer uses rocket frame

John and Mary each measure the lapse of time between "pen spark" and "fire-extinguisher spark." They use accurate and fast electronic clocks. John measures this time lapse as 33.6900 thousand-millionths of a second (0.0000000336900 second $= 33.6900 \times 10^{-9}$ second). This equals 33.6900 **nanoseconds** in the terminology of high-speed electronic circuitry. (One nanosecond $= 10^{-9}$ second.) Mary measures a slightly different value for the time lapse between the two sparks, 33.0228 nanoseconds. For John the fire-extinguisher spark is separated in space by 2.0000 meters from the pen spark. For Mary in the rocket the pen spark and fire-extinguisher spark occur at the same place, namely at the end of her antenna. Thus for her their space separation equals zero.

Later, laboratory and rocket observers compare their space and time measurements between the various sparks (Table 1-3). Space locations and time lapses in both frames are measured from the pen spark.

TABLE 1-3

SPACE AND TIME LOCATIONS OF THE SAME SPARKS AS SEEN BY TWO OBSERVERS

	Distance and time between sparks as measured by observer who is standing in laboratory (John)		*moving by in rocket (Mary)*	
	Distance (meters)	Time (nanoseconds)	Distance (meters)	Time (nanoseconds)
Reference spark (pen spark)	0	0	0	0
Spark *A* (fire-extinguisher spark)	2.0000	33.6900	0	33.0228
Spark *B*	3.0000	50.5350	0	49.5343
Spark *C*	5.0000	84.2250	0	82.5572
Spark *D*	8.0000	134.7600	0	132.0915

Discovery: Invariance of spacetime interval

The third point of comparison between the Parable of the Surveyors and the state of physics before special relativity is this: The mythical student's discovery of the concept of distance is matched by the Einstein–Poincaré discovery in 1905 of the **invariant spacetime interval** (formal name **Lorentz interval**, but we often say just **interval**), a central theme of this book. Let each time measurement in seconds be converted to meters by multiplying it by the "conversion factor c," the speed of light:

$$\begin{aligned} c &= 299{,}792{,}458 \text{ meters/second} = 2.99792458 \times 10^8 \text{ meters/second} \\ &= 0.299792458 \times 10^9 \text{ meters/second} = 0.299792458 \text{ meters/nanosecond} \end{aligned}$$

Then the square of the spacetime interval is calculated from the laboratory observer's measurements by *subtracting* the square of the space separation from the square of the time separation. Note the minus sign in equation (1-4).

$$(\text{interval})^2 = \Bigg[c \times \overbrace{\begin{pmatrix} \text{time} \\ \text{separation} \\ \text{(seconds)} \end{pmatrix}}^{\textbf{Laboratory}} \Bigg]^2 - \overbrace{\begin{bmatrix} \text{space} \\ \text{separation} \\ \text{(meters)} \end{bmatrix}^2}^{\textbf{Laboratory}} \qquad (1\text{-}4)$$

The rocket calculation gives exactly the same value of the interval as the laboratory calculation,

$$(\text{interval})^2 = \Bigg[c \times \overbrace{\begin{pmatrix} \text{time} \\ \text{separation} \\ \text{(seconds)} \end{pmatrix}}^{\textbf{Rocket}} \Bigg]^2 - \overbrace{\begin{bmatrix} \text{space} \\ \text{separation} \\ \text{(meters)} \end{bmatrix}^2}^{\textbf{Rocket}} \qquad (1\text{-}5)$$

even though the respective space and time separations are not the same. Two observers find different space and time separations, respectively, between pen spark and fire-extinguisher spark, but when they calculate the spacetime interval between these sparks their results agree (Table 1-4).

The student surveyor found that invariance of distance was most simply written with both northward and eastward separations expressed in the same unit, the meter. Likewise, invariance of the spacetime interval is most simply written with space and

TABLE 1-4

"INVARIANT SPACETIME INTERVAL" FROM REFERENCE SPARK TO SPARK A

(Data from Table 1-3)

Laboratory measurements		Rocket measurements	
Time lapse 33.6900×10^{-9} seconds $= 33.6900$ nanoseconds		Time lapse 33.0228×10^{-9} seconds $= 33.0228$ nanoseconds	
Multiply by $c = 0.299792458$ meters per nanosecond to convert to meters: 10.1000 meters		Multiply by $c = 0.299792458$ meters per nanosecond to convert to meters: 9.9000 meters	
Square the value	102.010 $(\text{meters})^2$	Square the value	98.010 $(\text{meters})^2$
Spatial separation 2.000 meters		Spatial separation zero	
Square the value and subtract	$-$ 4.000 $(\text{meters})^2$	Square the value and subtract	$-$ 0
Result of subtraction expressed as a number squared	$= 98.010\ (\text{meters})^2$ $= (9.900\ \text{meters})^2$	Result of subtraction expressed as a number squared	$= 98.010\ (\text{meters})^2$ $= (9.900\ \text{meters})^2$
This is the square of what measurement?	**9.900 meters**	This is the square of what measurement?	**9.900 meters**

SAME SPACETIME
INTERVAL
from the reference event

time separations expressed in the same unit. Time is converted to meters: t (meters) $=$ $c \times t$ (seconds). Then the interval appears in simplified form:

$$(\text{interval})^2 = \begin{bmatrix} \text{time} \\ \text{separation} \\ (\text{meters}) \end{bmatrix}^2 - \begin{bmatrix} \text{space} \\ \text{separation} \\ (\text{meters}) \end{bmatrix}^2 \qquad (1\text{-}6)$$

The **invariance of the spacetime interval** — its independence of the state of motion of the observer — forces us to recognize that time cannot be separated from space. Space and time are part of a single entity, **spacetime.** Space has three dimensions: northward, eastward, and upward. Time has one dimension: onward! The interval combines all four dimensions in a single expression. The geometry of spacetime is truly four-dimensional.

Space and time are part of spacetime

To recognize the unity of spacetime we follow the procedure that makes a landscape take on depth — we look at it from several angles. That is why we compare space and time separations between events A and B as recorded by two different observers in relative motion.

Why the minus sign in the equation for the interval? Pythagoras tells us to ADD the squares of northward and eastward separations to get the square of the distance. Who tells us to SUBTRACT the square of the space separation between events from the square of their time separation in order to get the square of the spacetime interval?