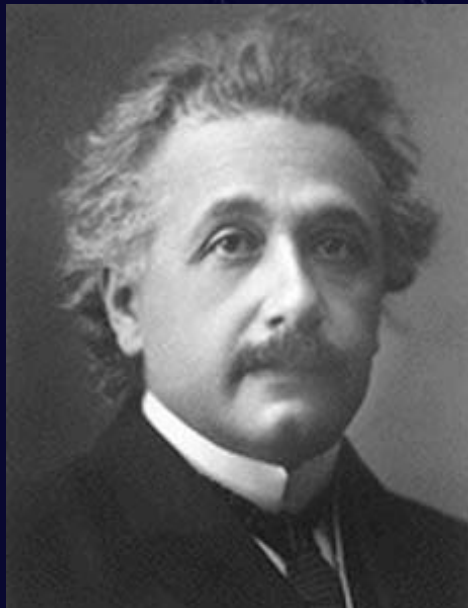


# Einstein's Universe



UNIVERSITY  
of  
GLASGOW



**Dr Martin Hendry**

*Dept of Physics and Astronomy  
University of Glasgow, UK*

# Plan for remaining sessions

- 6) GR, Cosmology and Black Holes
  - 7) Hot topics in Cosmology
  - 8) The Search for Gravitational Waves
  - 9) Einstein and the Quantum Revolution
  - 10) The post-Einstein Universe
- 

# Relativity: the Twins Paradox



1. Twins, **Terence** and **Stella**, on Earth, aged 20

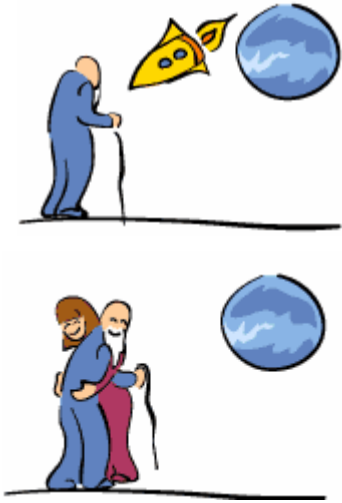


2. **Stella** sets off on spaceship to **Pollux** (distance 33.73 l.y.) travelling at  $0.999c$



3. From **Terence's** point of view, **Stella's** journey takes **33 years and 9 months** while from **Stella's** point of view it takes only **18 months**

# Relativity: the Twins Paradox



4. After a few days (from her point of view) at Pollux, **Stella** heads for home. When she arrives back, she is **23 years old**. There she greets her 'twin' Terence, who is now **87½ years old**.

# Relativity: the Twins Paradox



4. After a few days (from her point of view) at Pollux, **Stella** heads for home. When she arrives back, she is **23 years old**. There she greets her ‘twin’ Terence, who is now **87½ years old**.

*The ‘paradox’ is the argument that says:*

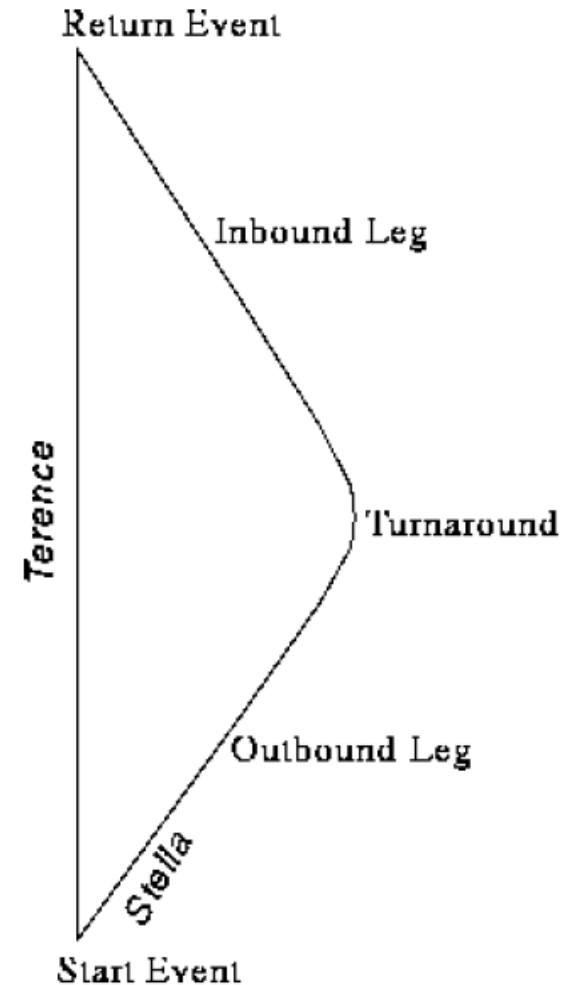
*“Why can’t we think about Stella as being the ‘stay at home’ twin, and Stella as the traveller?*

*Wouldn’t that mean Stella is the twin that ages?”*

# Resolving the Twins Paradox

The 'paradox' is resolved when we realise that the roles of Terence and Stella are **not** fully symmetrical.

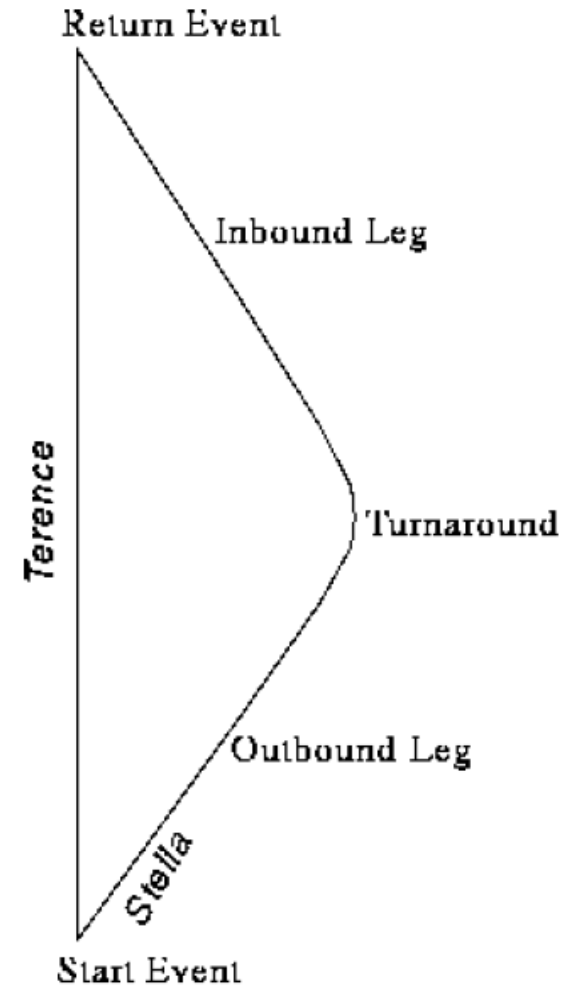
- Terence remains in the **same** inertial reference frame throughout Stella's journey
- Stella **changes** inertial frame between the outbound and inbound legs.



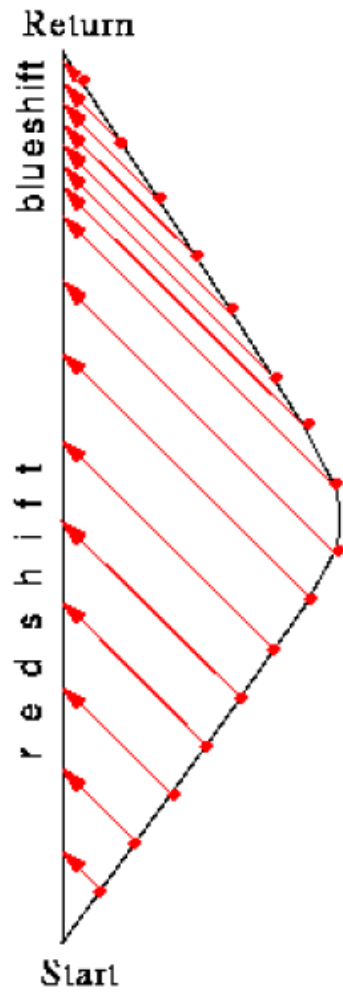
# Resolving the Twins Paradox

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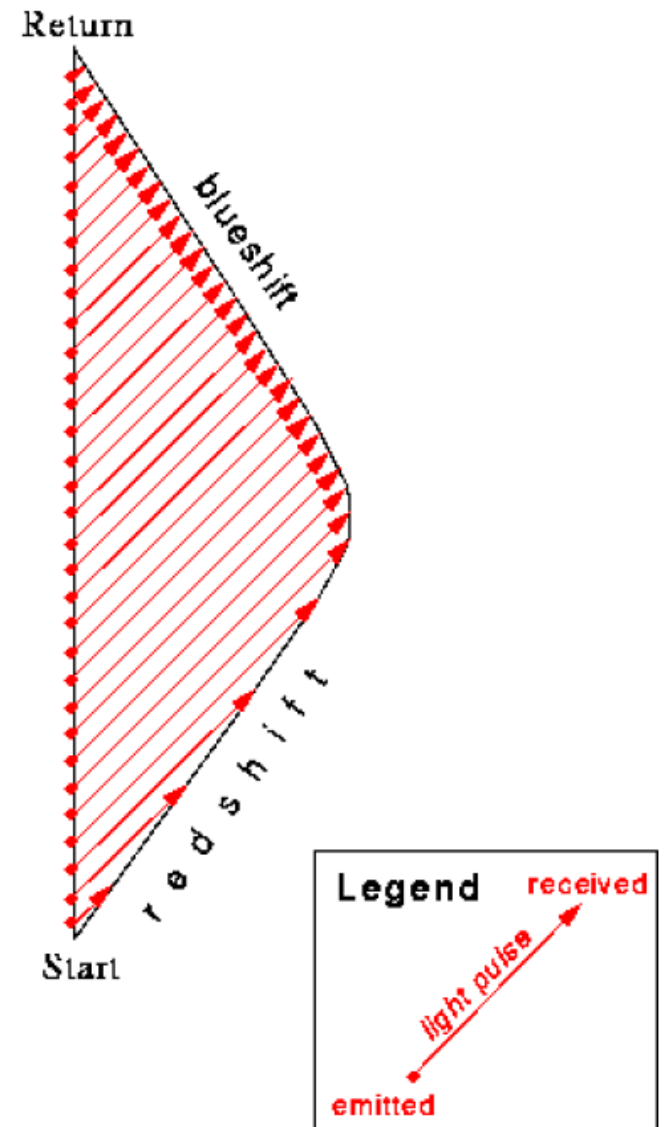
- Terence remains in the **same** inertial reference frame throughout Stella's journey
- Stella **changes** inertial frame between the outbound and inbound legs.
- The **relativity of simultaneity** between these **three** inertial frames results in the difference between the elapsed **proper time** for Terence and Stella.



Stella sends pulses to Terence



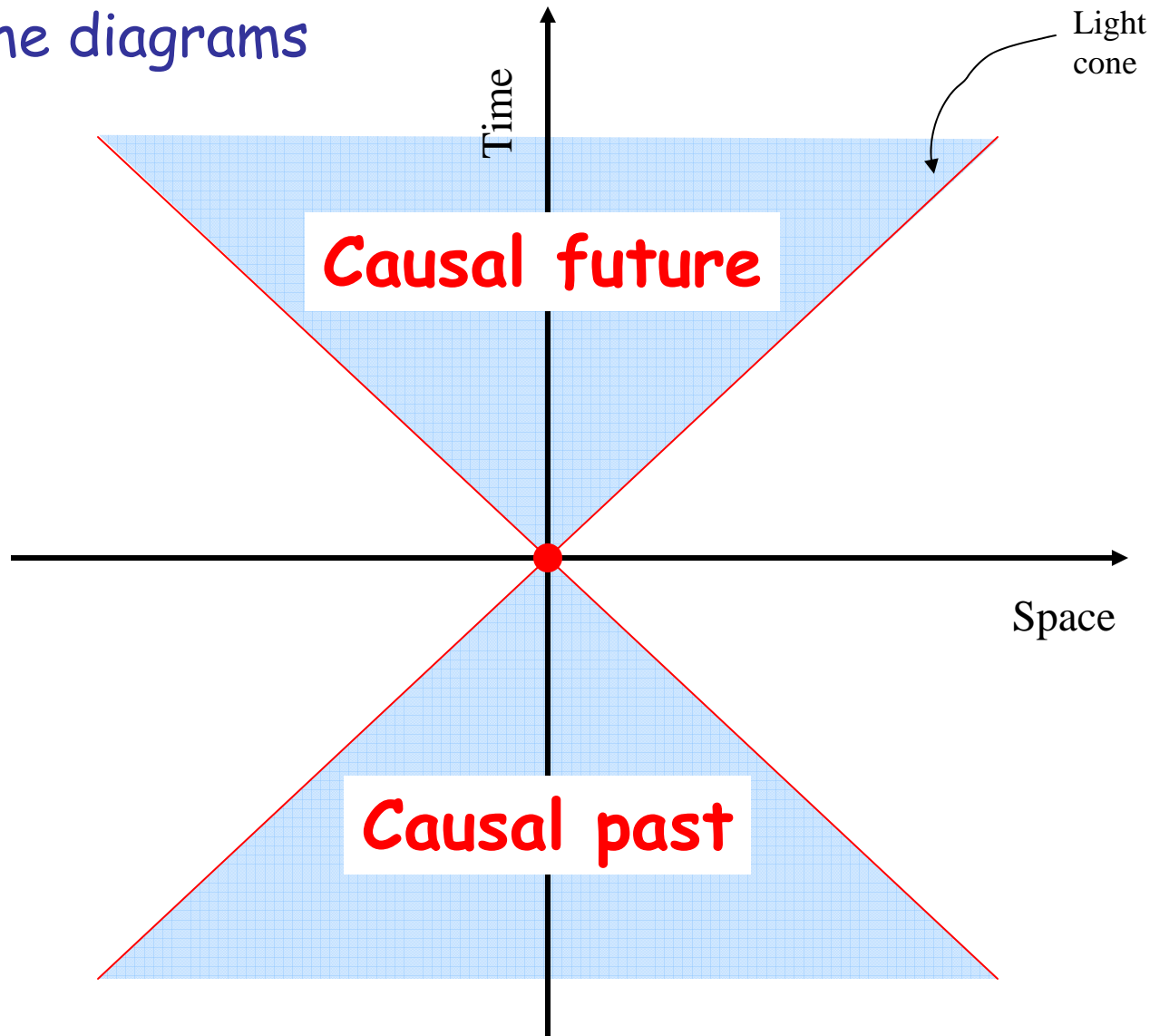
Terence sends pulses to Stella



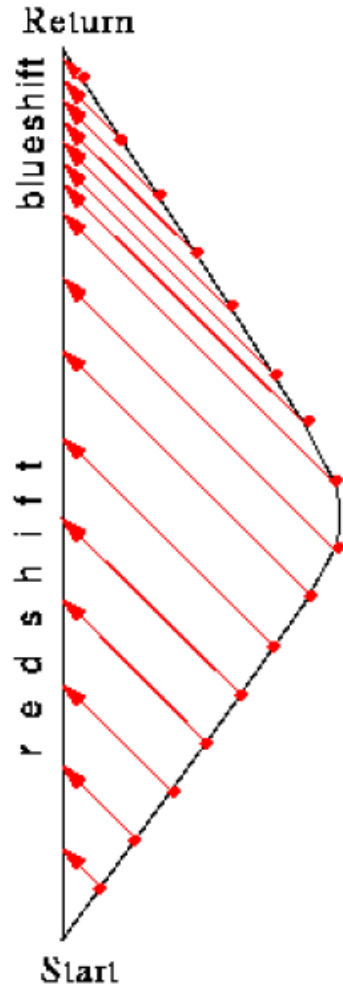


# Making Sense of Einstein's Universe

Spacetime diagrams



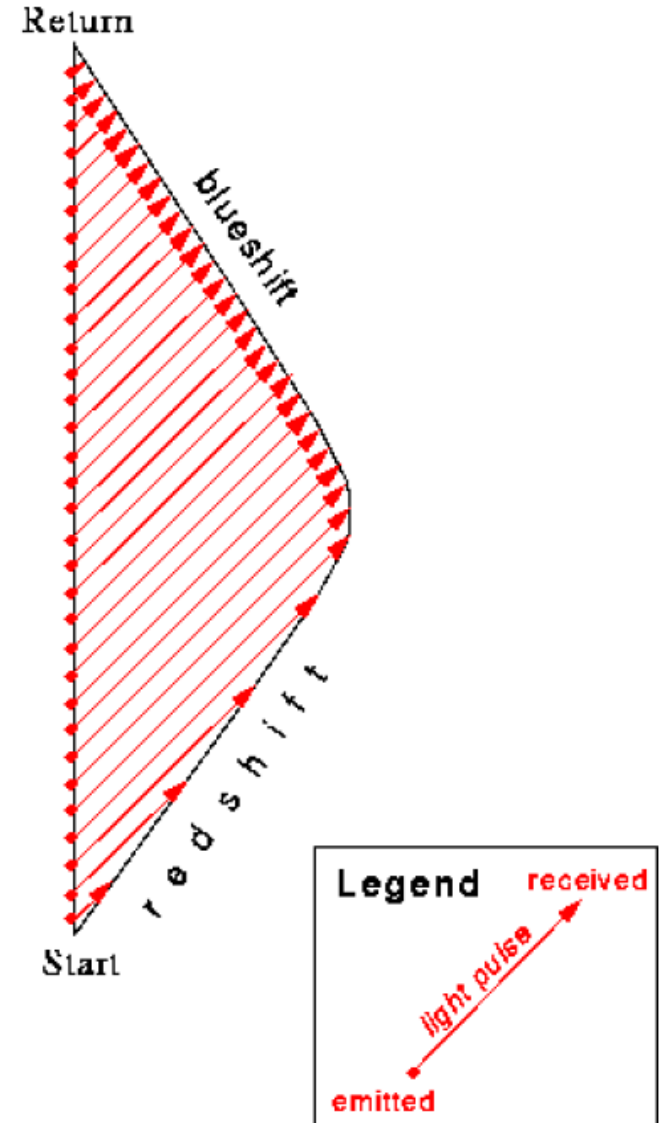
Stella sends pulses to Terence



Pulses are sent at **equal** proper time intervals in the **sender's** frame.

These pulses **don't** arrive at equal time intervals in the **receiver's** frame.

Terence sends pulses to Stella

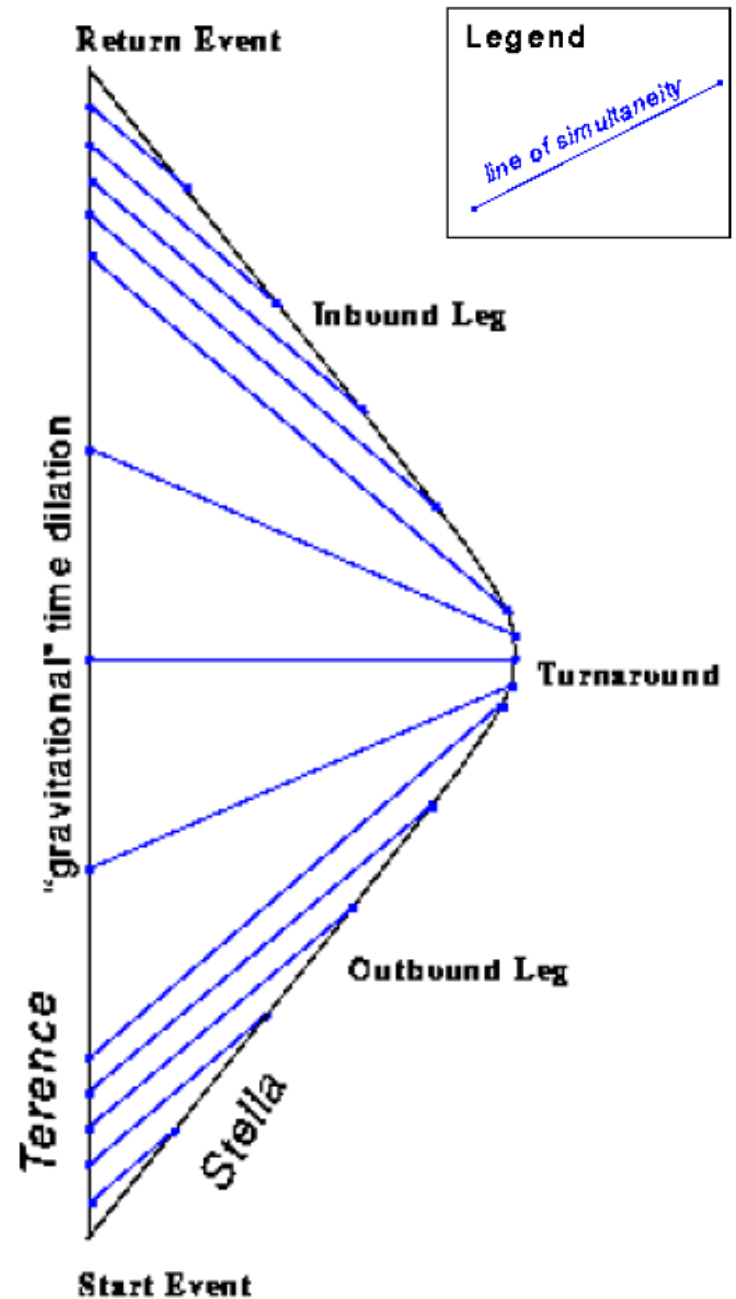


We can also look at this in terms of spacetime events which *are* simultaneous.

We see that, from Stella's point of view, Terence does most of his aging close to Stella's 'turnaround', when she changes inertial frame (i.e. she changes **velocity**).

Change in velocity = **acceleration**.

This makes it a GR problem!



We can also look at this in terms of spacetime events which *are* simultaneous.

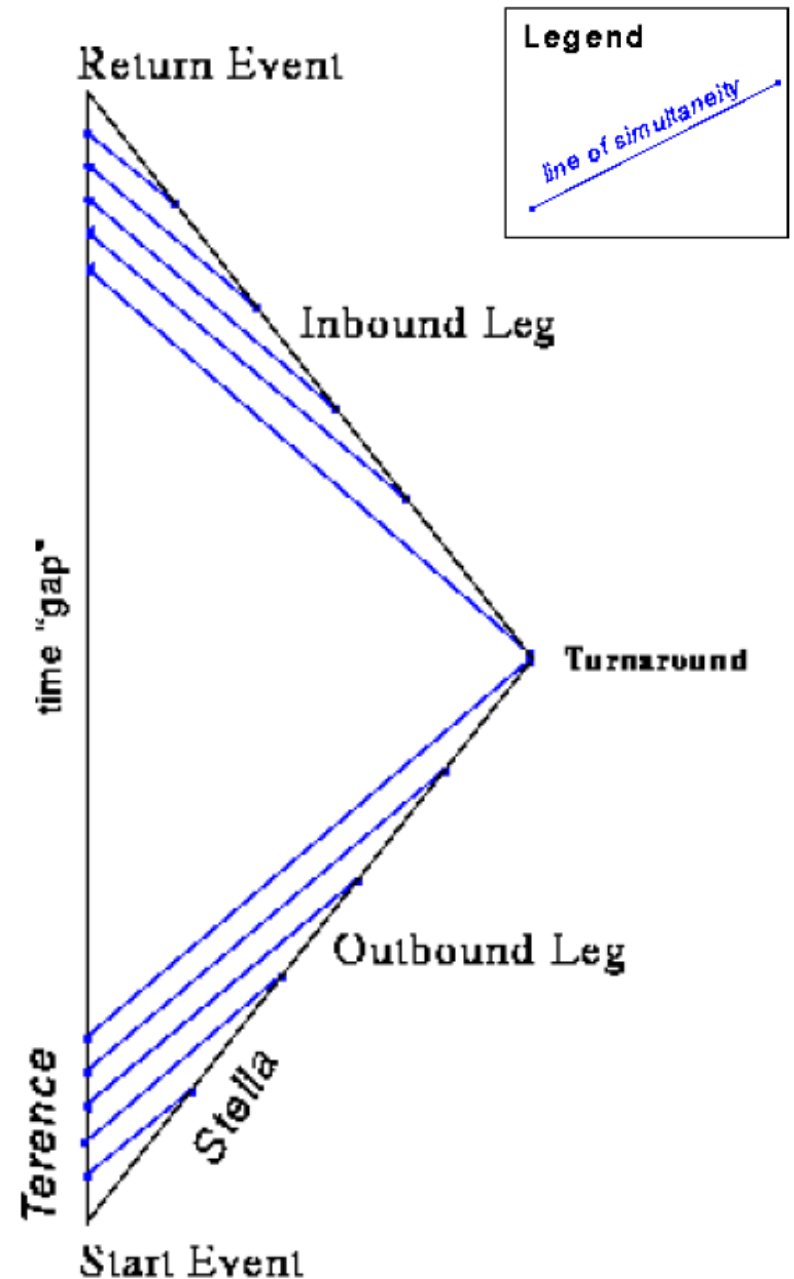
We see that, from Stella's point of view, Terence does most of his aging close to Stella's 'turnaround', when she changes inertial frame (i.e. she changes **velocity**).

Change in velocity = **acceleration**.

This makes it a GR problem!

But we can make the 'turnaround' as sharp as we like, so that the maths of SR will be enough.

The 'time gap' is still clearly there



## From Special to General Relativity...

*How do we 'stitch'  
all the Local Inertial  
Frames together?*

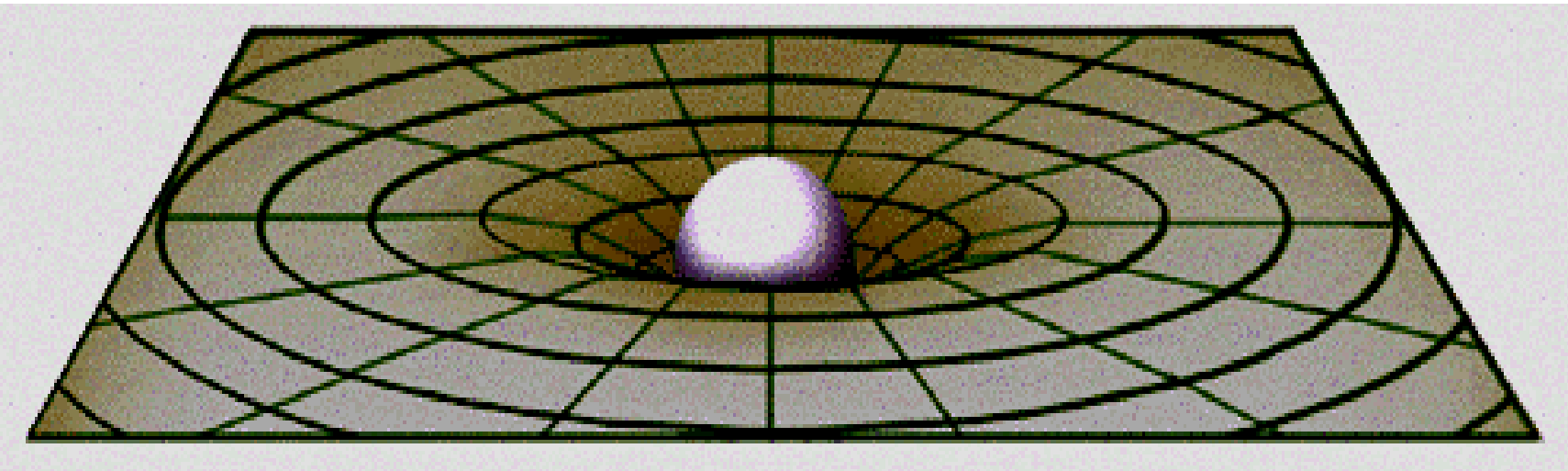
*Can we find a  
mathematical  
description that  
makes sense for  
any observer?*



# Gravity in Einstein's Universe: 1912



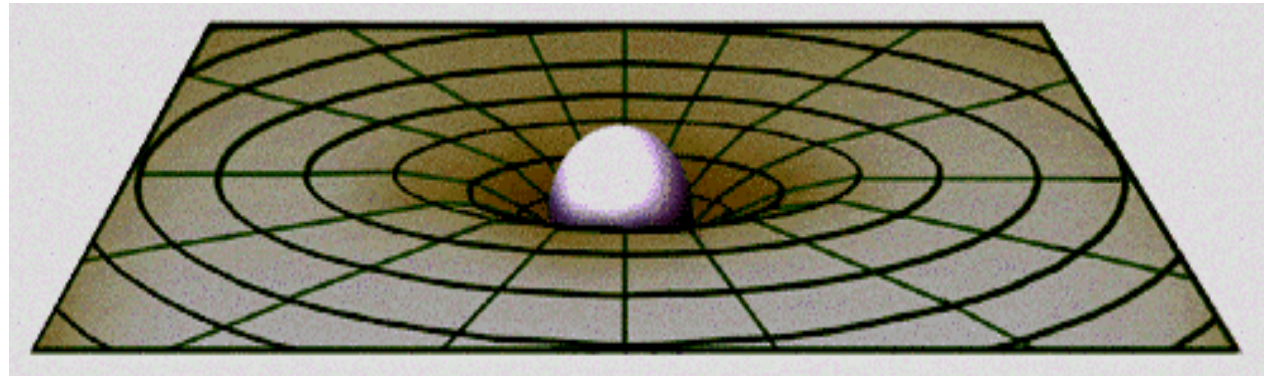
“Spacetime tells matter how to move, and matter tells spacetime how to curve”







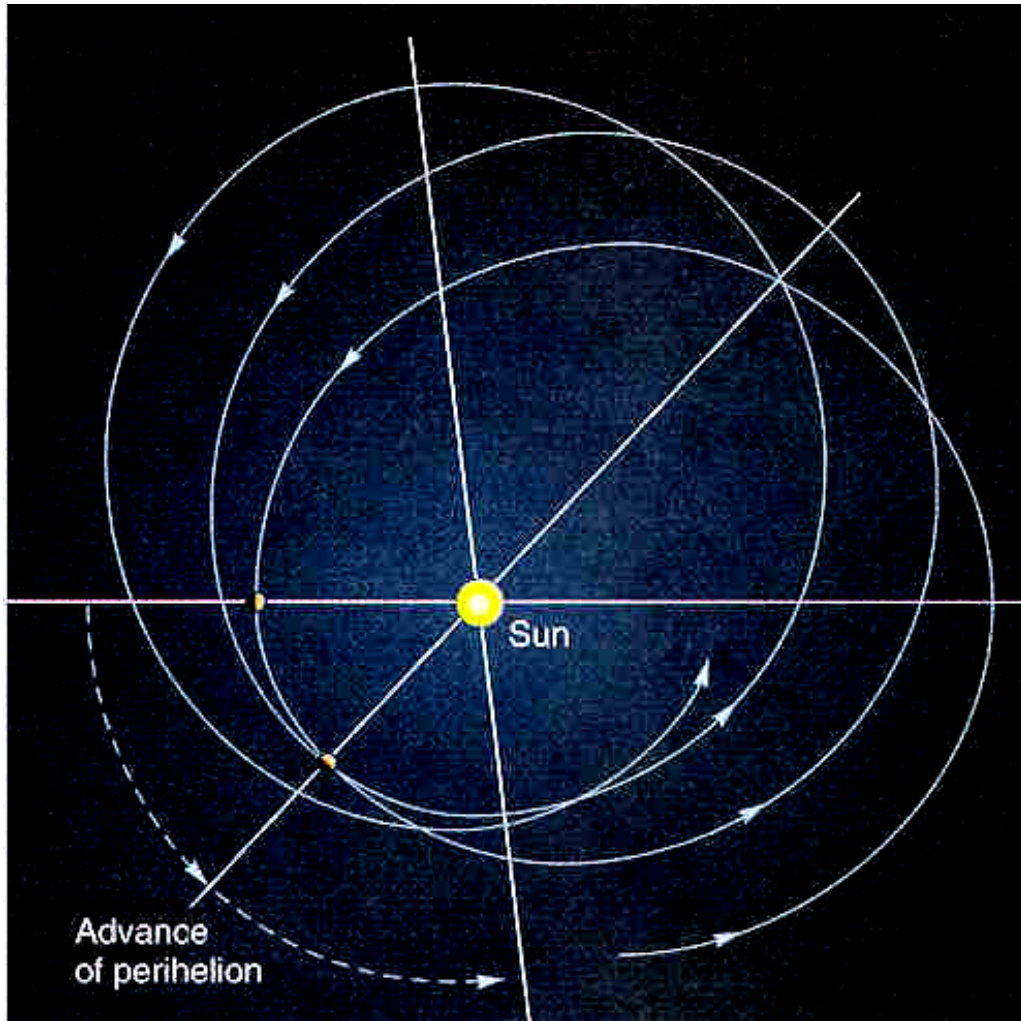
*“Since the mathematicians have invaded the theory of relativity, I do not understand it myself anymore.”*



Abandoned geometric approach in 1913, but returned to it in 1915. In October 1915 published **field equations** relating geometry to matter-energy content.

Looked promising! Successfully predicted **perihelion advance**





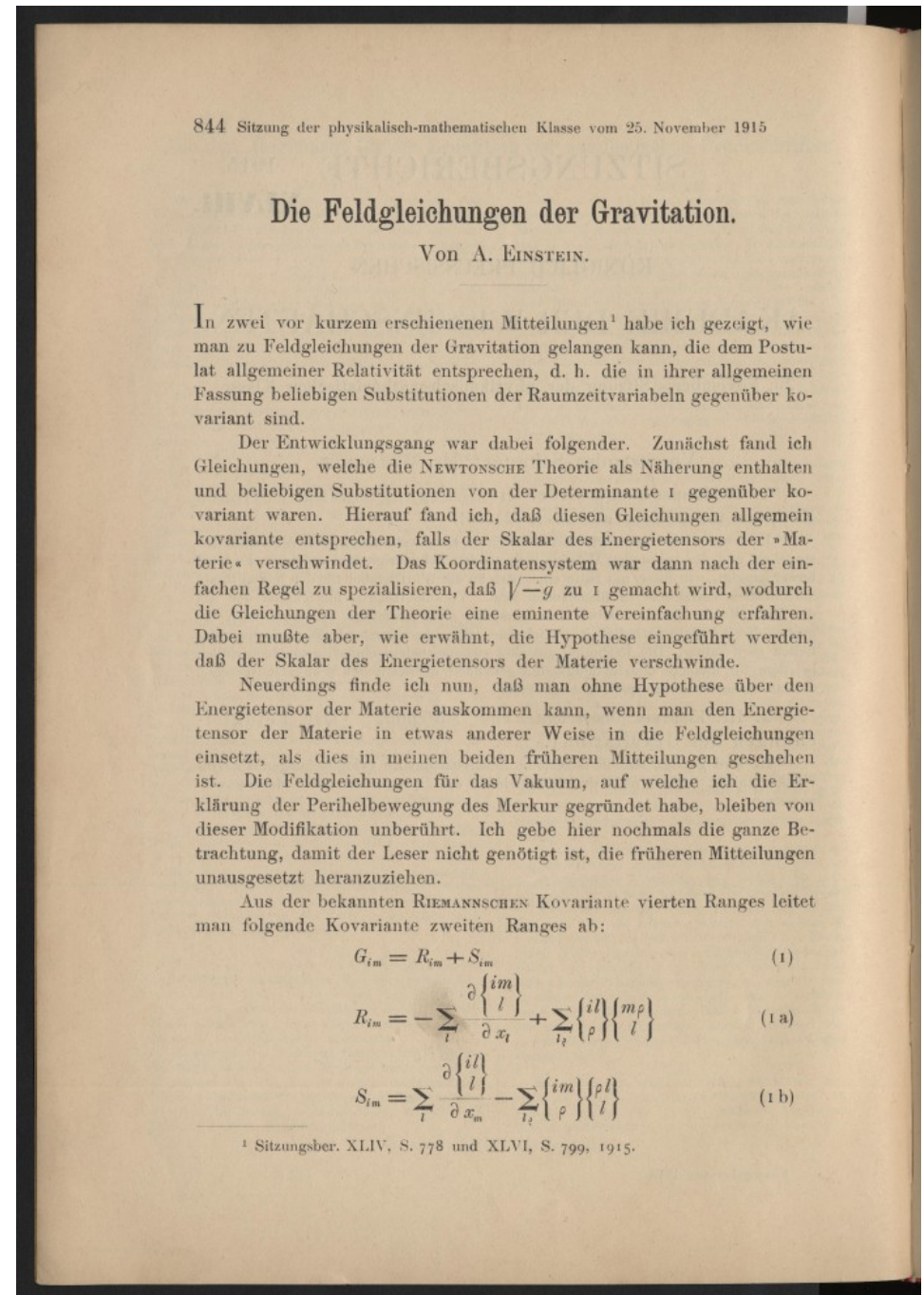
Le Verrier

GR explained the advance of Perihelion of Mercury, observed since the 19<sup>th</sup> Century.



Formulated correct field equations in Nov 1915 paper. (Also published by David Hilbert).

Very complicated!  
Einstein thought there were no exact solutions



## Die Feldgleichungen der Gravitation.

VON A. EINSTEIN.

In zwei vor kurzem erschienenen Mitteilungen<sup>1</sup> habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Fassung beliebigen Substitutionen der Raumzeitvariablen gegenüber kovariant sind.

Der Entwicklungsgang war dabei folgender. Zunächst fand ich Gleichungen, welche die NEWTONSCHE Theorie als Näherung enthalten und beliebigen Substitutionen von der Determinante 1 gegenüber kovariant waren. Hierauf fand ich, daß diesen Gleichungen allgemein kovariante entsprechen, falls der Skalar des Energietensors der „Materie“ verschwindet. Das Koordinatensystem war dann nach der einfachen Regel zu spezialisieren, daß  $\sqrt{-g}$  zu 1 gemacht wird, wodurch die Gleichungen der Theorie eine eminente Vereinfachung erfahren. Dabei mußte aber, wie erwähnt, die Hypothese eingeführt werden, daß der Skalar des Energietensors der Materie verschwinde.

Neuerdings finde ich nun, daß man ohne Hypothese über den Energietensor der Materie auskommen kann, wenn man den Energietensor der Materie in etwas anderer Weise in die Feldgleichungen einsetzt, als dies in meinen beiden früheren Mitteilungen geschehen ist. Die Feldgleichungen für das Vakuum, auf welche ich die Erklärung der Perihelbewegung des Merkur gegründet habe, bleiben von dieser Modifikation unberührt. Ich gebe hier nochmals die ganze Betrachtung, damit der Leser nicht genötigt ist, die früheren Mitteilungen unausgesetzt heranzuziehen.

Aus der bekannten RIEMANNSCHEM Kovariante vierten Ranges leitet man folgende Kovariante zweiten Ranges ab:

$$G_{im} = R_{im} + S_{im} \quad (1)$$

$$R_{im} = - \sum_l \frac{\partial \{im\}}{\partial x_l} + \sum_{l_2} \{il\} \{m\rho\} \quad (1a)$$

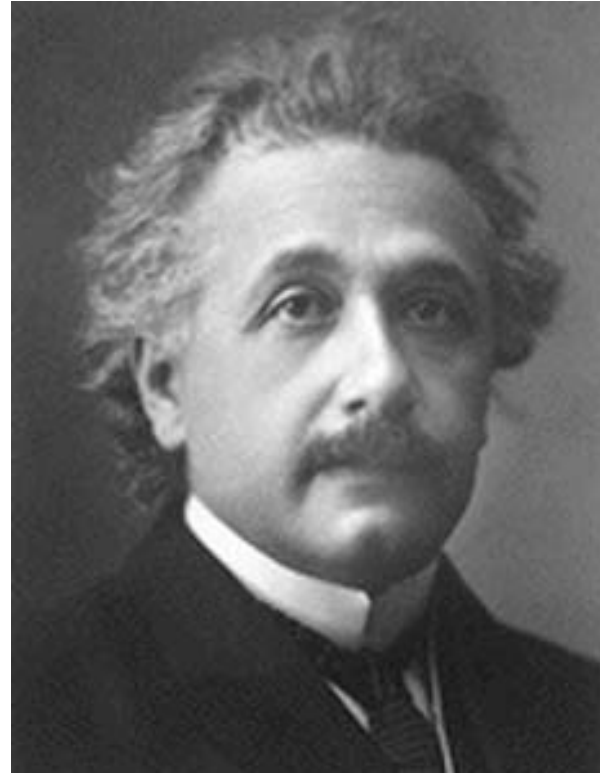
$$S_{im} = \sum_l \frac{\partial \{il\}}{\partial x_m} - \sum_{l_2} \{im\} \{l\rho\} \quad (1b)$$

<sup>1</sup> Sitzungsber. XLIV, S. 778 und XLVI, S. 799, 1915.



Colleague: *"Professor Eddington, you must be one of only three persons in the world who understand relativity!"*

Eddington: *"oh, I don't know..."*



Colleague: *"Professor Eddington, you must be one of only three persons in the world who understand relativity!"*

Eddington: *"oh, I don't know..."*

Colleague: *"Don't be modest Eddington."*

Eddington: ***"On the contrary, I am trying to think who the third person is."***



*“...joy and amazement at the beauty and grandeur of this world of which man can just form a faint notion.”*

$$G_{\mu\nu} = \kappa T_{\mu\nu}$$

Spacetime  
curvature

Matter  
(and energy)

1916.

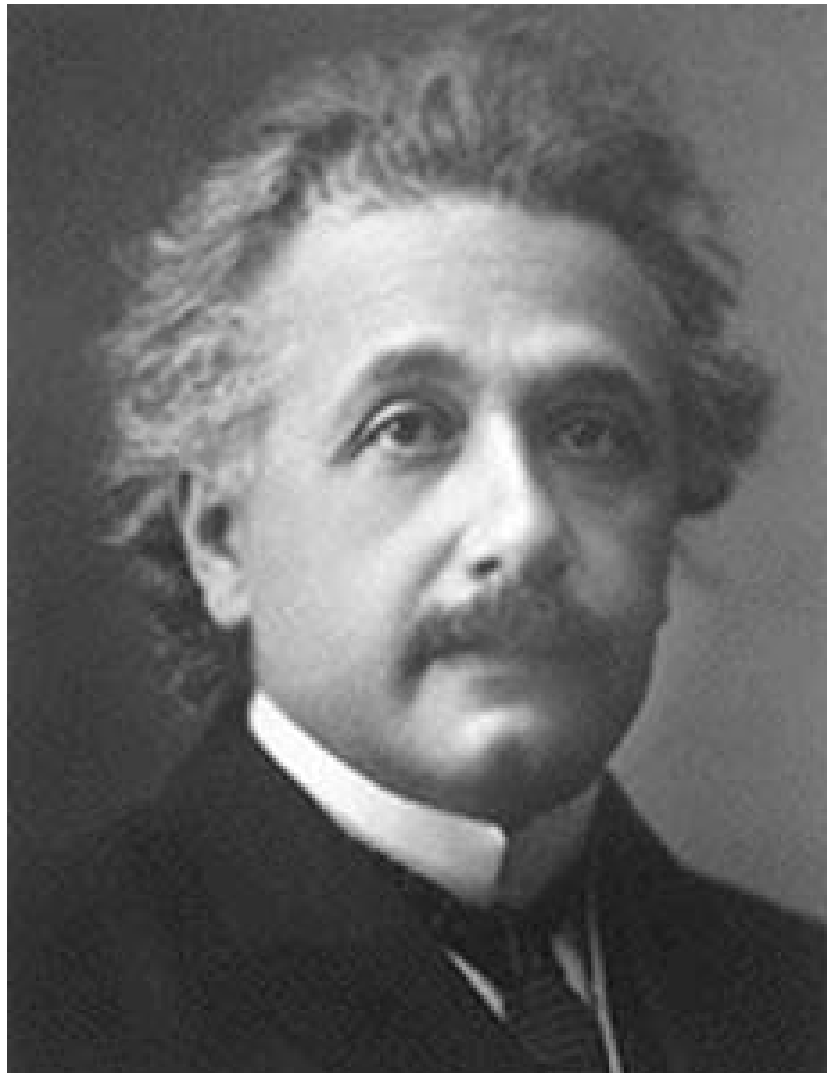
№ 7.

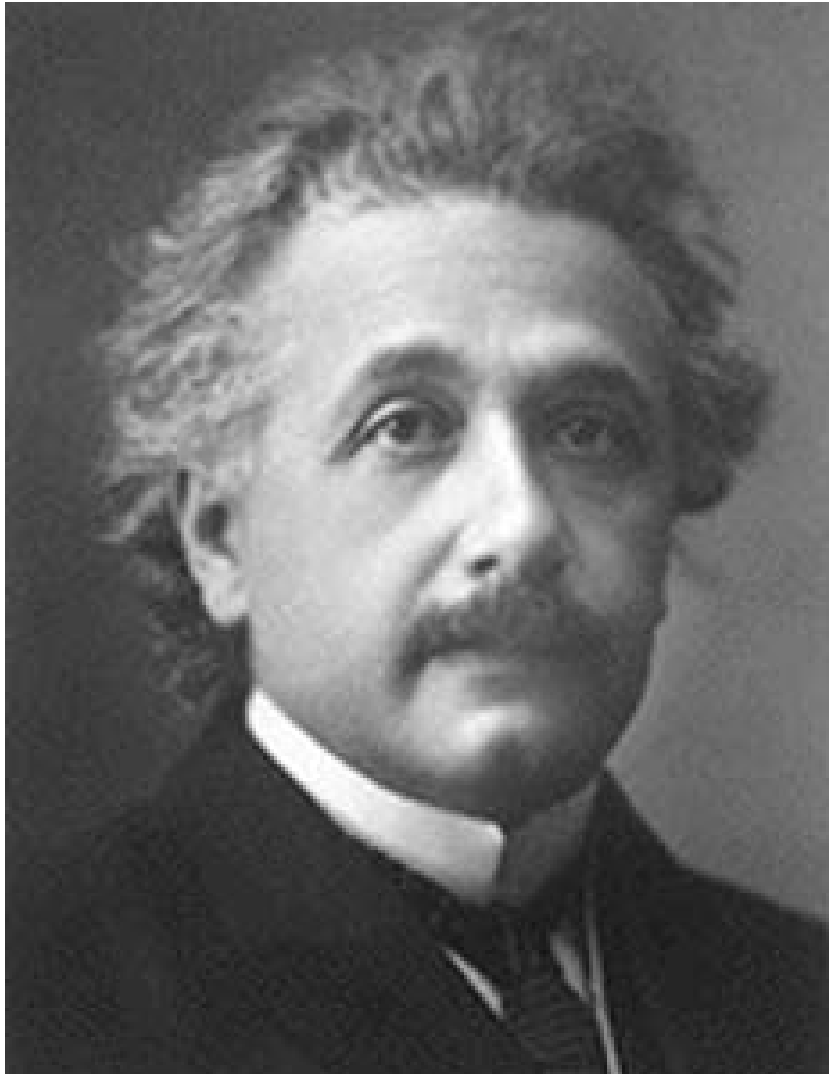
## ANNALEN DER PHYSIK.

VIERTE FOLGE. BAND 49.

### 1. Die Grundlage der allgemeinen Relativitätstheorie; von A. Einstein.

Die im nachfolgenden dargelegte Theorie bildet die denkbar weitgehendste Verallgemeinerung der heute allgemein als „Relativitätstheorie“ bezeichneten Theorie; die letztere nenne ich im folgenden zur Unterscheidung von der ersteren „spezielle Relativitätstheorie“ und setze sie als bekannt voraus. Die Verallgemeinerung der Relativitätstheorie wurde sehr erleichtert durch die Gestalt, welche der speziellen Relativitätstheorie durch Minkowski gegeben wurde, welcher Mathematiker zuerst die formale Gleichwertigkeit der räumlichen Koordinaten und der Zeitkoordinate klar erkannte und für den Aufbau der Theorie nutzbar machte. Die für die allgemeine Relativitätstheorie nötigen mathematischen Hilfsmittel lagen fertig bereit in dem „absoluten Differentialkalkül“, welcher auf den Forschungen von Gauss, Riemann und Christoffel über nichteuklidische Mannigfaltigkeiten ruht und von Ricci und Levi-Civita in ein System gebracht und bereits auf Probleme der theoretischen Physik angewendet wurde. Ich habe im Abschnitt B der vorliegenden Abhandlung alle für uns nötigen, bei dem Physiker nicht als bekannt vorauszusetzenden mathematischen Hilfsmittel in möglichst einfacher und durchsichtiger Weise entwickelt, so daß ein Studium mathematischer Literatur für das Verständnis der vorliegenden Abhandlung nicht erforderlich ist. Endlich sei an dieser Stelle dankbar meines Freundes, des Mathematikers Grossmann, gedacht, der mir durch seine Hilfe nicht nur das Studium der einschlägigen mathematischen Literatur ersparte, sondern mich auch beim Suchen nach den Feldgleichungen der Gravitation unterstützte.





“The greatest feat of human thinking about nature, the most amazing combination of philosophical penetration, physical intuition and mathematical skill.”

*Max Born*

## Schwarzschild solution for a spherical star



Karl Schwarzschild:  
1873 - 1916

*Einstein, on K.S.*

“I have read your paper with the utmost interest. I had not expected that one could formulate the exact solution of the problem in such a simple way. I liked very much your mathematical treatment of the subject. Next Thursday I shall present the work to the Academy with a few words of explanation.”



Schwarzschild' solution for the spacetime metric exterior to a point mass,  $M$

$$ds^2 = - \left( 1 - \frac{2M}{r} \right) dt^2 + \frac{dr^2}{\left( 1 - \frac{2M}{r} \right)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$$

Compare with the 'flat' spacetime of Minkowski (SR)

$$ds^2 = -dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$$

Schwarzschild' solution for the spacetime metric exterior to a point mass,  $M$

$$ds^2 = - \left( 1 - \frac{2M}{r} \right) dt^2 + \frac{dr^2}{\left( 1 - \frac{2M}{r} \right)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$$

This term means spacetime isn't flat close to the point mass

Compare with the 'flat' spacetime of Minkowski (SR)

$$ds^2 = -dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$$

## Four classical 'tests' of the Schwarzschild GR solution:

1) Advance of pericentre of orbits

2) Gravitational lensing

3) Gravitational redshift

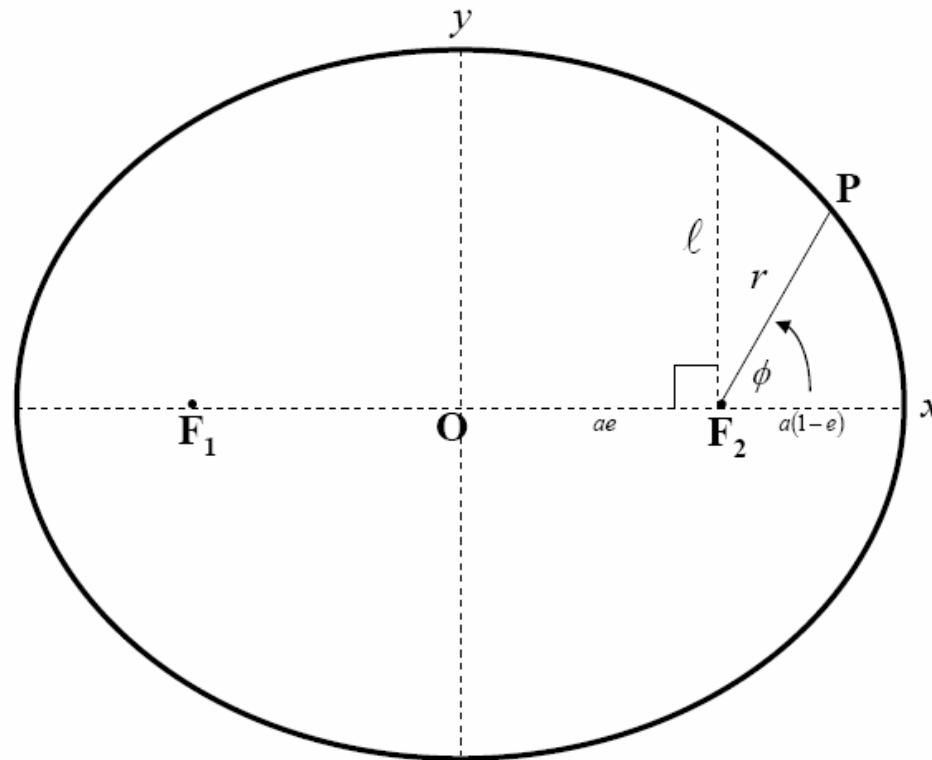
4) Gravitational time-delay



Essentially the same effect

# 1) Advance of orbit pericentre

Newtonian solution:  
Elliptical orbit



Foci at  $F_1$  and  $F_2$

$Ox = a =$  semi-major axis

$Oy = b =$  semi-minor axis

$$b^2 = a^2(1 - e^2)$$

$e =$  eccentricity

$P$  defined by

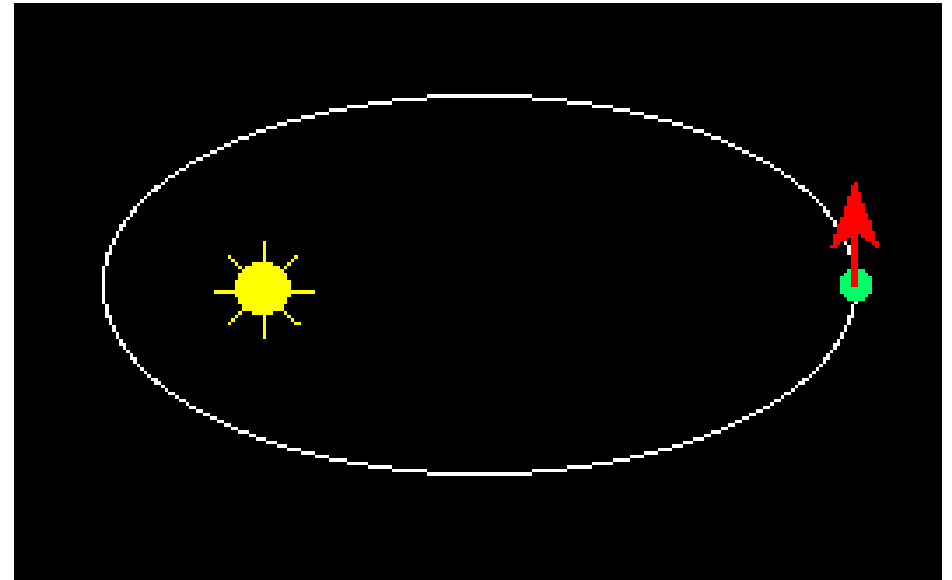
$$r = \frac{l}{1 + e \cos \phi}$$

$l =$  semi-latus rectum

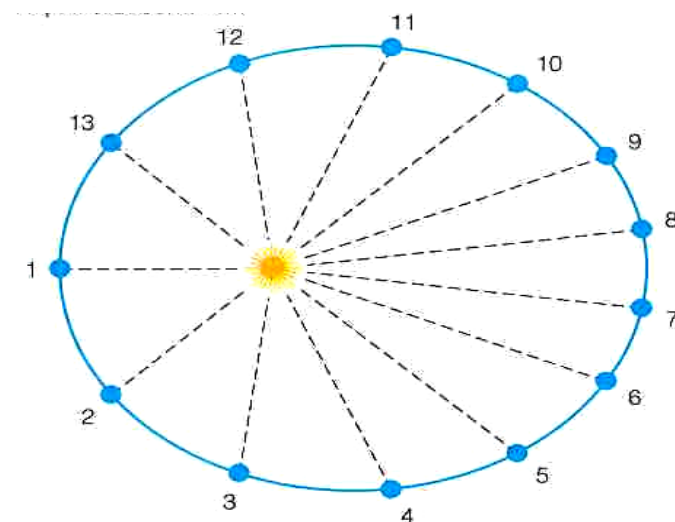
$$= b^2 / a$$

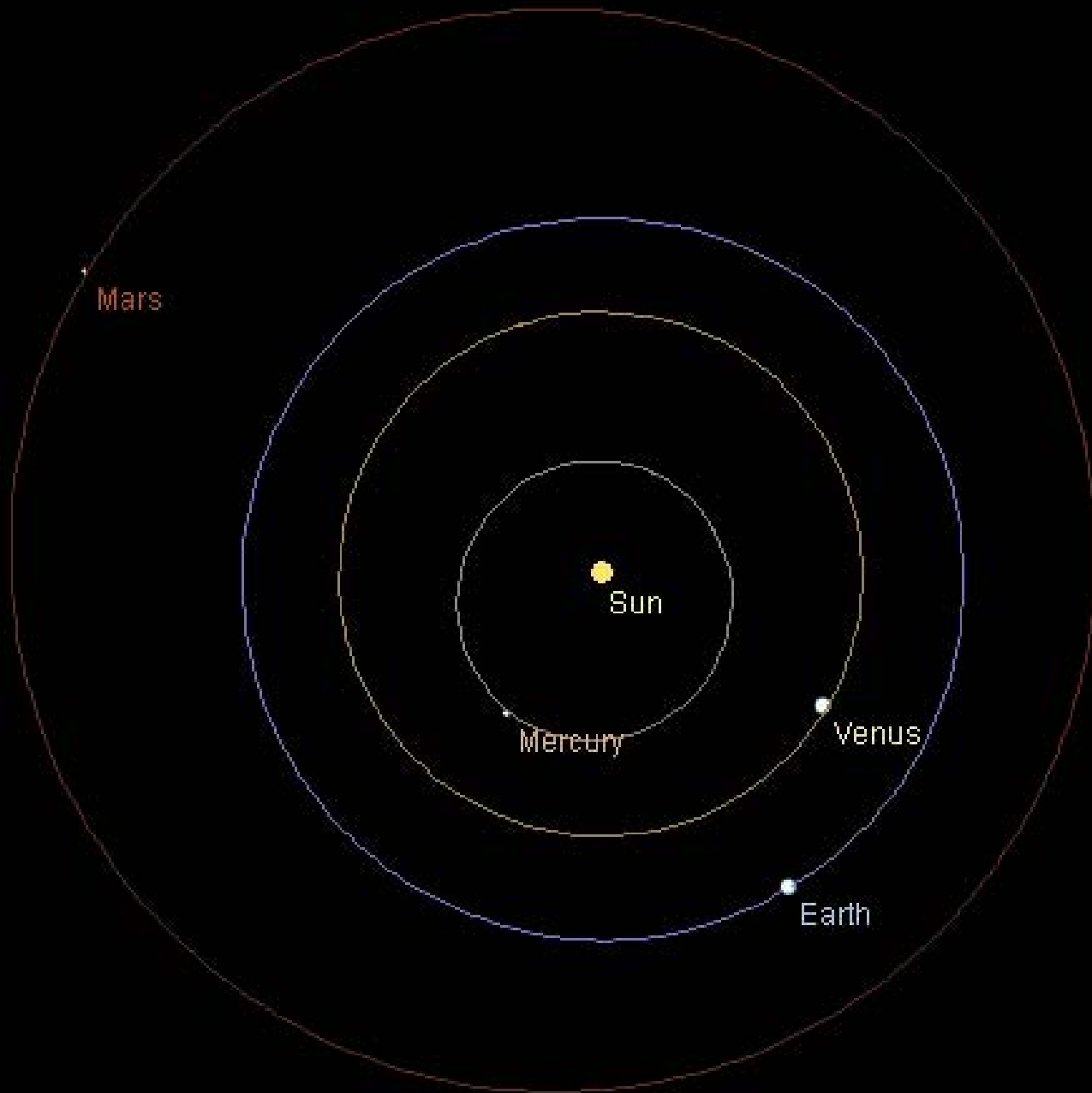


Johannes Kepler  
(1571-1630)



'New Astronomy'  
published in 1609





GR solution:

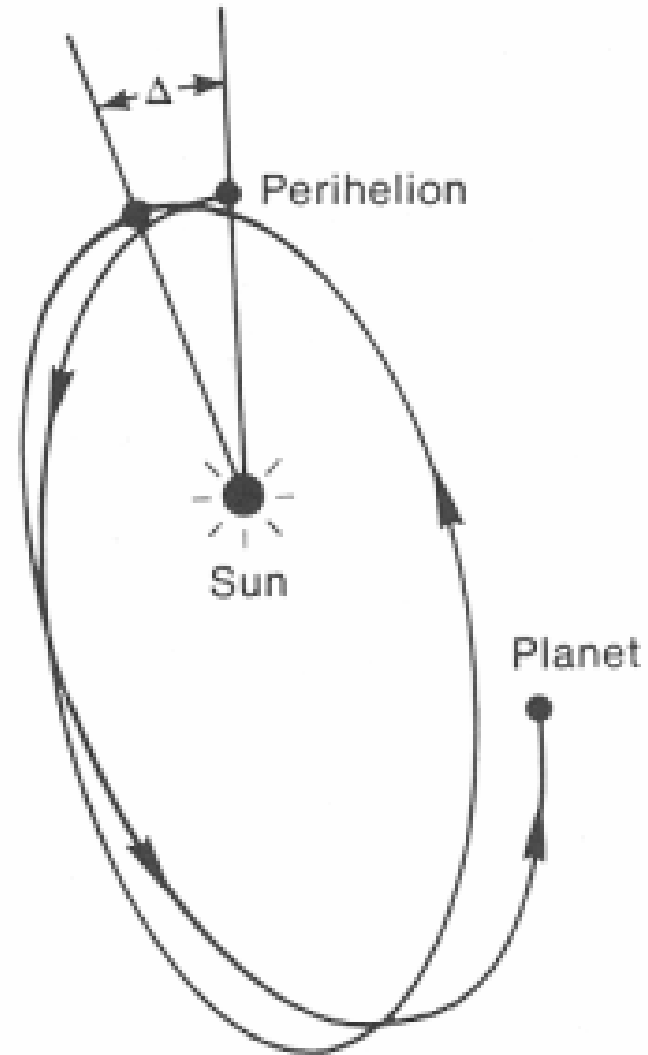
Precessing ellipse

$$u = \frac{M}{h^2} \left[ 1 + e \cos \left( 1 - \frac{3M^2}{h^2} \right) \phi \right]$$

Here

$$P = \frac{2\pi}{1 - 3M^2/h^2} > 2\pi$$

$$\Delta = \frac{6\pi M}{a(1 - e^2)}$$



GR solution:

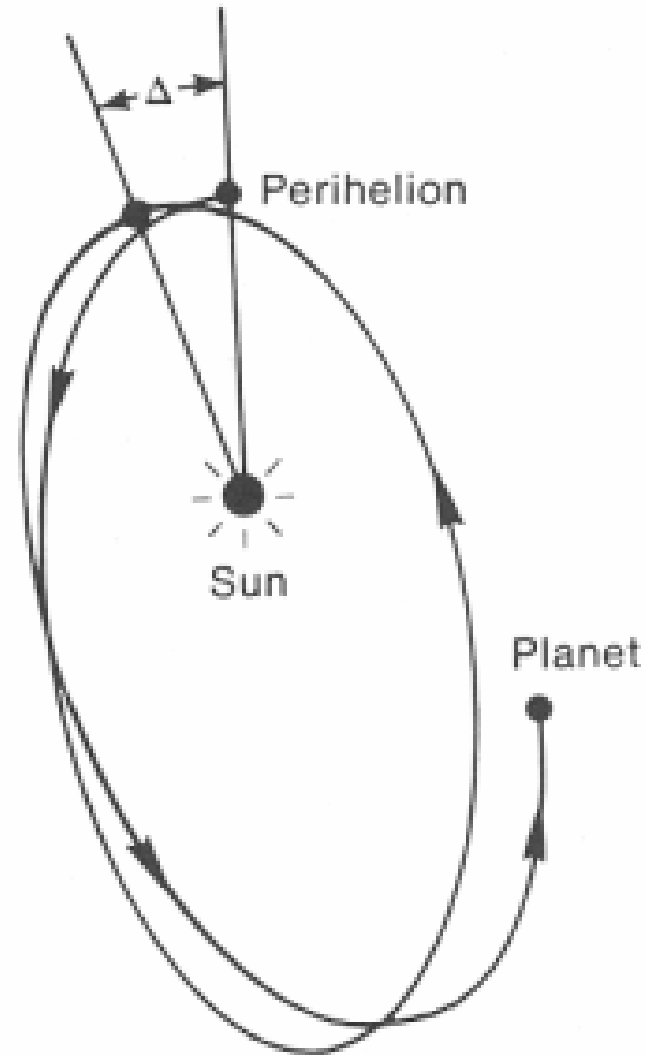
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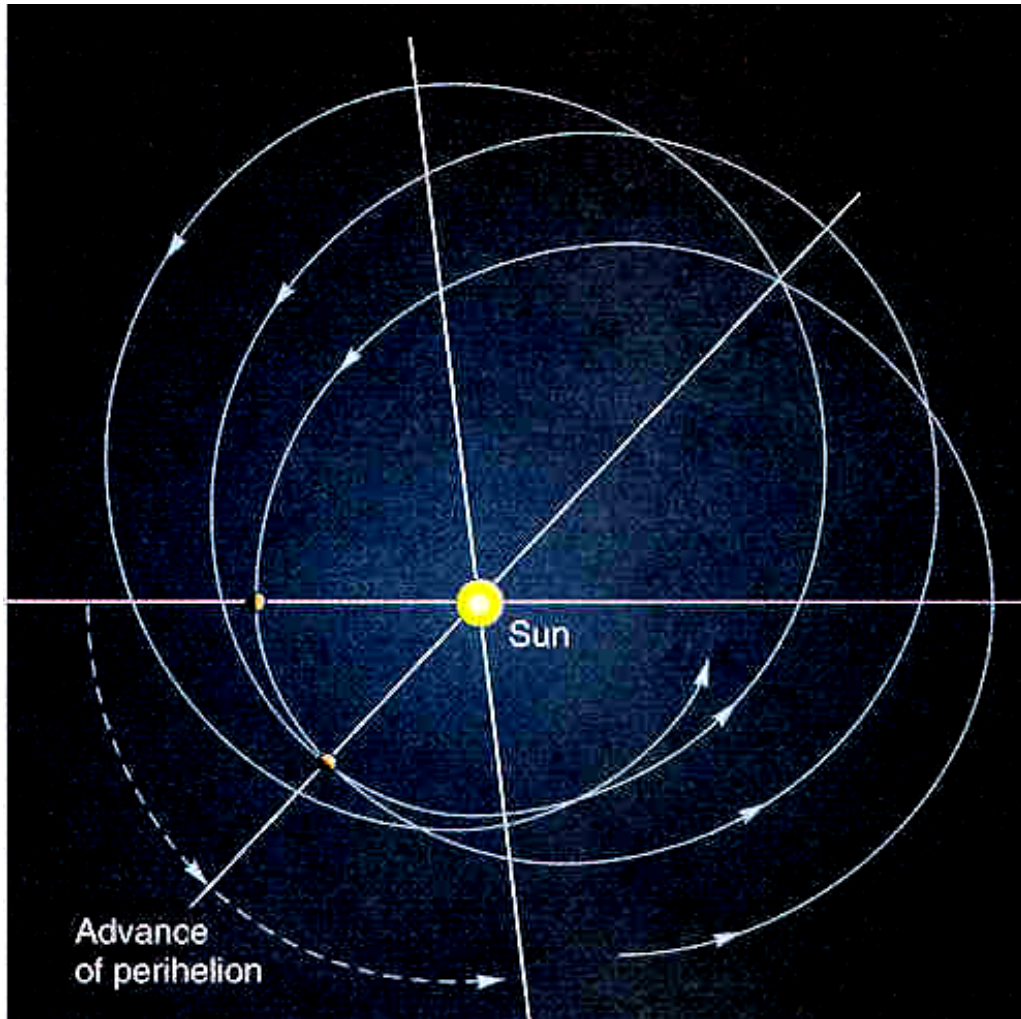
$$P = \frac{2\pi}{1 - 3M^2/h^2} > 2\pi$$

$$\Delta = \frac{6\pi M}{a(1 - e^2)}$$



If we apply this equation to the orbit of Mercury, we obtain a perihelion advance which builds up to about 43 seconds of arc per century.





Le Verrier

GR explained the **advance of Perihelion of Mercury**, observed since the 19<sup>th</sup> Century.

GR solution:

Precessing ellipse

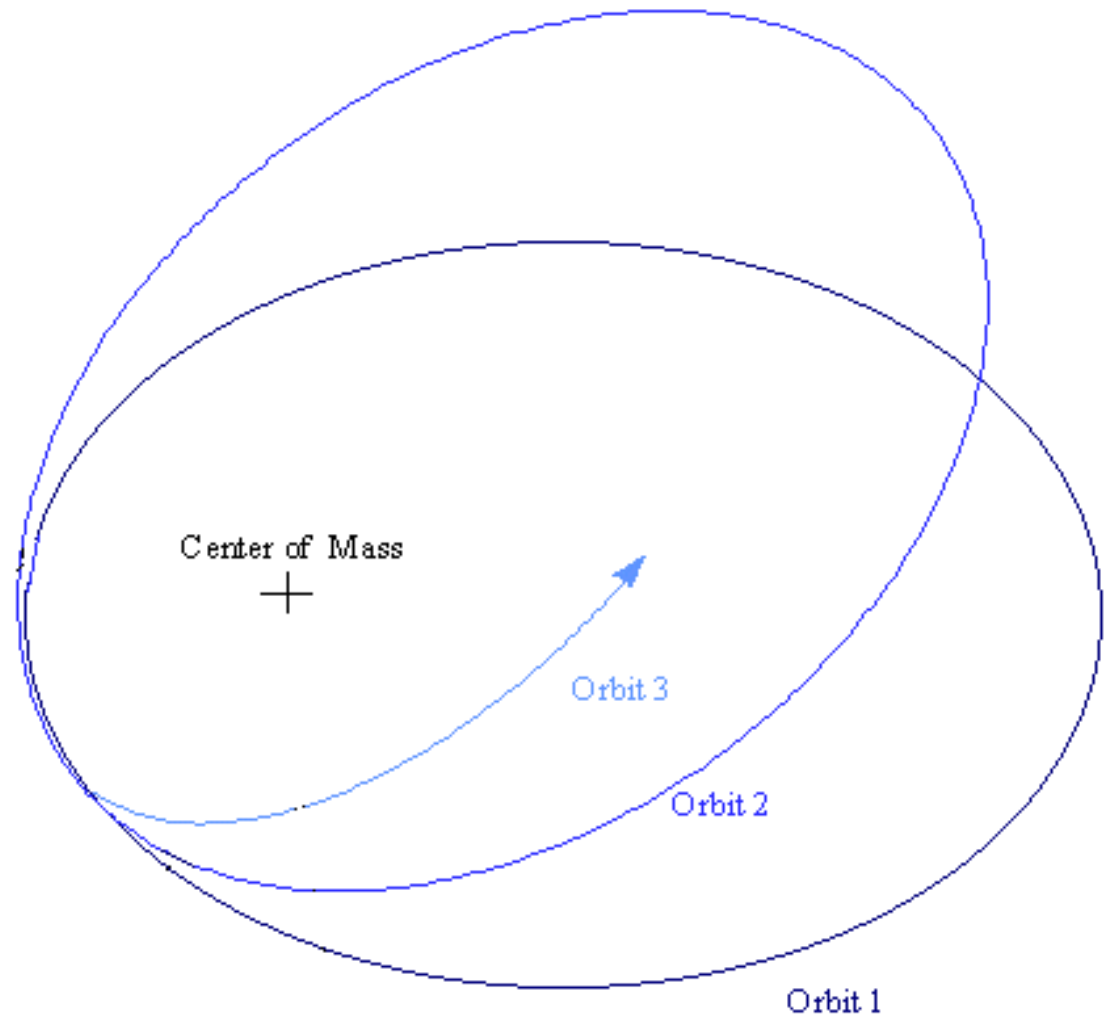
Seen much more  
dramatically in the

**binary pulsar**

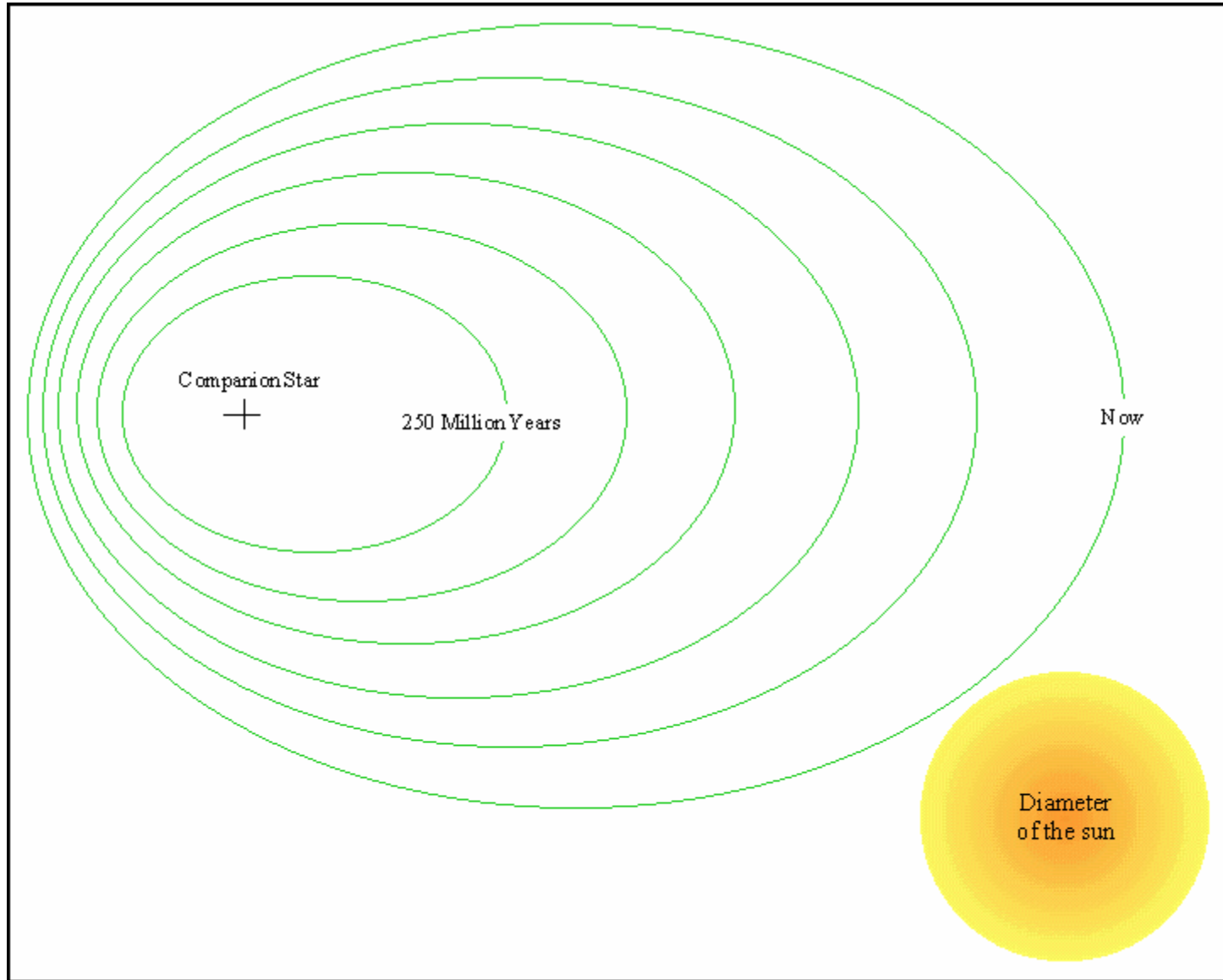
PSR 1913+16.

Periastron is

advancing at a rate of  
~4 degrees per year!

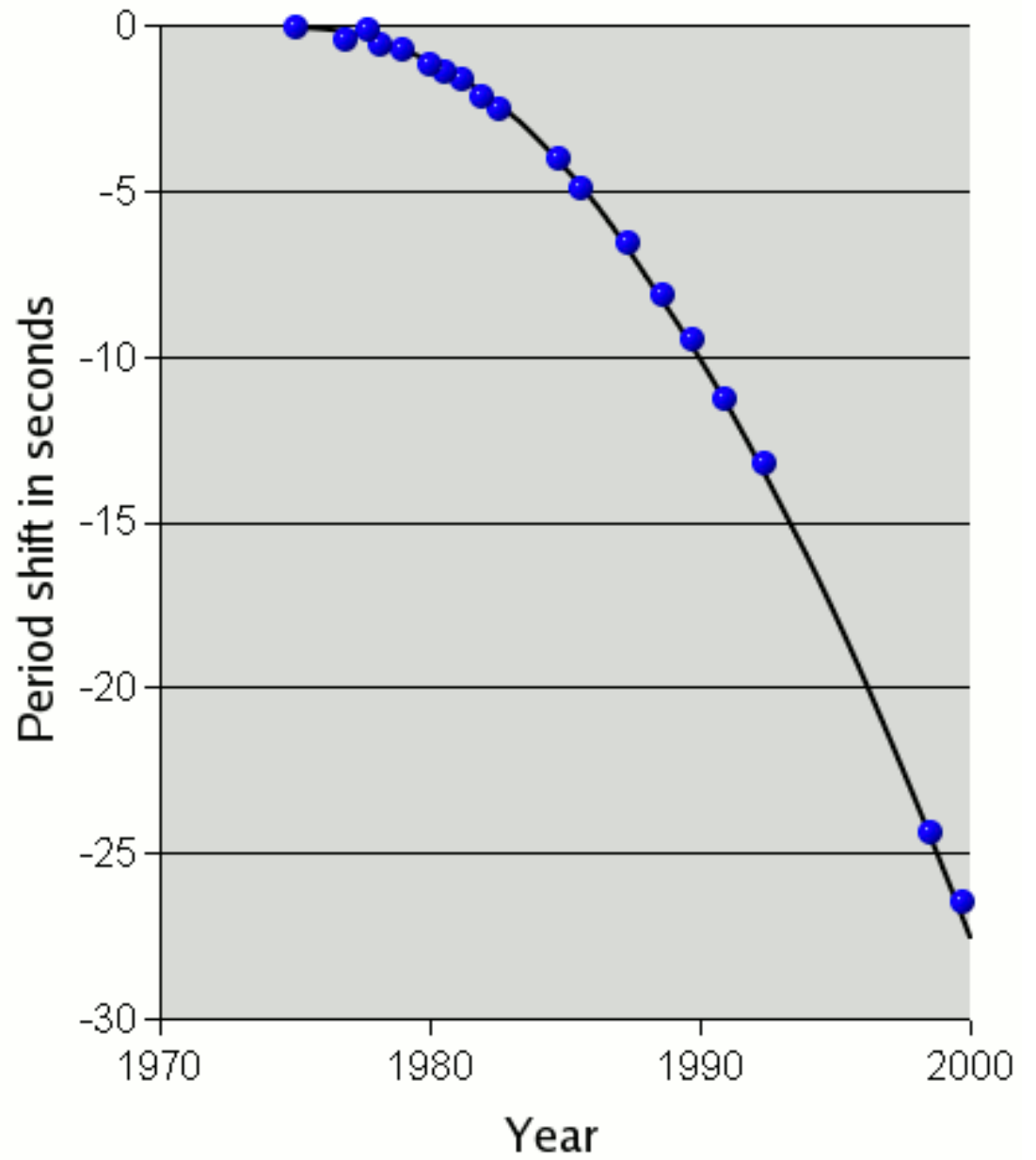


# The binary pulsar system PSR 1913+16





Russell Hulse and Joe Taylor



## 2) Gravitational light deflection / lensing

Radial geodesic for a photon

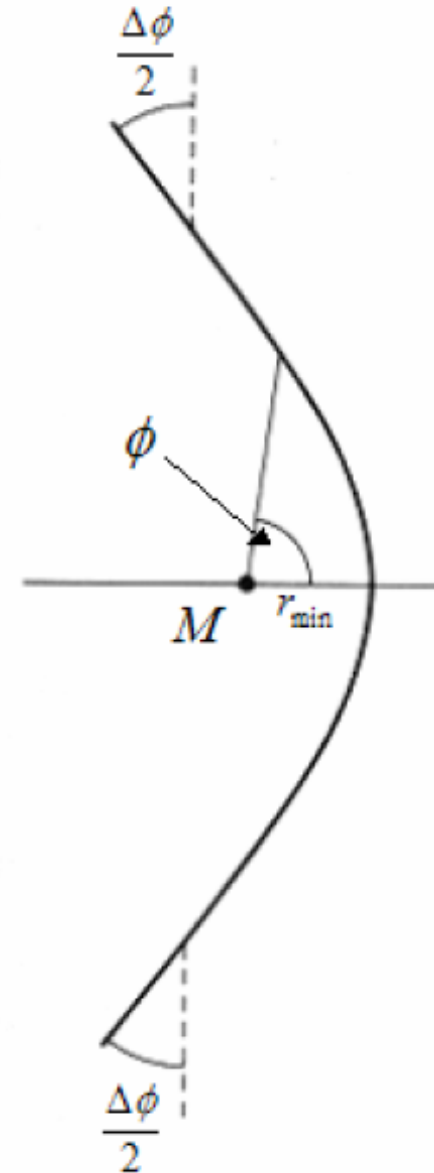
$$\left(\frac{dr}{d\lambda}\right)^2 = k^2 - \frac{h^2}{r^2} + \frac{2Mh^2}{r^3}$$

or 
$$\frac{d^2u}{d\phi^2} + u = 3Mu^2$$

Solution reduces to 
$$u = -\frac{\Delta\phi}{2r_{\min}} + \frac{2M}{r_{\min}^2}$$

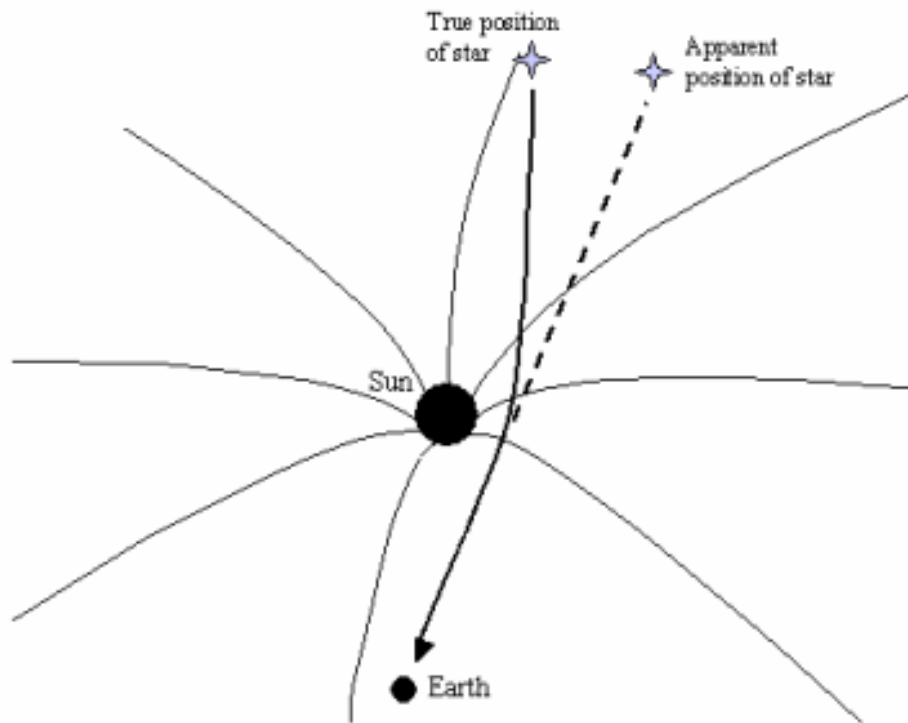
So that

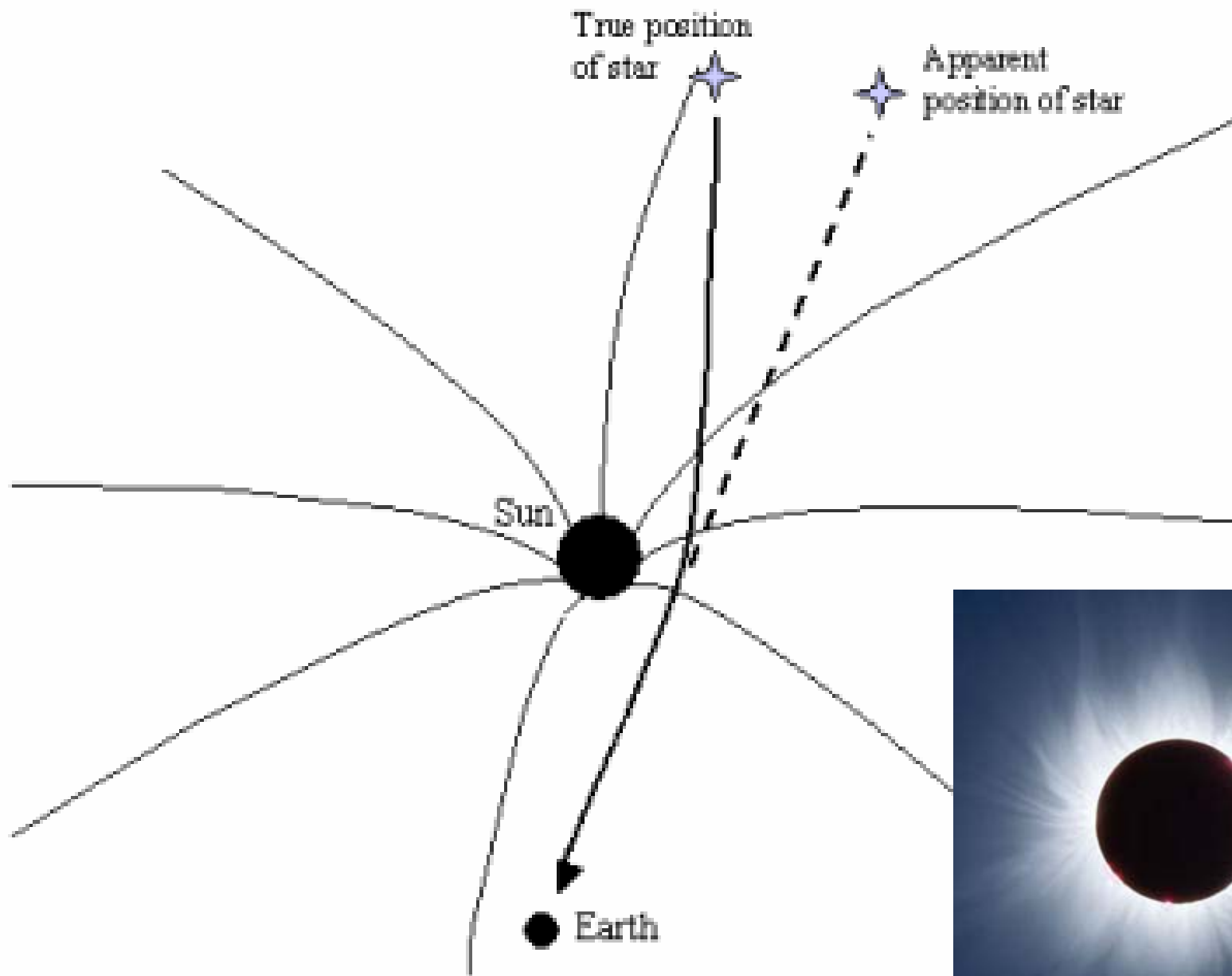
$$\Delta\phi = \frac{4M}{r_{\min}} \equiv \frac{4GM}{c^2 r_{\min}}$$

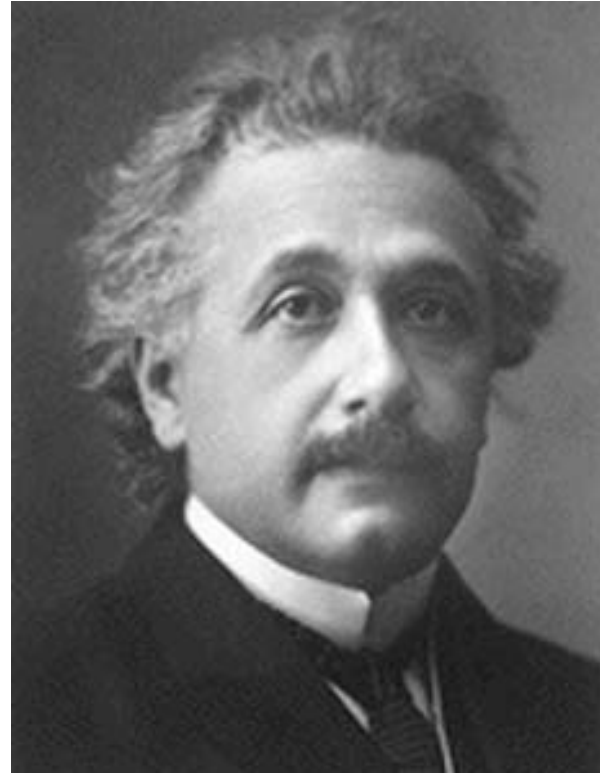


This is exactly twice the deflection angle predicted by a Newtonian treatment. If we take  $r_{\min}$  to be the radius of the Sun (which would correspond to a light ray grazing the limb of the Sun from a background star observed during a total solar eclipse) then we find that

$$\Delta\phi = \frac{4 \times 1.5 \times 10^3}{6.95 \times 10^8} = 8.62 \times 10^{-6} \text{ radians} = 1.77 \text{ arcsec}$$







1919 expedition, led by Arthur Eddington, to observe total solar eclipse, and measure light deflection.

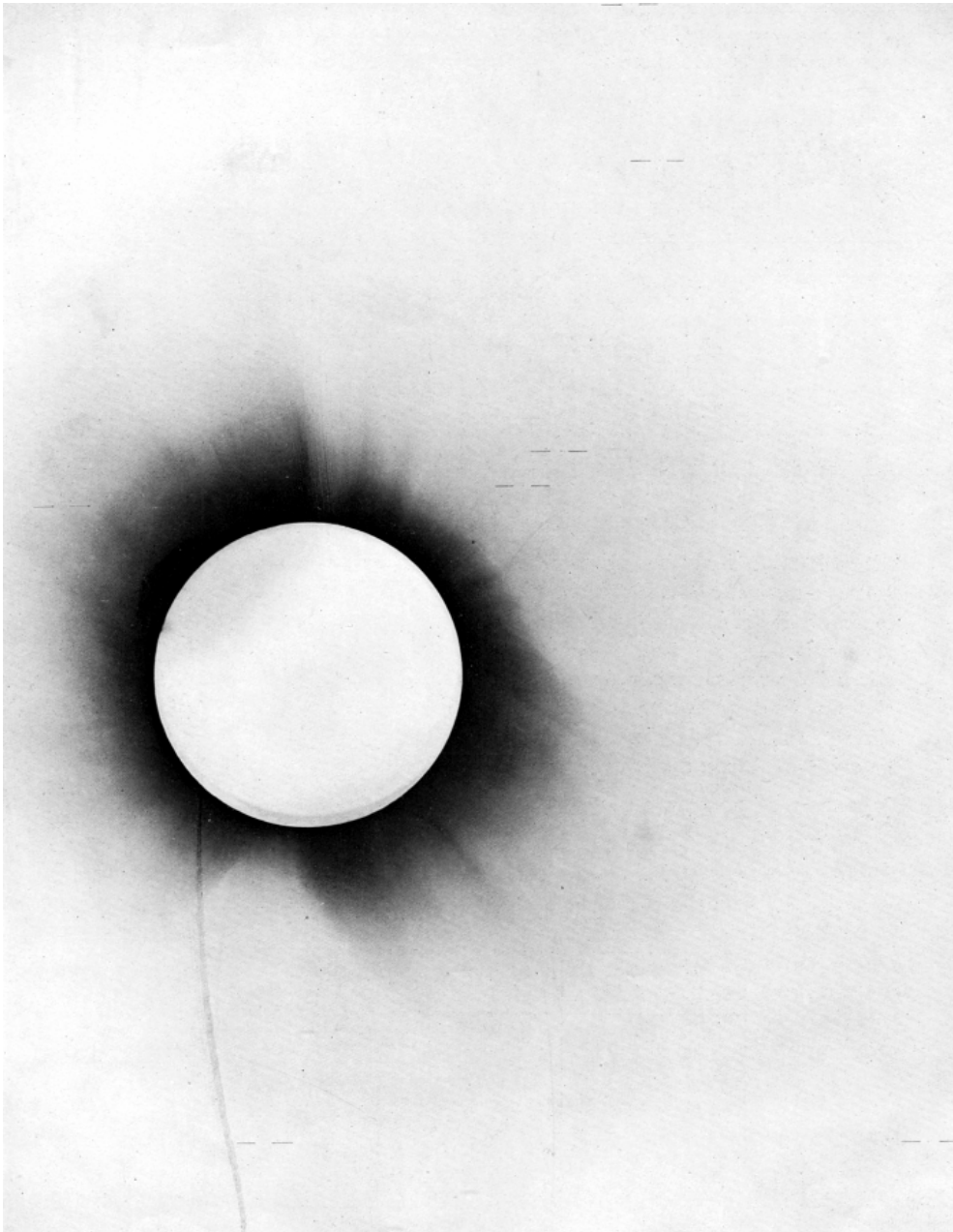
**GR passed the test!**







*“He was one of the finest people I have ever known....but he didn’t really understand physics. During the eclipse...he stayed up all night to see if it would confirm the bending of light by the gravitational field. If he had really understood general relativity, he would have gone to bed the way I did.”*



## **ECLIPSE SHOWED GRAVITY VARIATION**

**Diversion of Light Rays Ac-  
cepted as Affecting New-  
ton's Principles.**

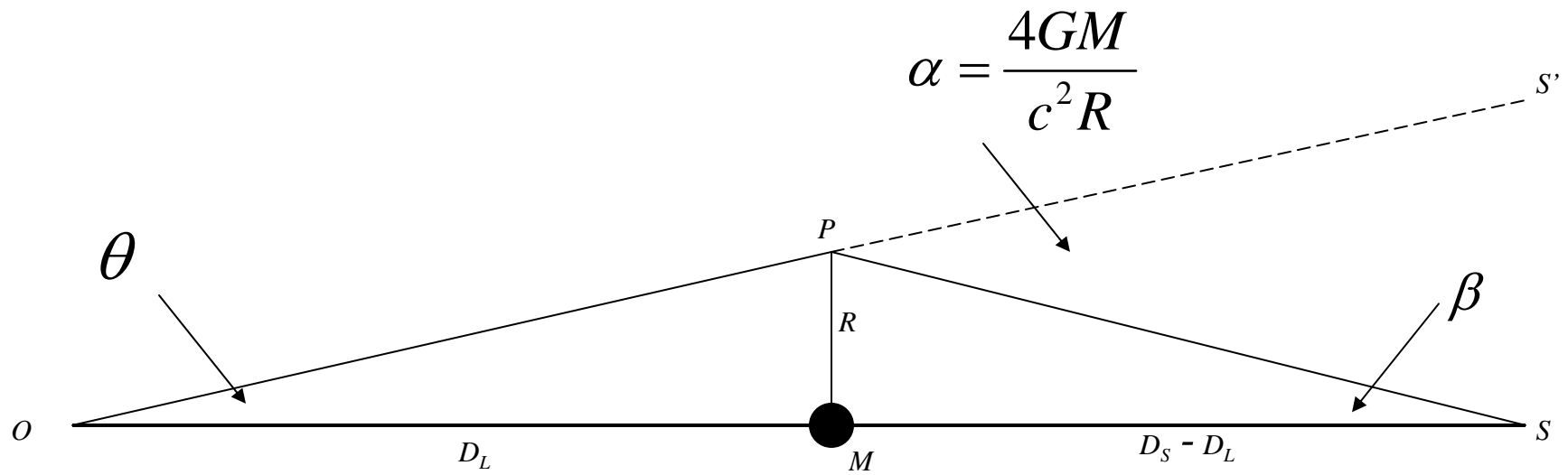
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**HAILED AS EPOCHMAKING**

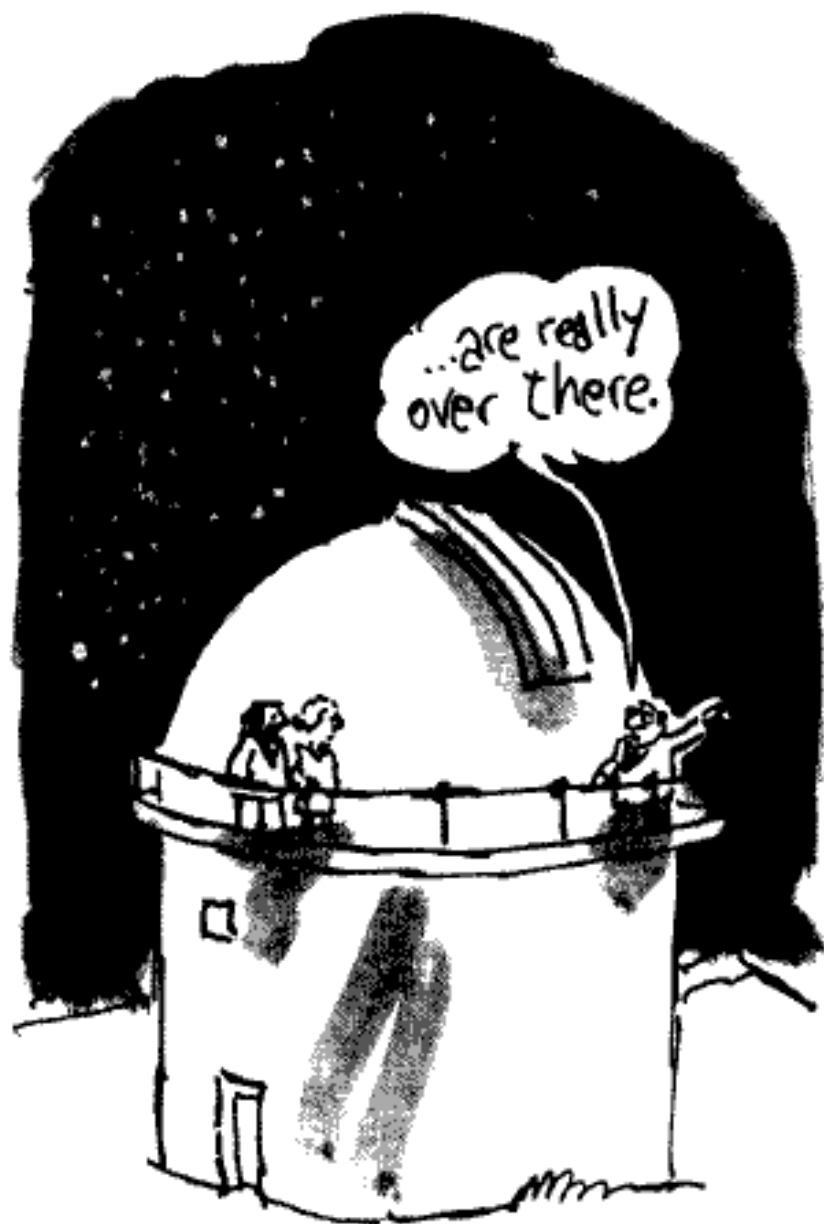
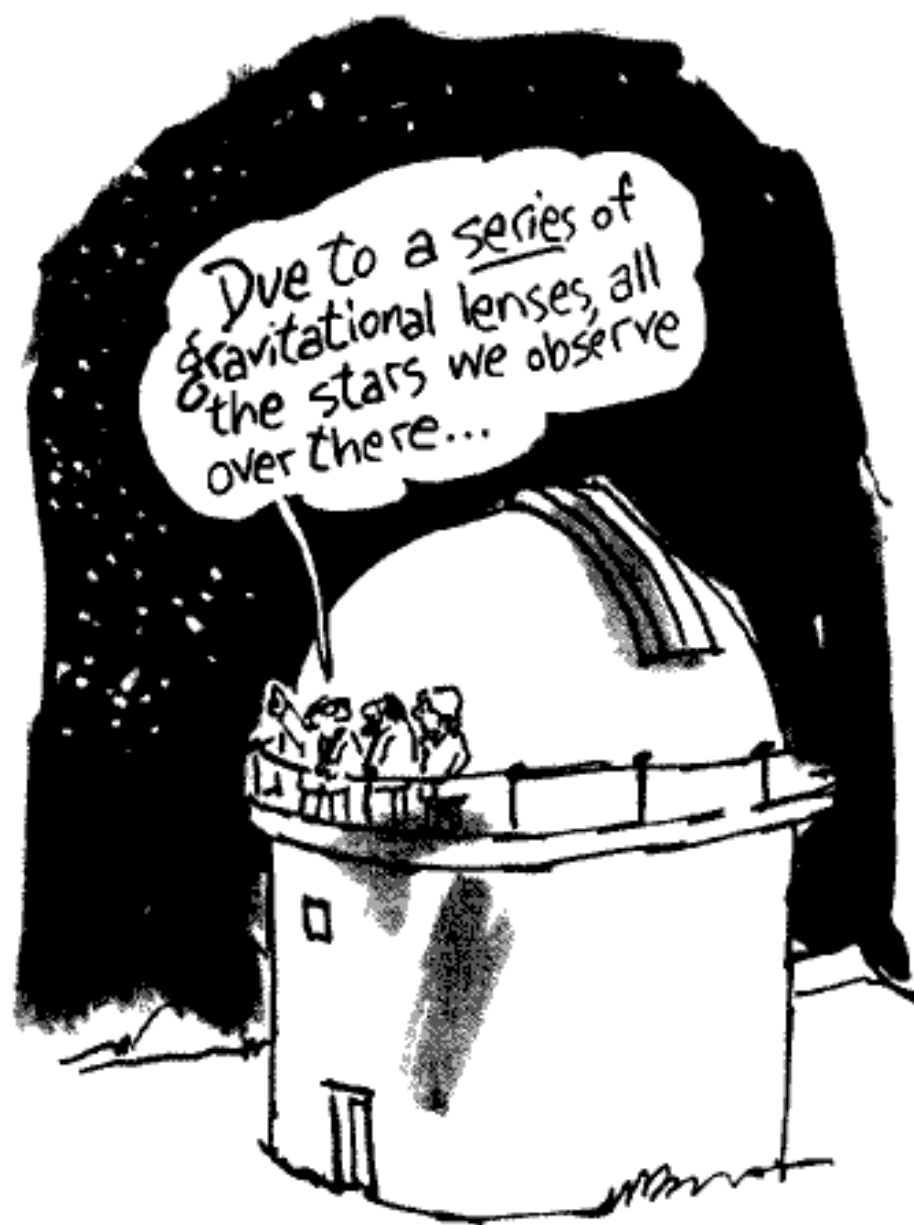
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**British Scientist Calls the Discov-  
ery One of the Greatest of  
Human Achievements.**

New York Times  
Nov 9<sup>th</sup> 1919

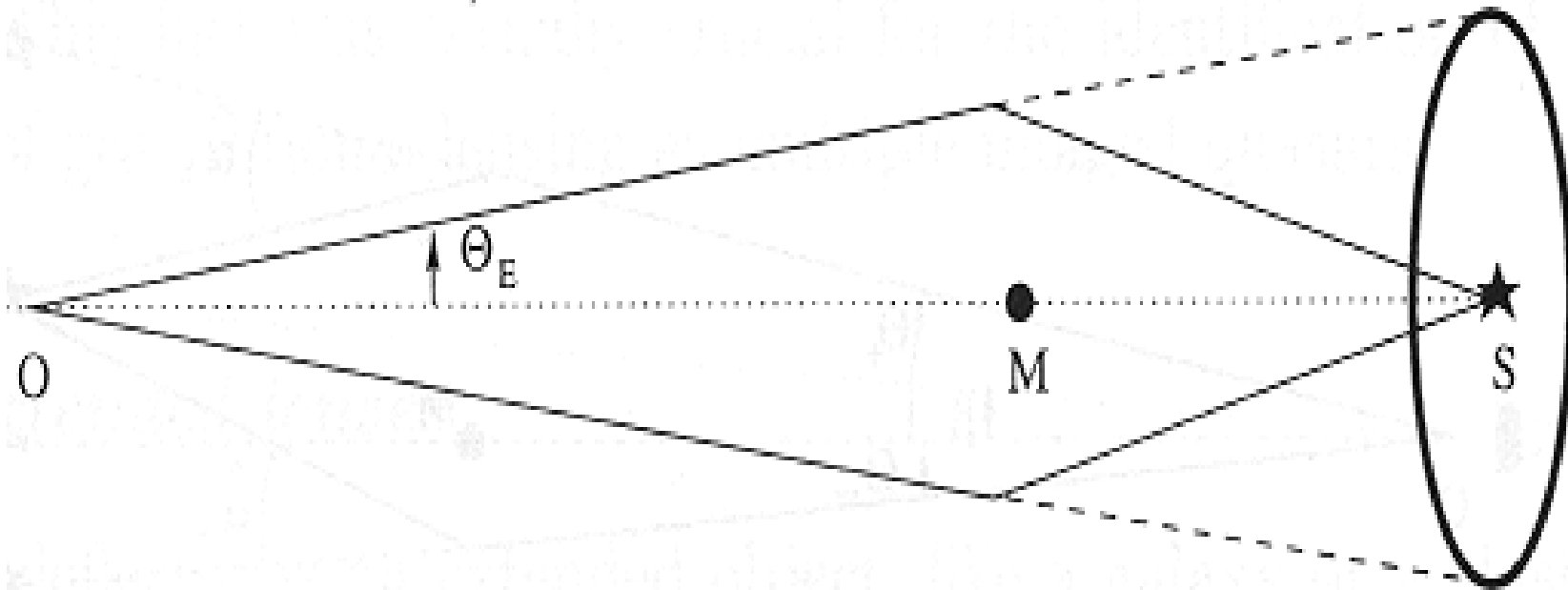


We can use (fairly) simple mathematics to work out the positions of the multiple images of a lensed source

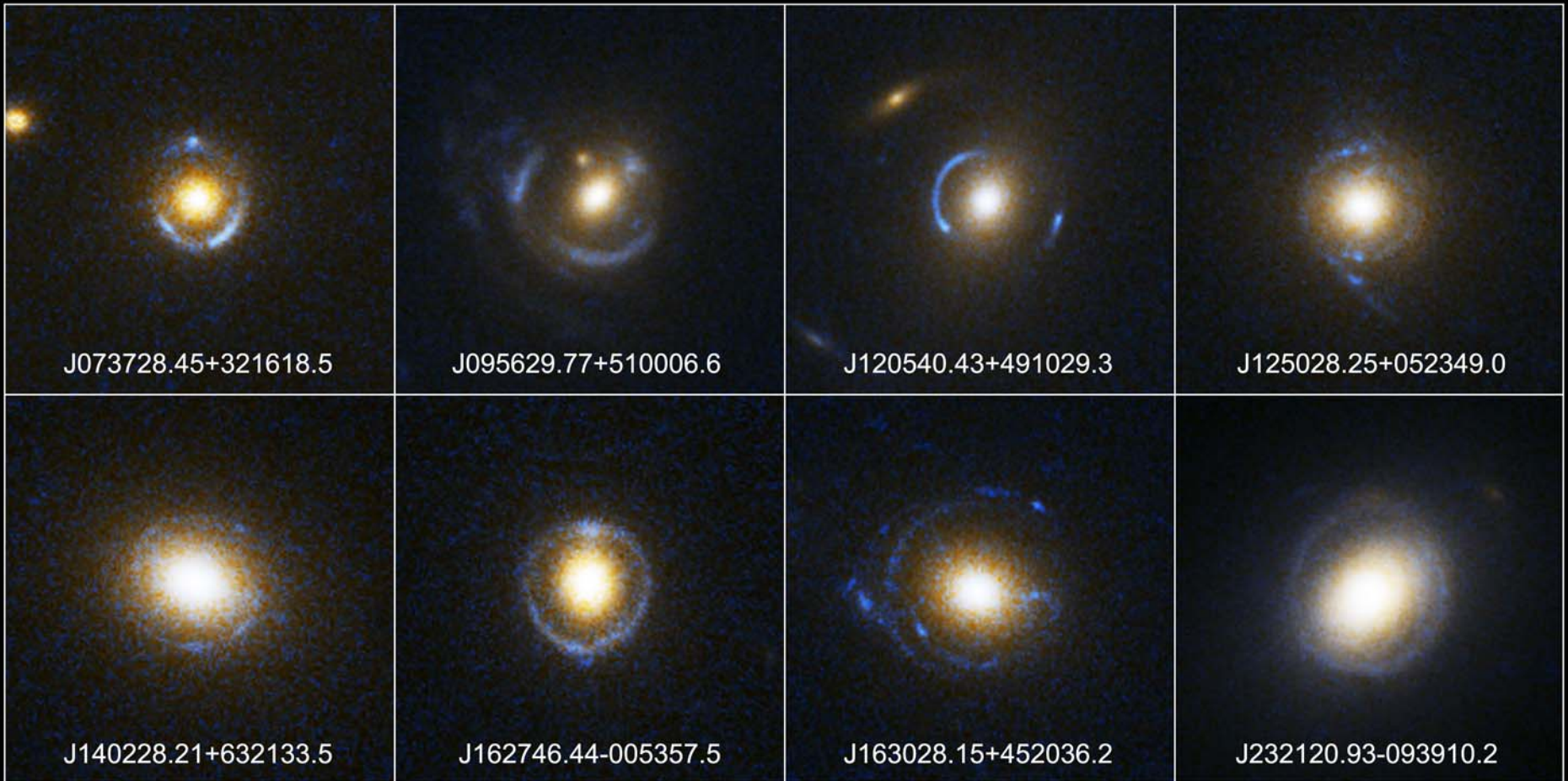


J. K. ...

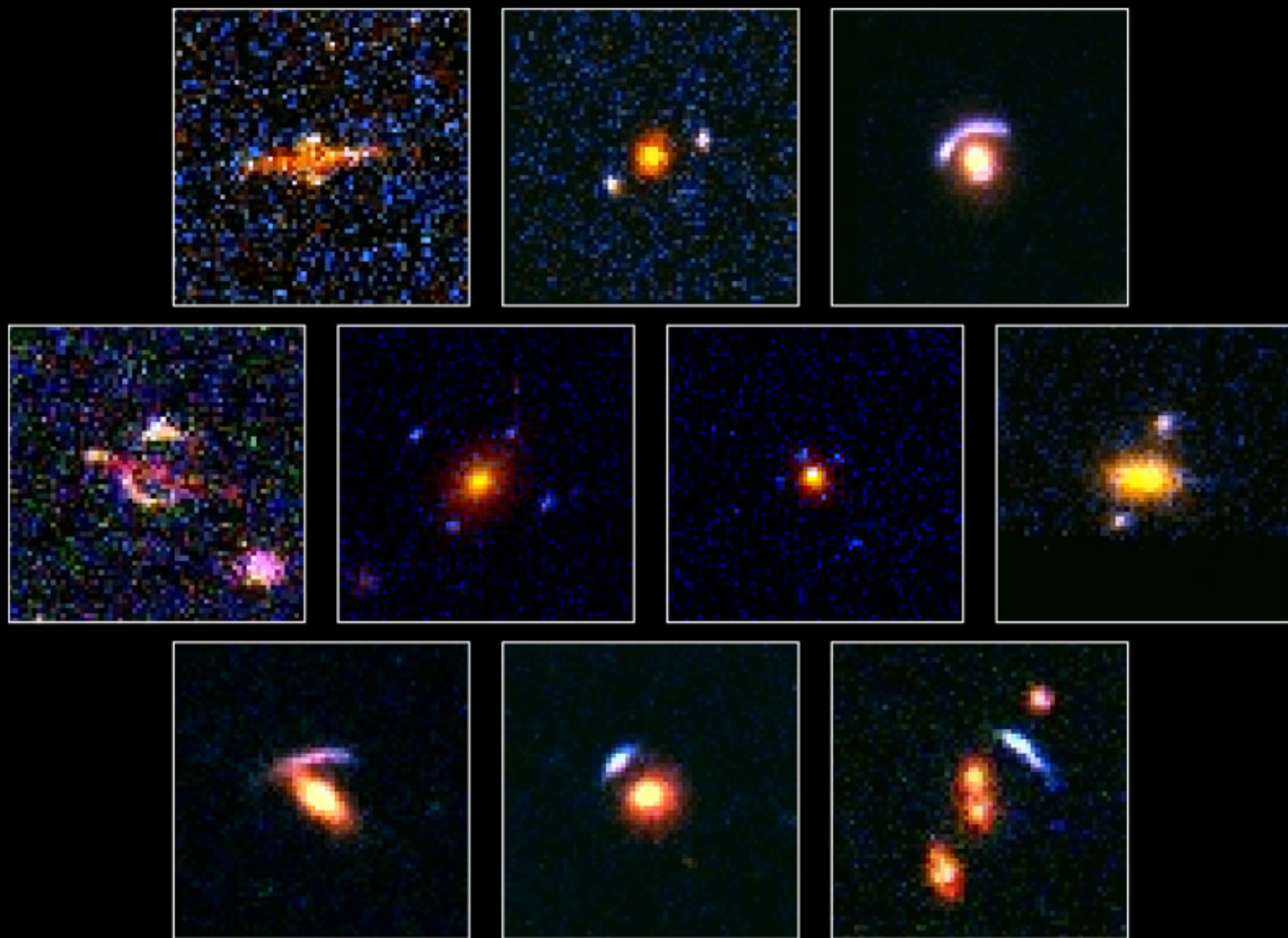
If the observer, lens and source are exactly aligned, images form an **Einstein Ring**



Chwolson (1924), Einstein (1936), Zwicky (1937)



**Einstein Ring Gravitational Lenses**  
*Hubble Space Telescope • Advanced Camera for Surveys*



**Gallery of Gravitational Lenses**

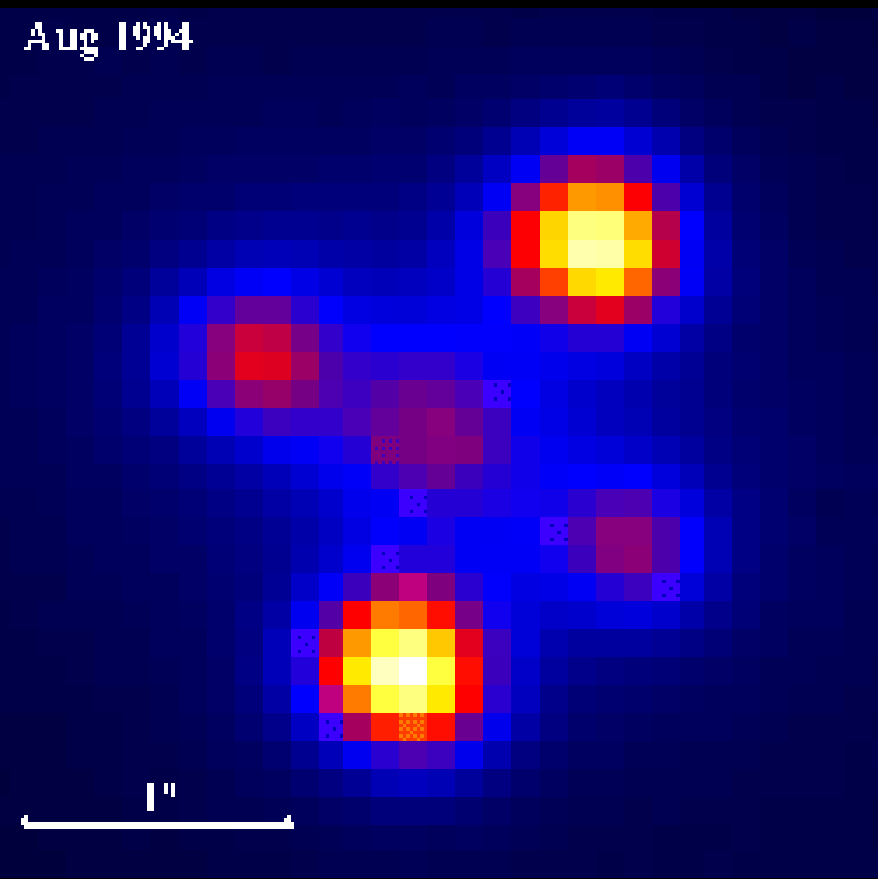
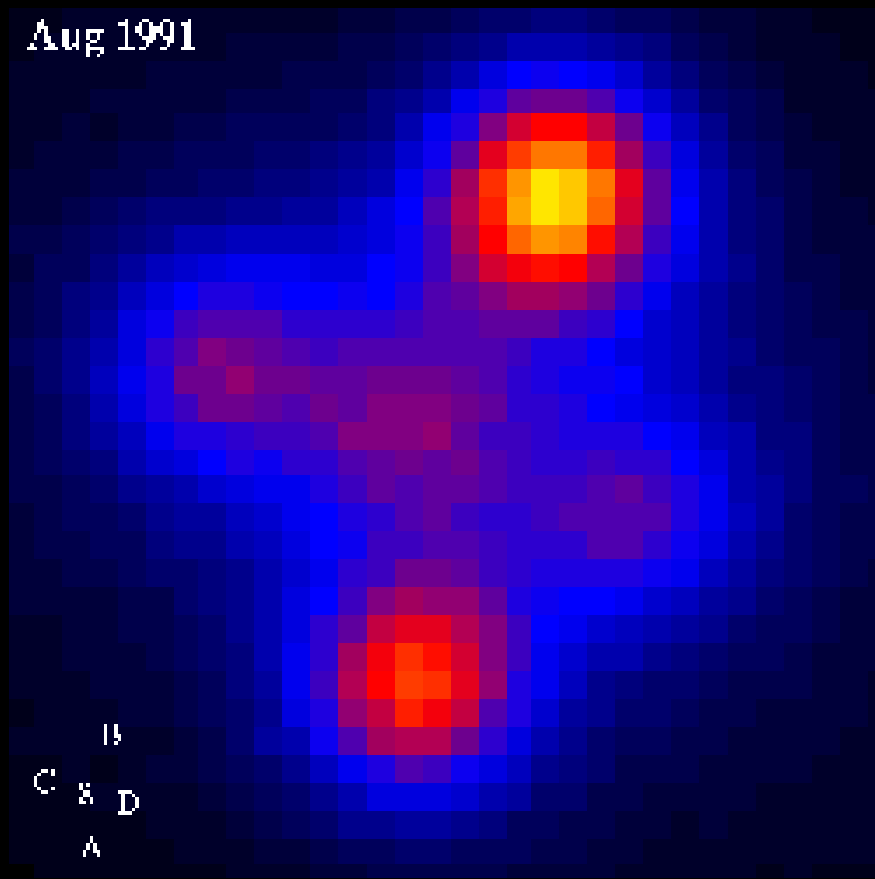
PRC99-18 • STScI OPO • K. Ratnatunga (Carnegie Mellon University) and NASA

**HST • WFPC2**





"Einstein Cross" 2237+0305:  
Foreground galaxy lensing distant quasar



Gravitational microlensing of the Einstein Cross:  
The individual images of the quasar change in brightness as they are lensed by individual stars in the lensing galaxy

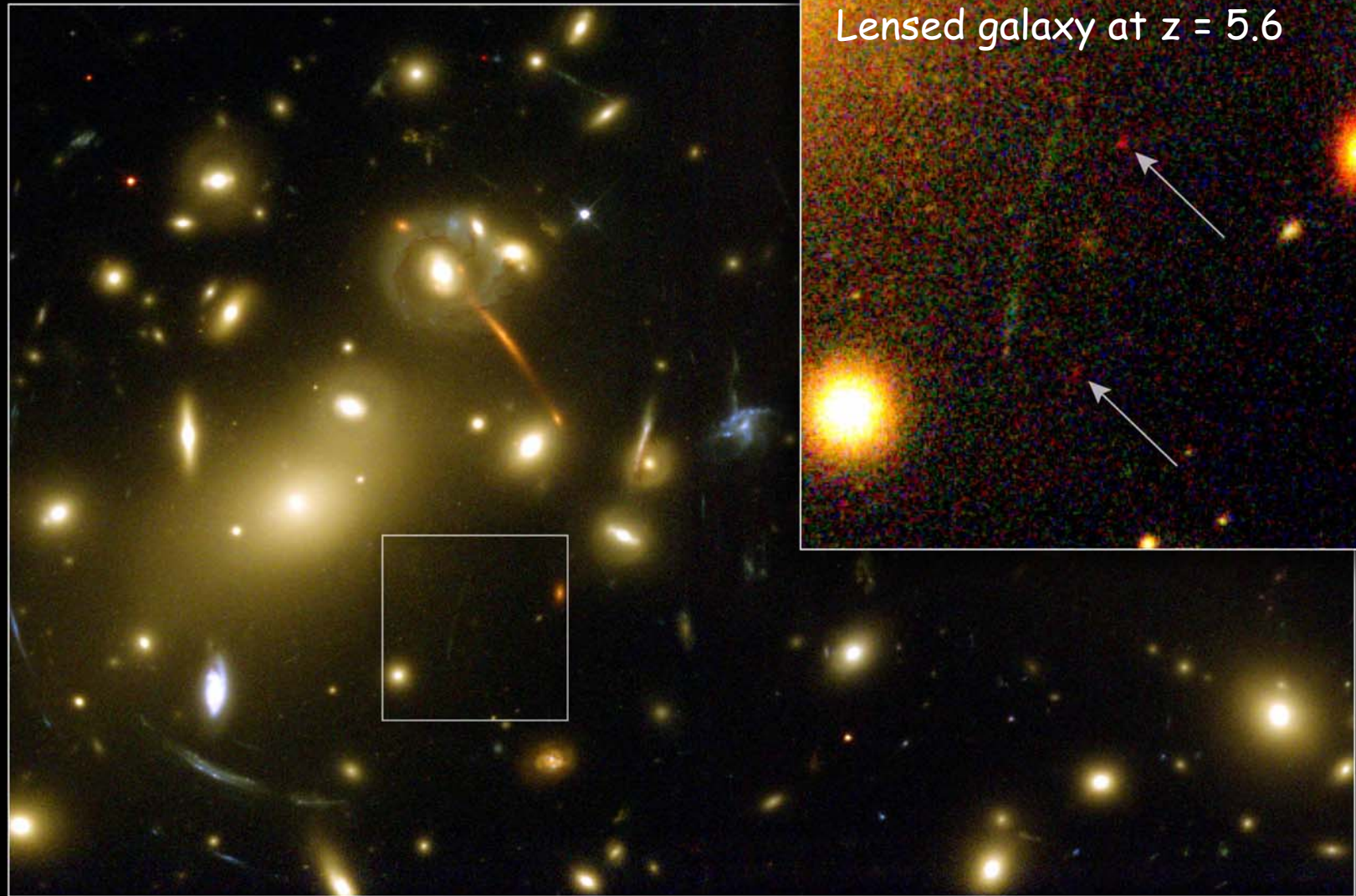
Lewis & Irwin (1994)



## Gravitational Lens in Abell 2218

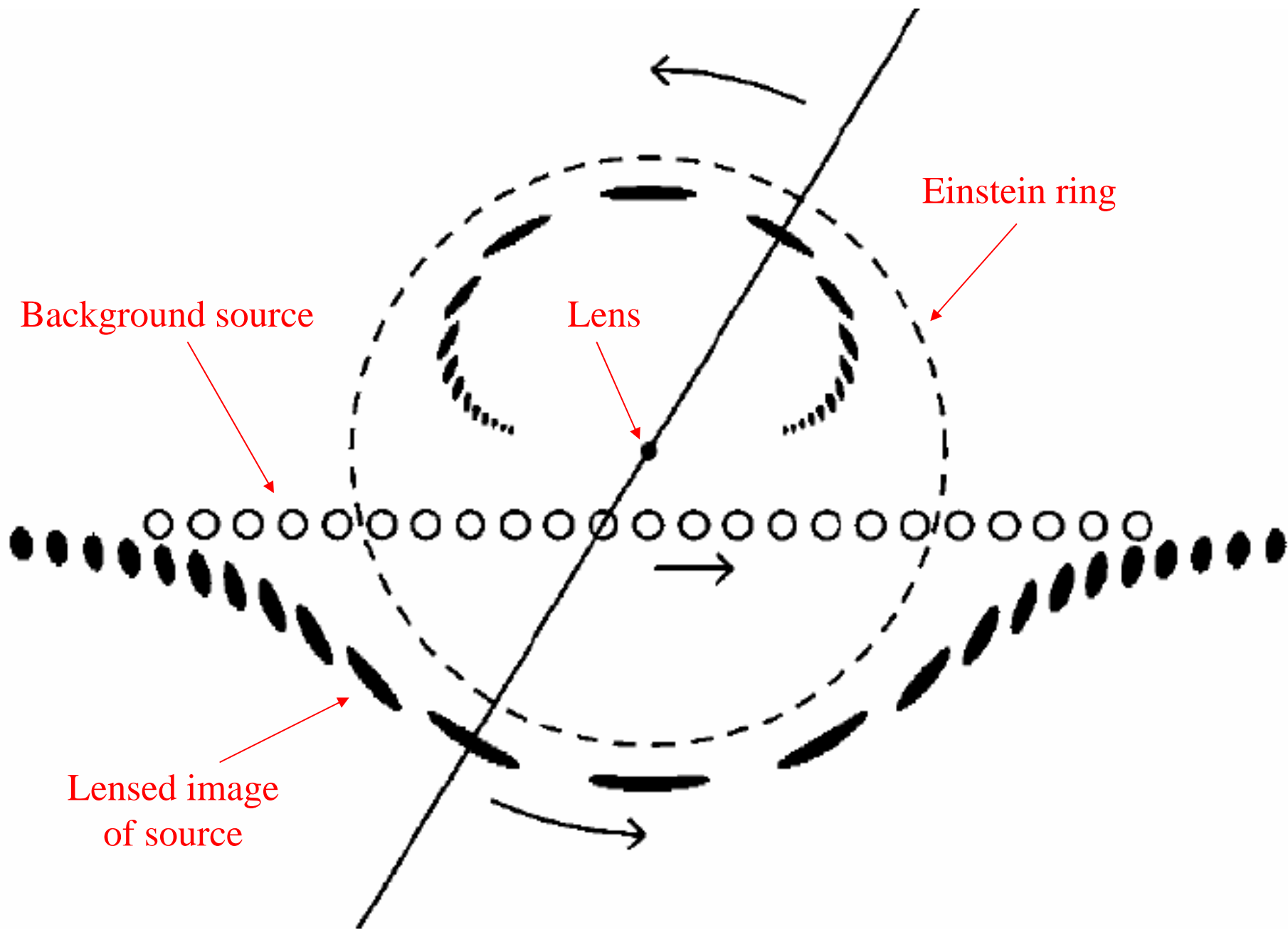
HST · WFPC2

PF95-14 · ST ScI OPO · April 5, 1995 · W. Couch (UNSW), NASA



**Distant Object Gravitationally Lensed by Galaxy Cluster Abell 2218**  
**Hubble Space Telescope • WFPC2**

NASA, ESA, R. Ellis (Caltech) and J.-P. Kneib (Observatoire Midi-Pyrenees) • STScI-PRC01-32



Background source

Lens

Einstein ring

Lensed image  
of source

Large Magellanic Cloud

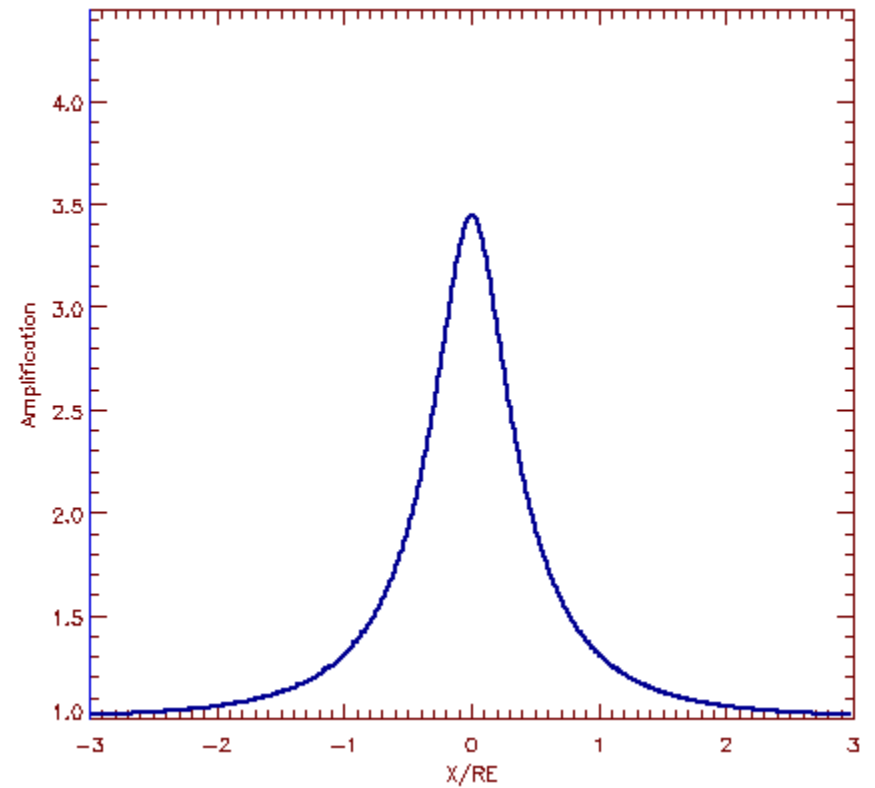
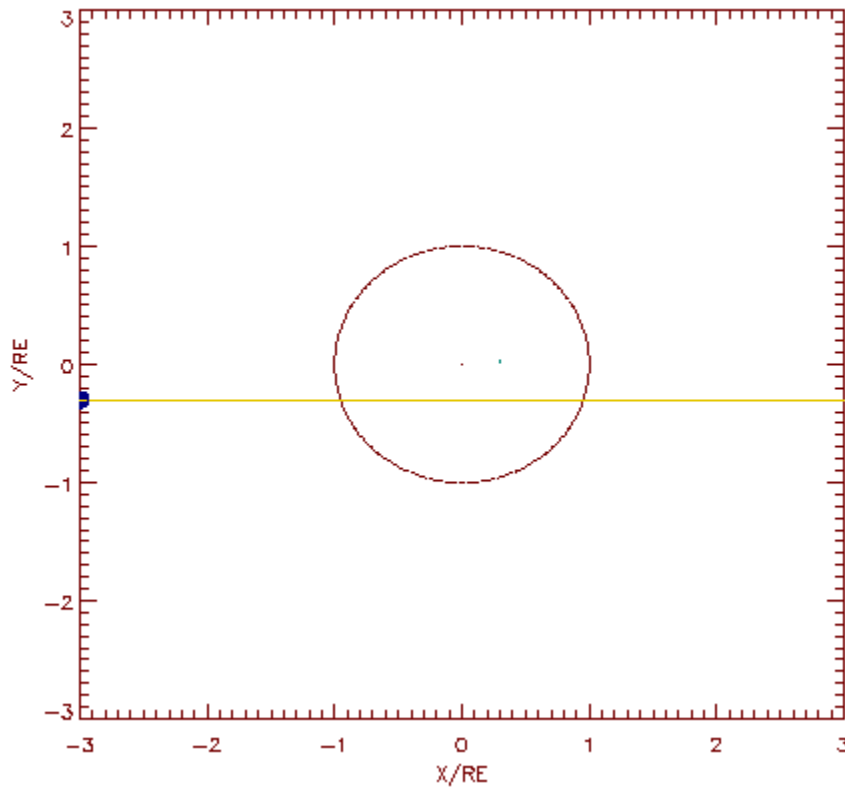
MACHO's gravity focuses  
the light of the background  
star on the Earth

A MACHO

So the background star  
briefly appears brighter

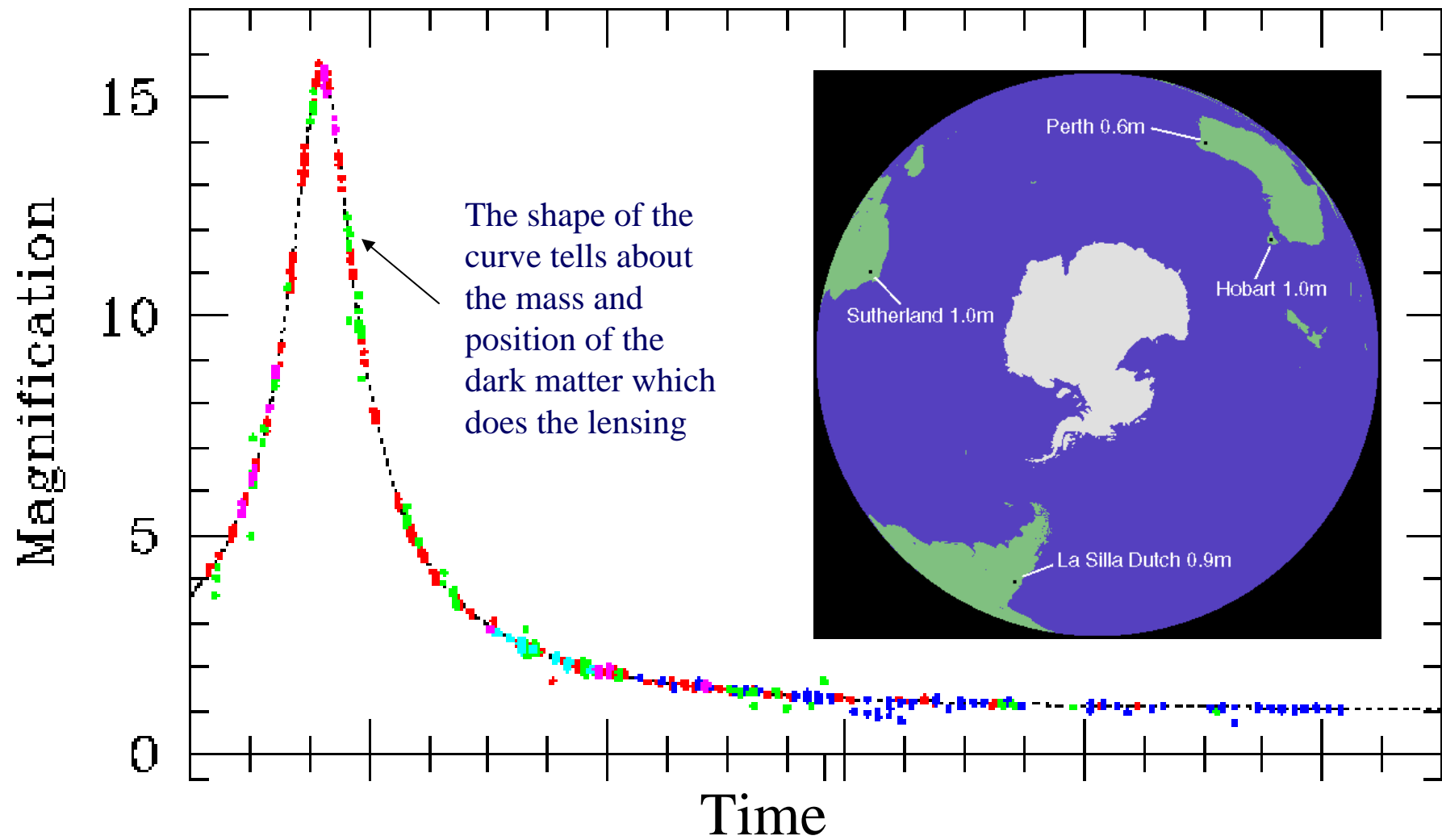


Courtesy: B.S. Gaudi



As the microlens crosses in front of the source, the brightness of the source increases by a significant factor, then decreases again.

# Lightcurve of a microlensing event

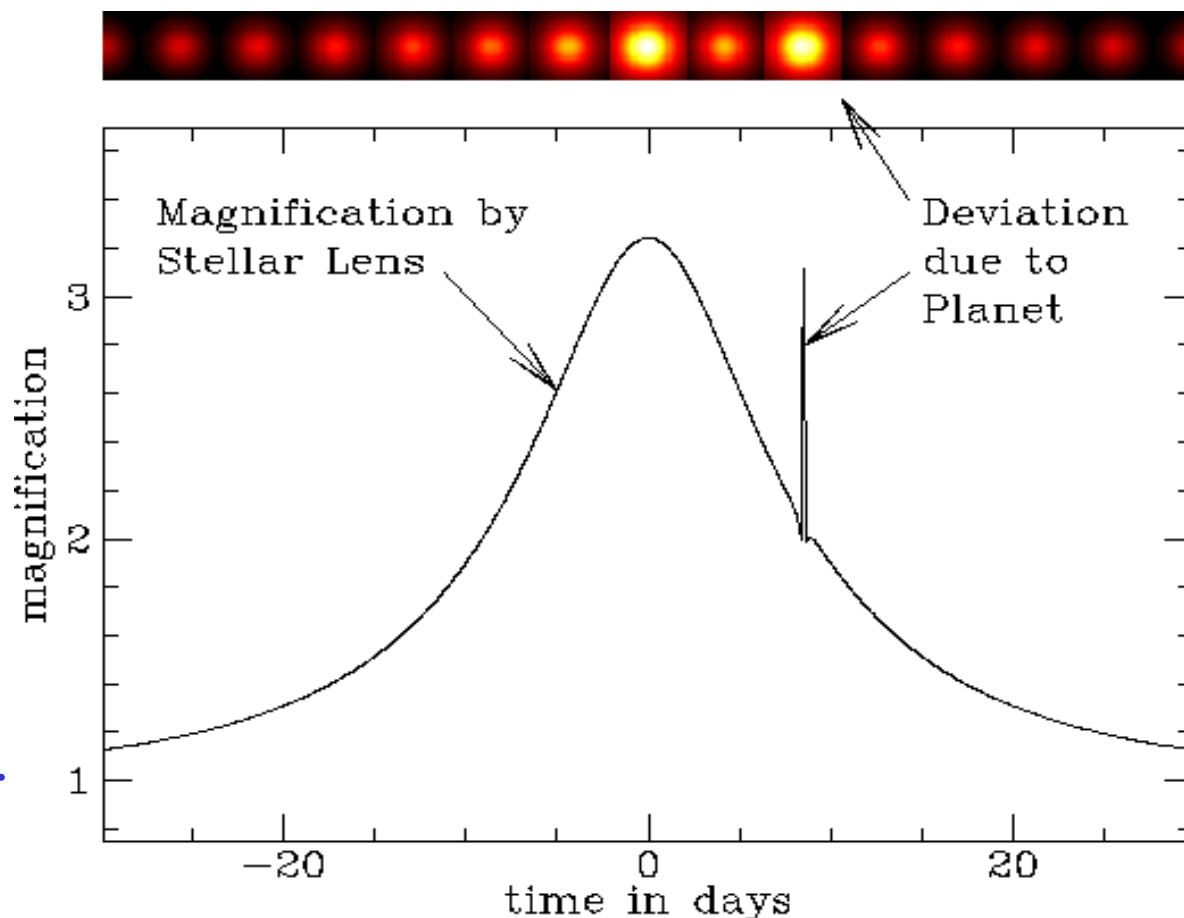




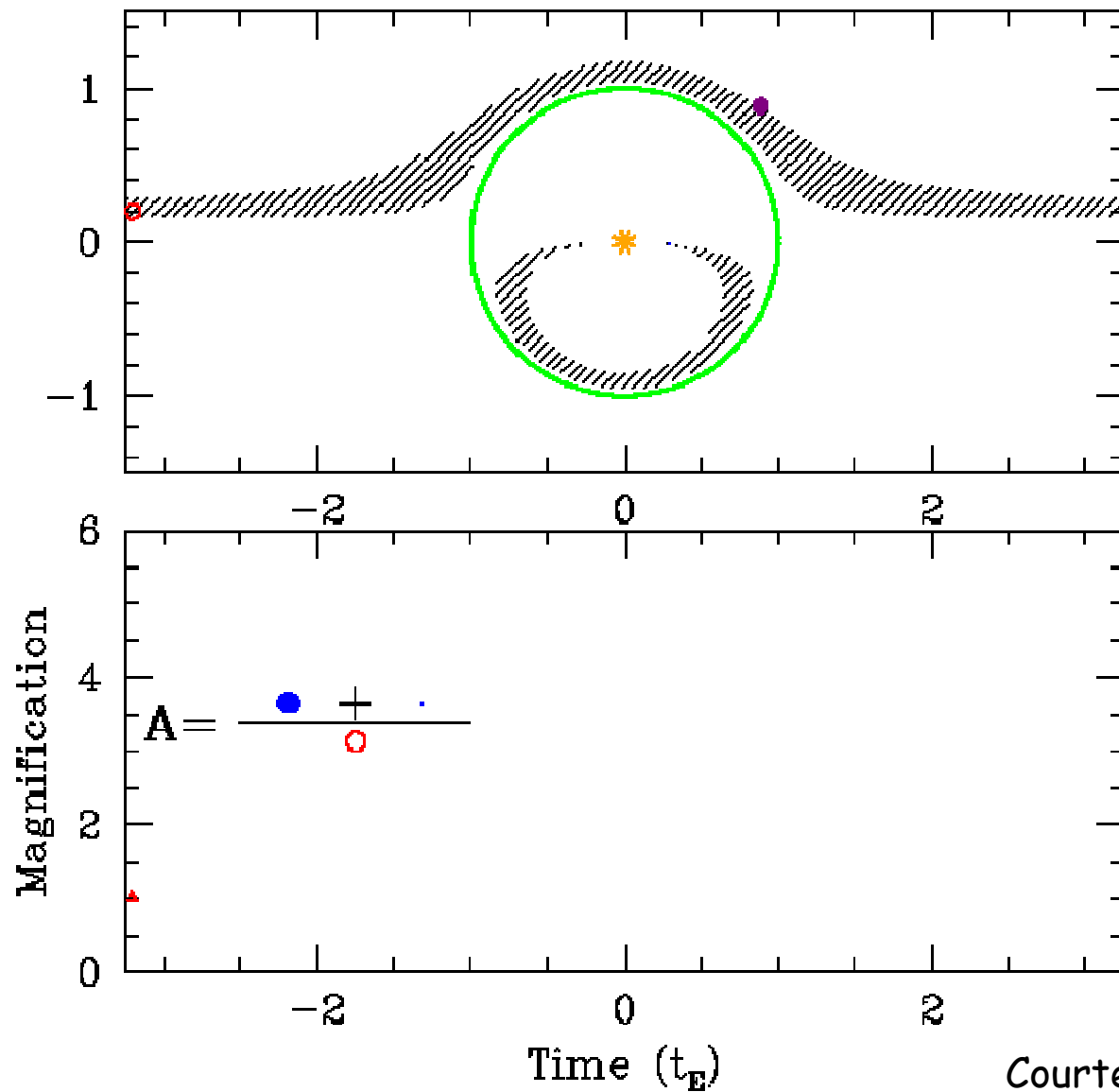
# Detecting planets with microlensing

If the lensing star has a planet which *also* passes exactly between us and the background source, then the light curve will show a **second peak**.

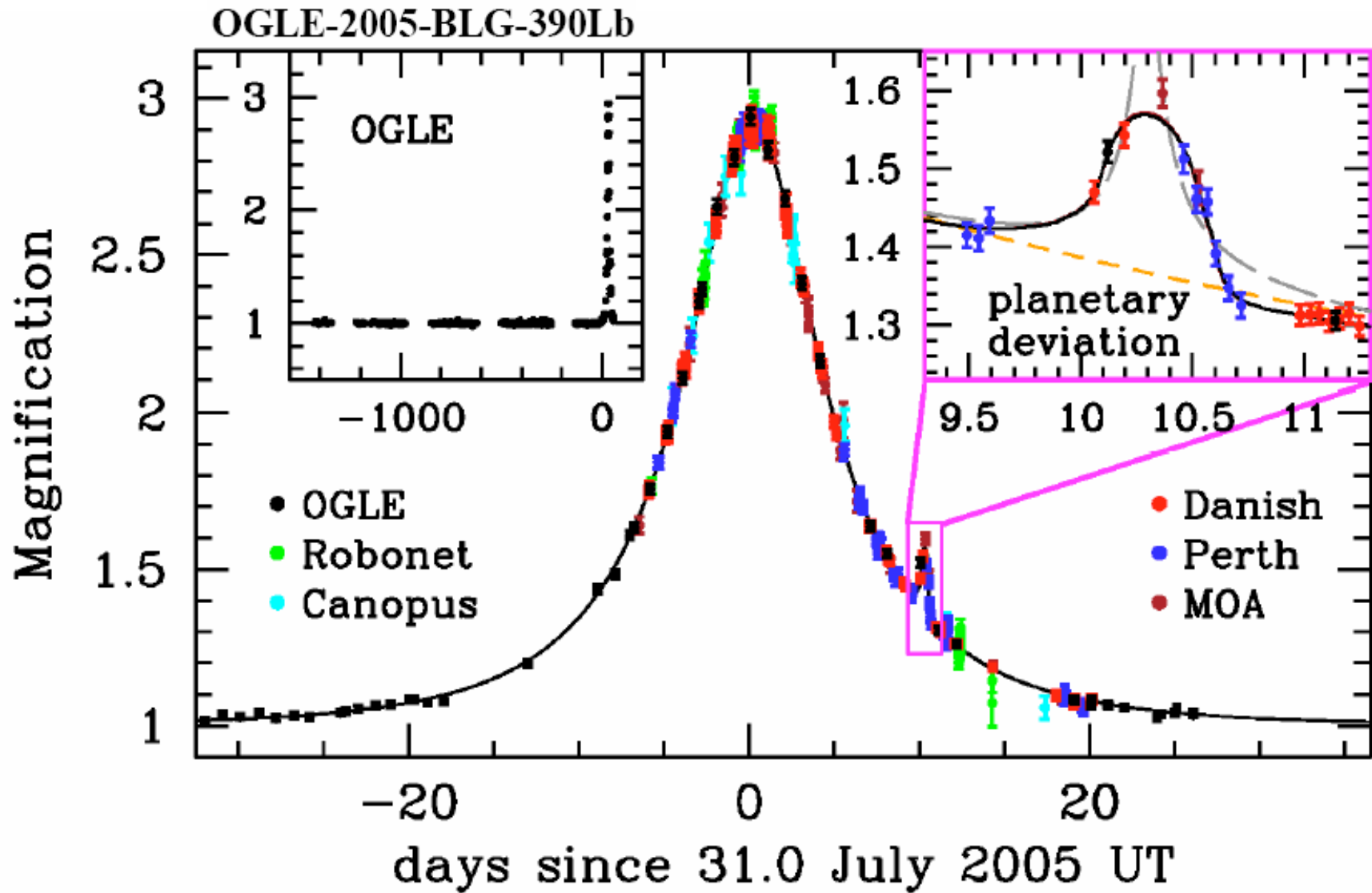
Can in principle detect Earth mass planets!



# Detecting planets with microlensing



Courtesy: B.S. Gaudi

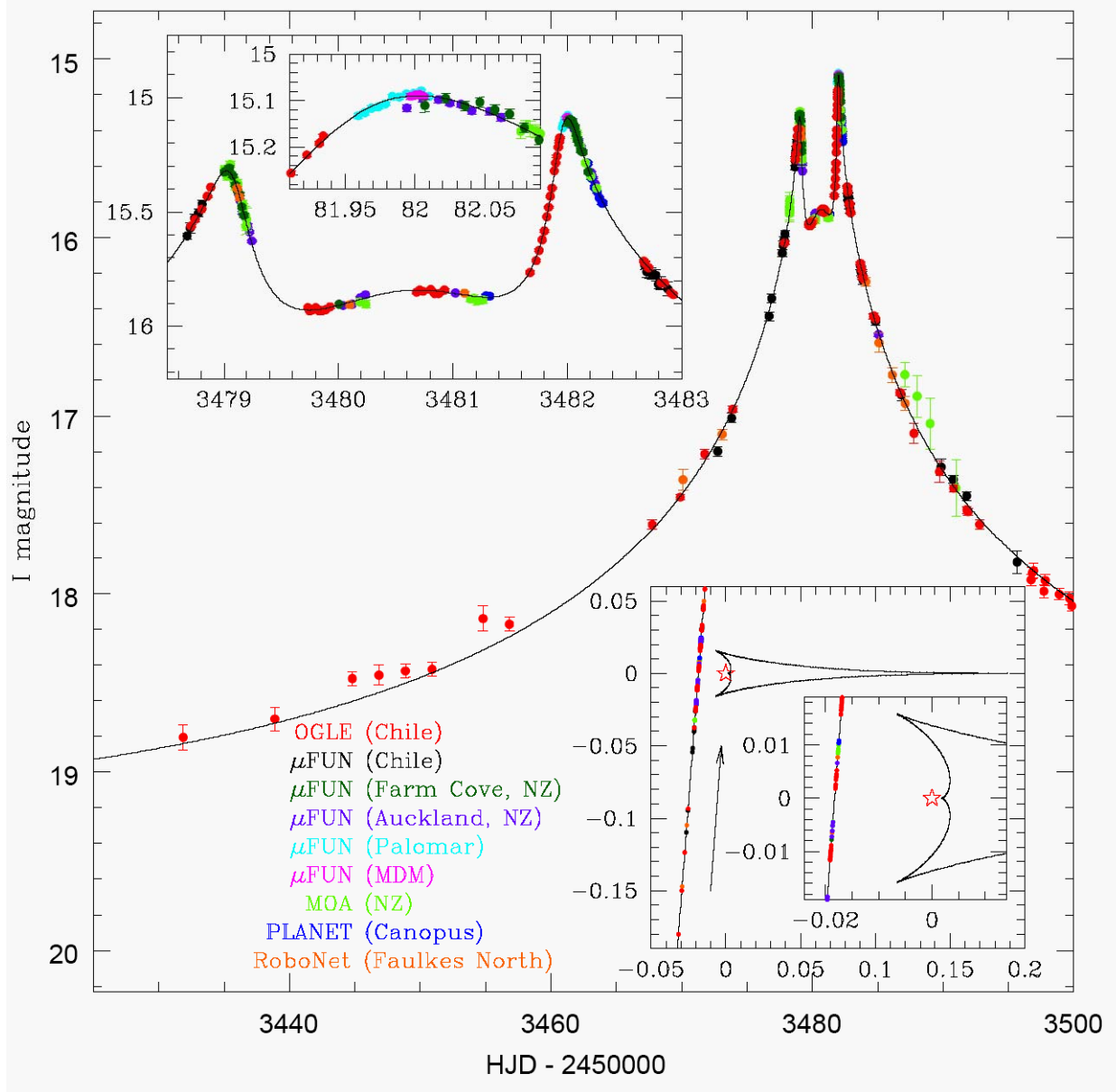


Beaulieu et al. (Jan 2006):

5.5 Earth mass planet,  
2.6 AU from a 0.22 solar mass M-dwarf



# OGLE-2005-BLG-071



Udalski et al.  
(Oct 2006)

~Jovian mass planet

### 3) Gravitational time delay / redshift

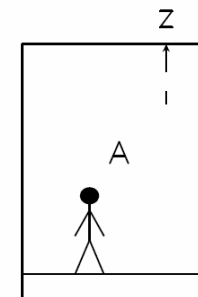
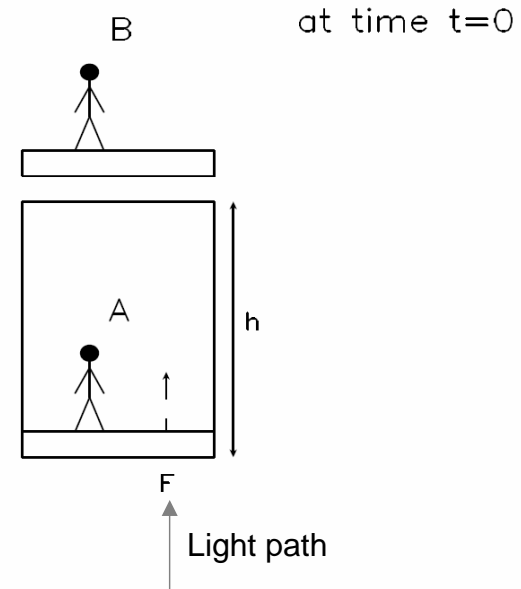


**GLOBAL POSITIONING SYSTEM**  
*Serving the World*

[www.gps.gov](http://www.gps.gov)

*But the Earth is **rotating** – which leads to interesting additional effect: **frame-dragging***

See links on course website.



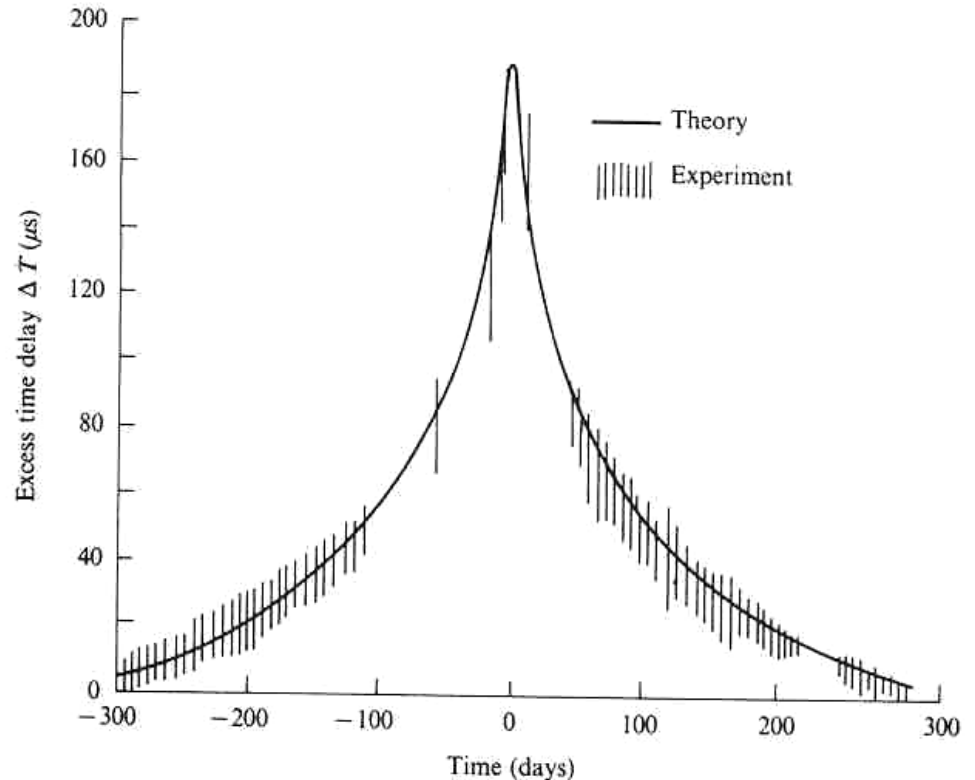
# Gravitational time delay



Irwin Shapiro

Bounced RADAR echoes from Venus in 1968, to measure A.U.

'Shapiro Effect' time delay also a test of General Relativity



1 A.U. = 92,958,329 miles  
= 149,597,870 km

Schwarzschild' solution for the spacetime metric exterior to a point mass,  $M$

$$ds^2 = - \left( 1 - \frac{2M}{r} \right) dt^2 + \frac{dr^2}{\left( 1 - \frac{2M}{r} \right)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$$

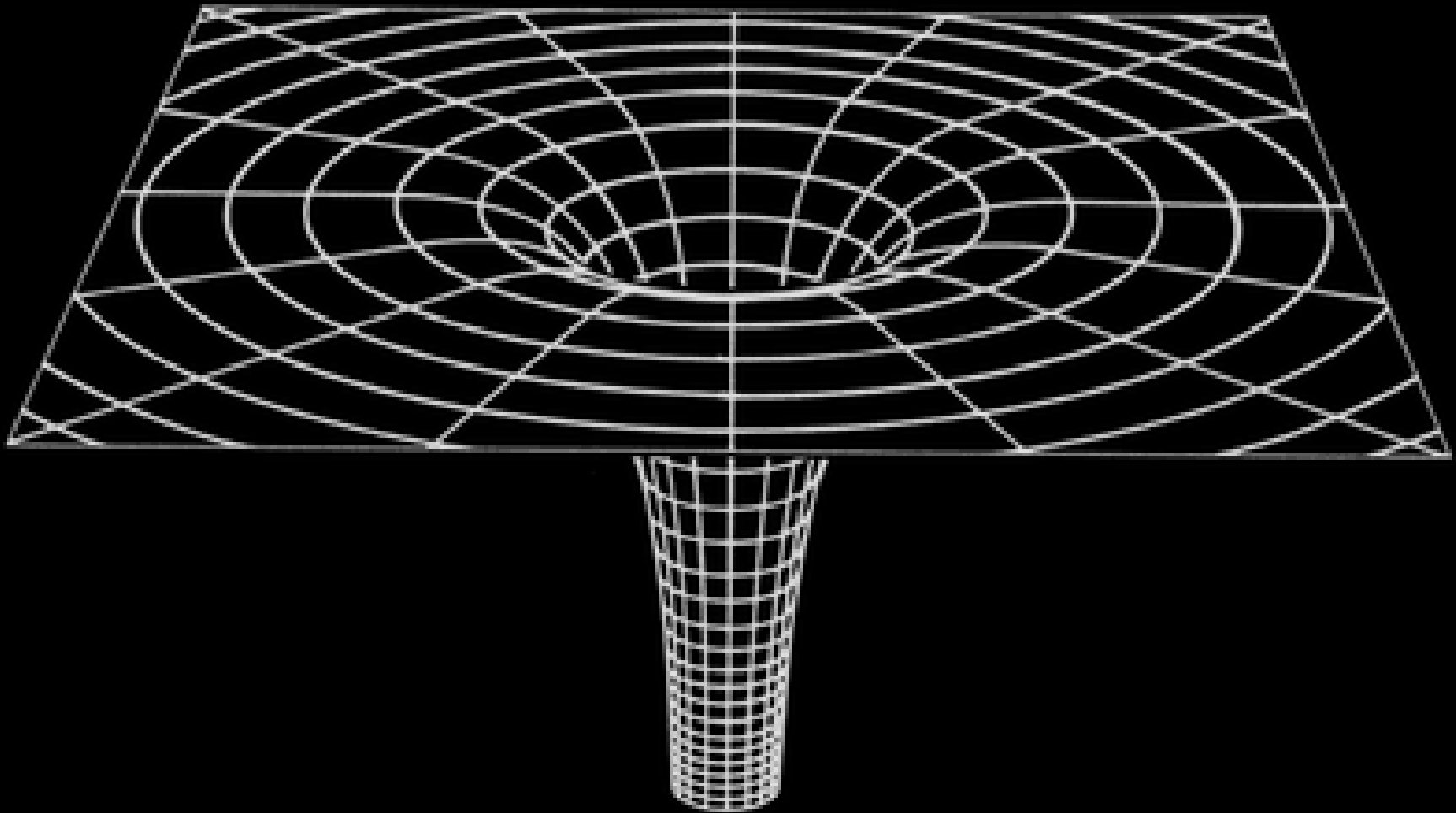
Coordinate singularity at  $r=2M$

Compare with the 'flat' spacetime of Minkowski (SR)

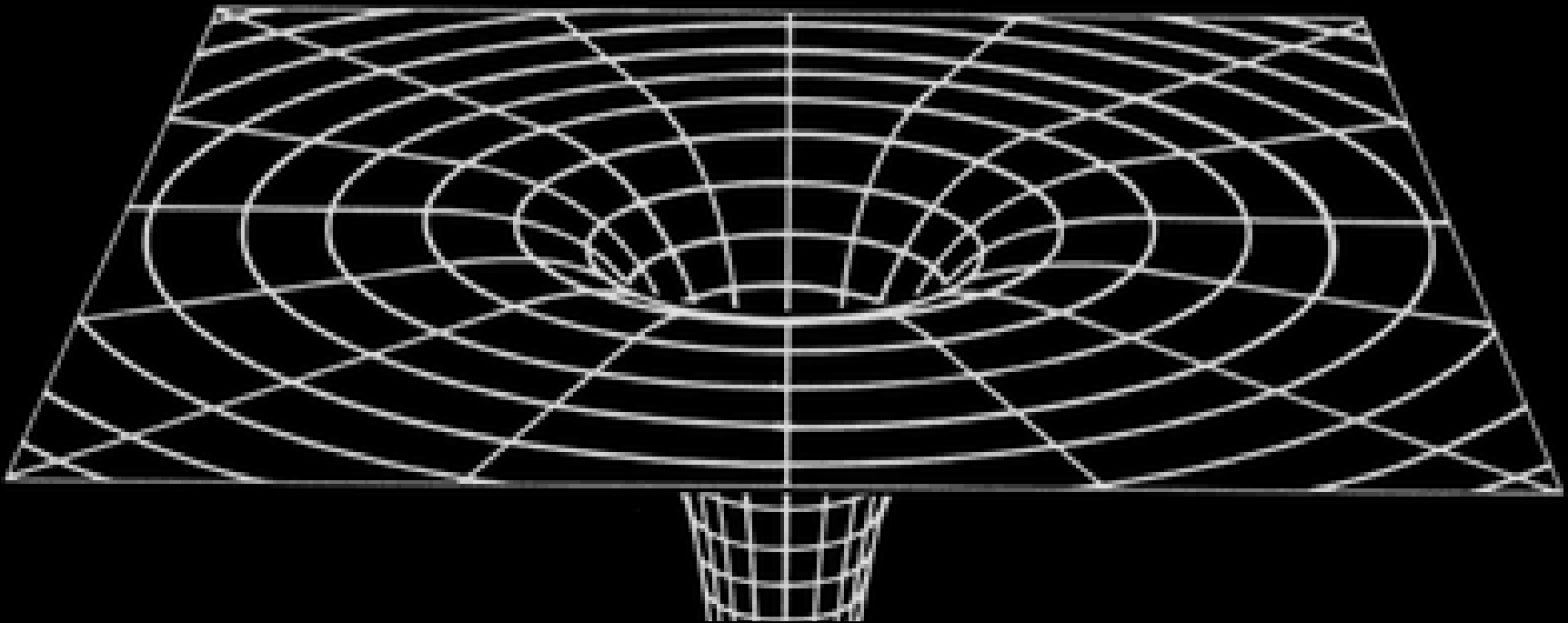
$$ds^2 = -dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$$



A Schwarzschild black hole. What happens at  $r = \frac{2GM}{c^2}$  ?



A Schwarzschild black hole. What happens at  $r = \frac{2GM}{c^2}$  ?



We call  $r = \frac{2GM}{c^2}$  the **Schwarzschild radius**.

The Schwarzschild radius defines an **event horizon** beyond which nothing (not even light) can escape, according to General Relativity.

For the Sun  $r = \frac{2GM}{c^2} = 3\text{km}$

For the Earth  $r = \frac{2GM}{c^2} = 7\text{mm}$

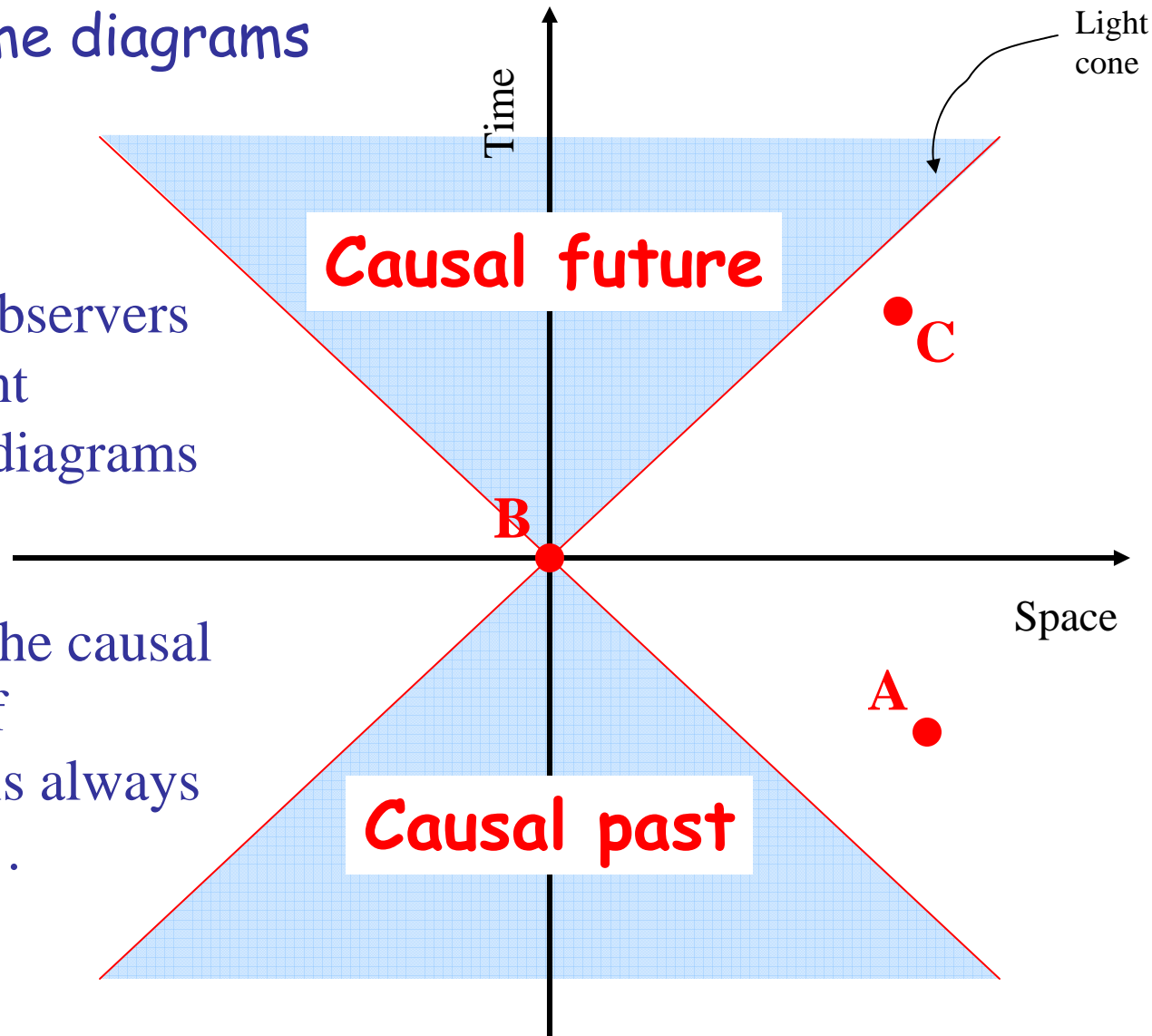
*But what if we could compress the Sun or the Earth down to be smaller than the Schwarzschild radius?...*

# Making Sense of Einstein's Universe

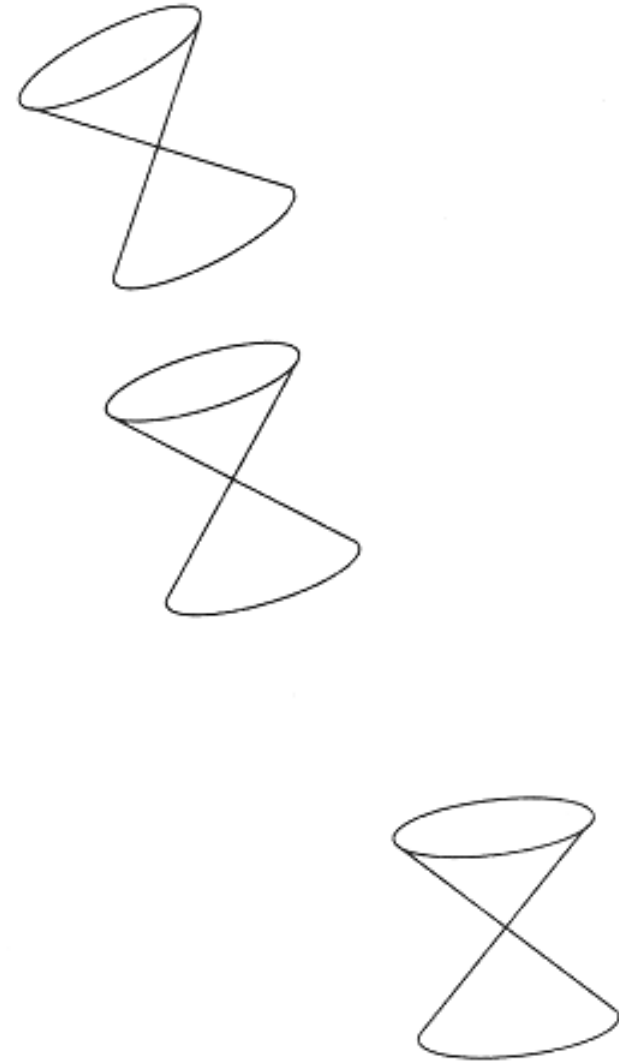
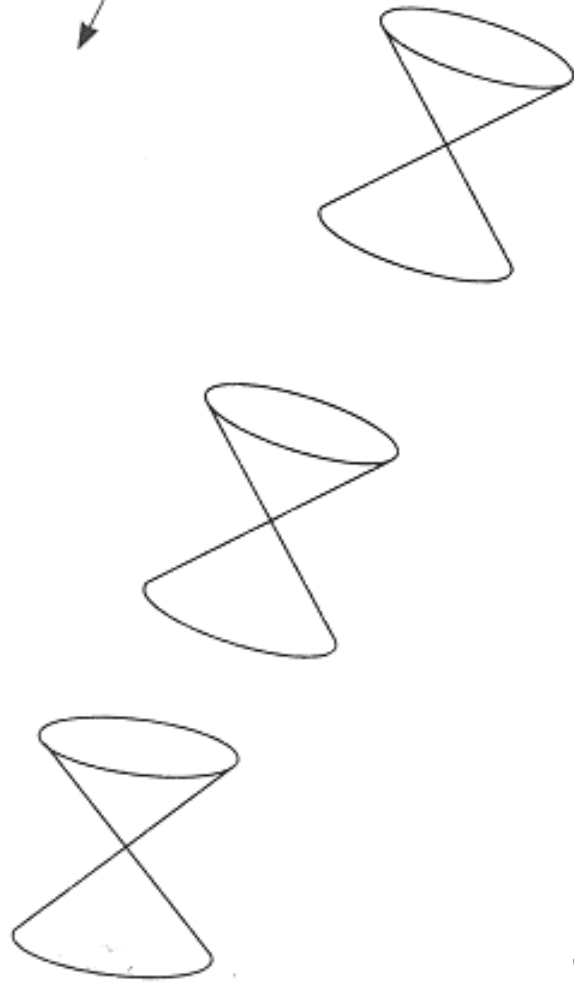
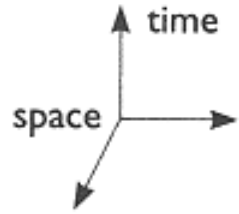
Spacetime diagrams

Different observers  
see different  
spacetime diagrams

However, the causal  
structure of  
spacetime is always  
preserved...



# Gravity 'tilts' light cones



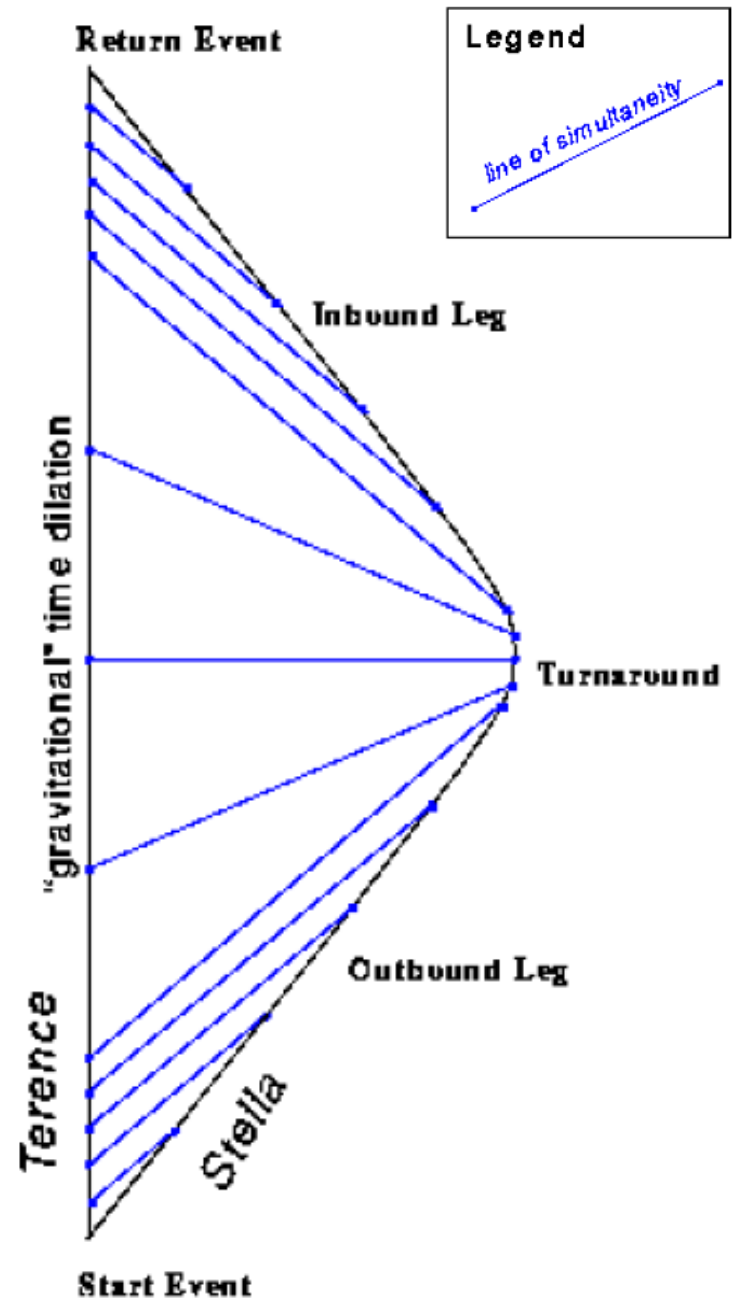
'World line' of a star

We can also look at this in terms of spacetime events which *are* simultaneous.

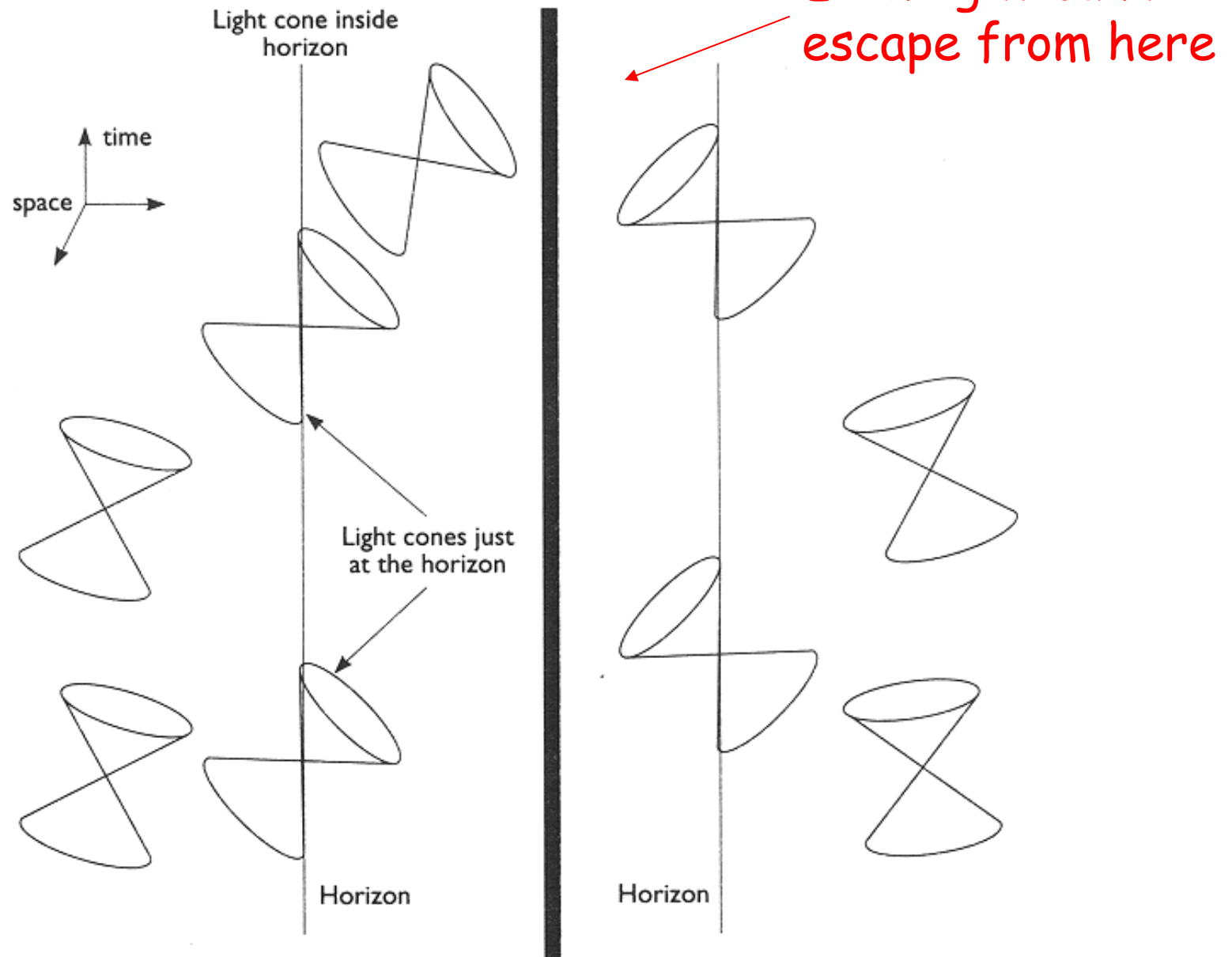
We see that, from Stella's point of view, Terence does most of his aging close to Stella's 'turnaround', when she changes inertial frame (i.e. she changes **velocity**).

Change in velocity = **acceleration**.

This makes it a GR problem!



# Gravity 'tilts' light cones



'World line' of a black hole

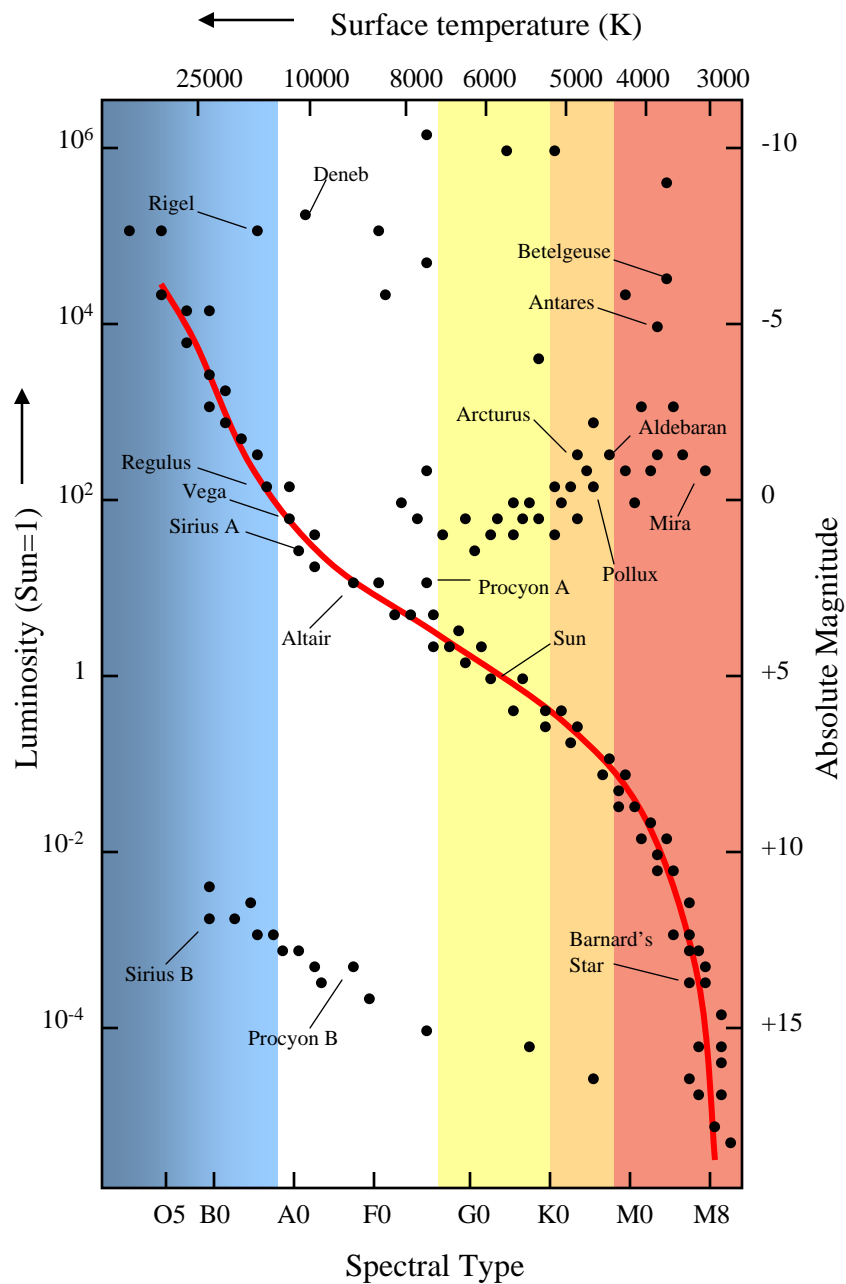
# Do black holes really exist?

Evidence (so far) is indirect, but still strong...

- (1) X-ray binary systems (stellar mass black holes)
- (2) AGN and quasars (supermassive black holes)
- (3) Intermediate mass black holes?
- (4) Primordial black holes?

*See also Week 8 discussion of Gravitational Waves*

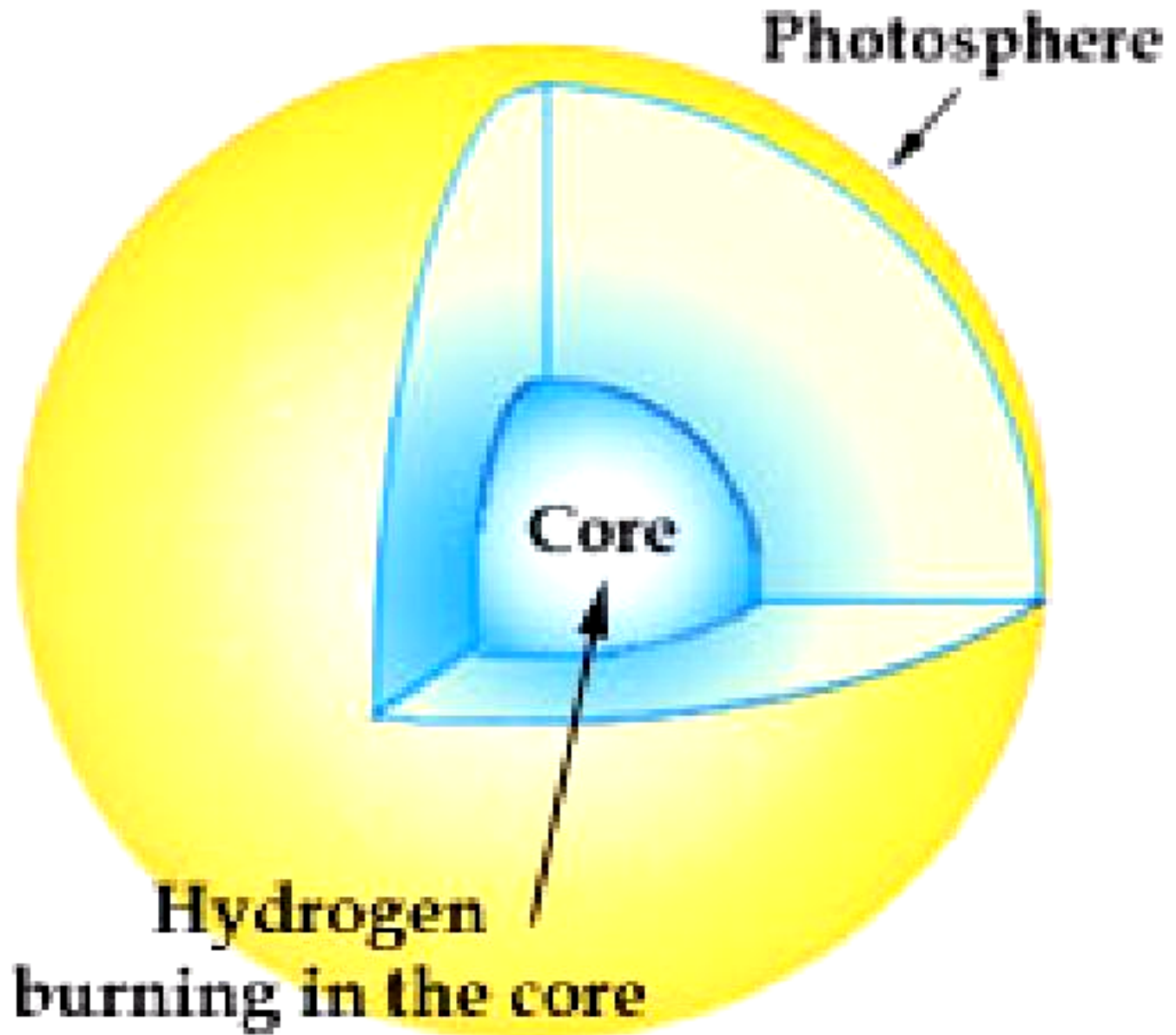




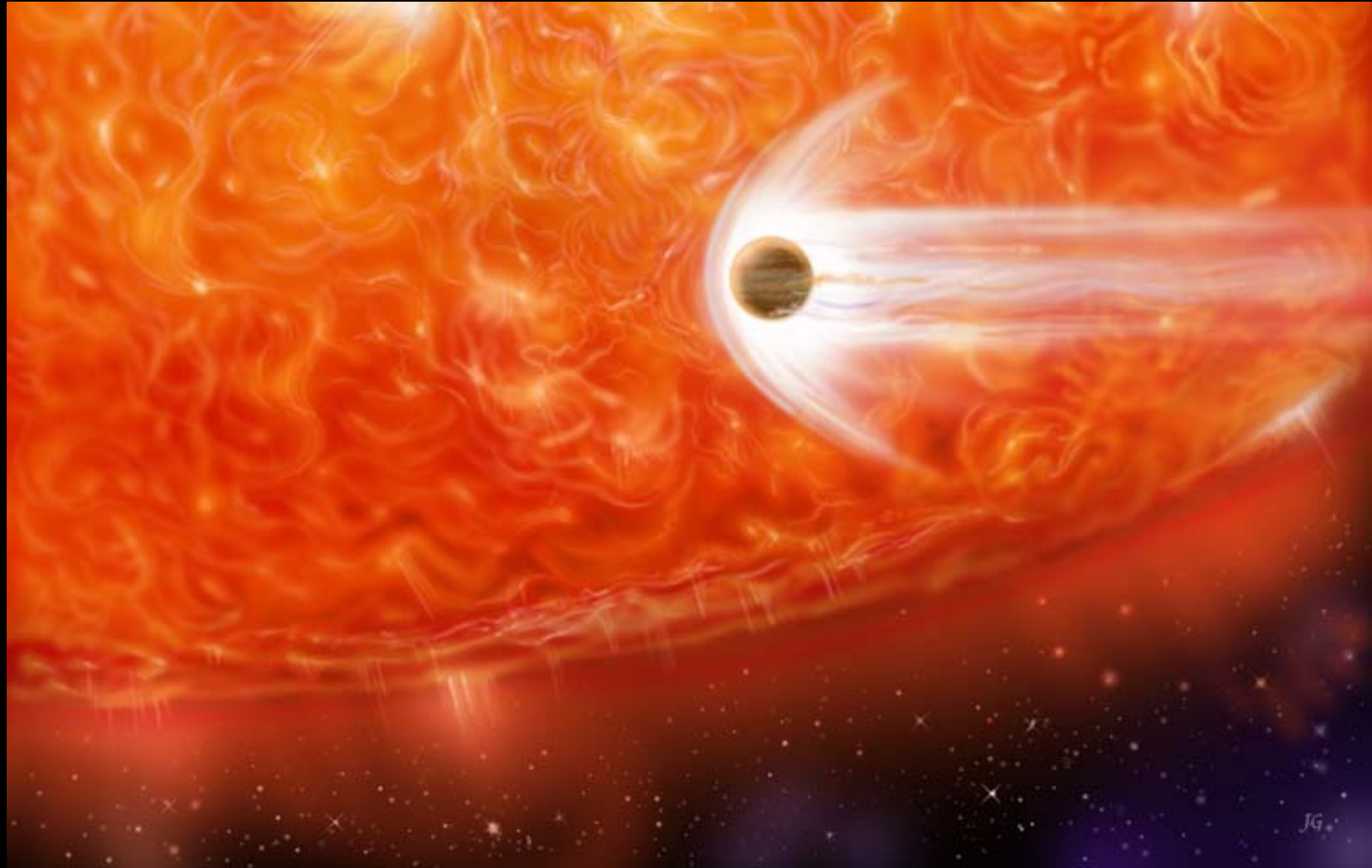
Stars found on the **Main Sequence** convert hydrogen into helium.

Stars like the Sun can do this for many billions of years.

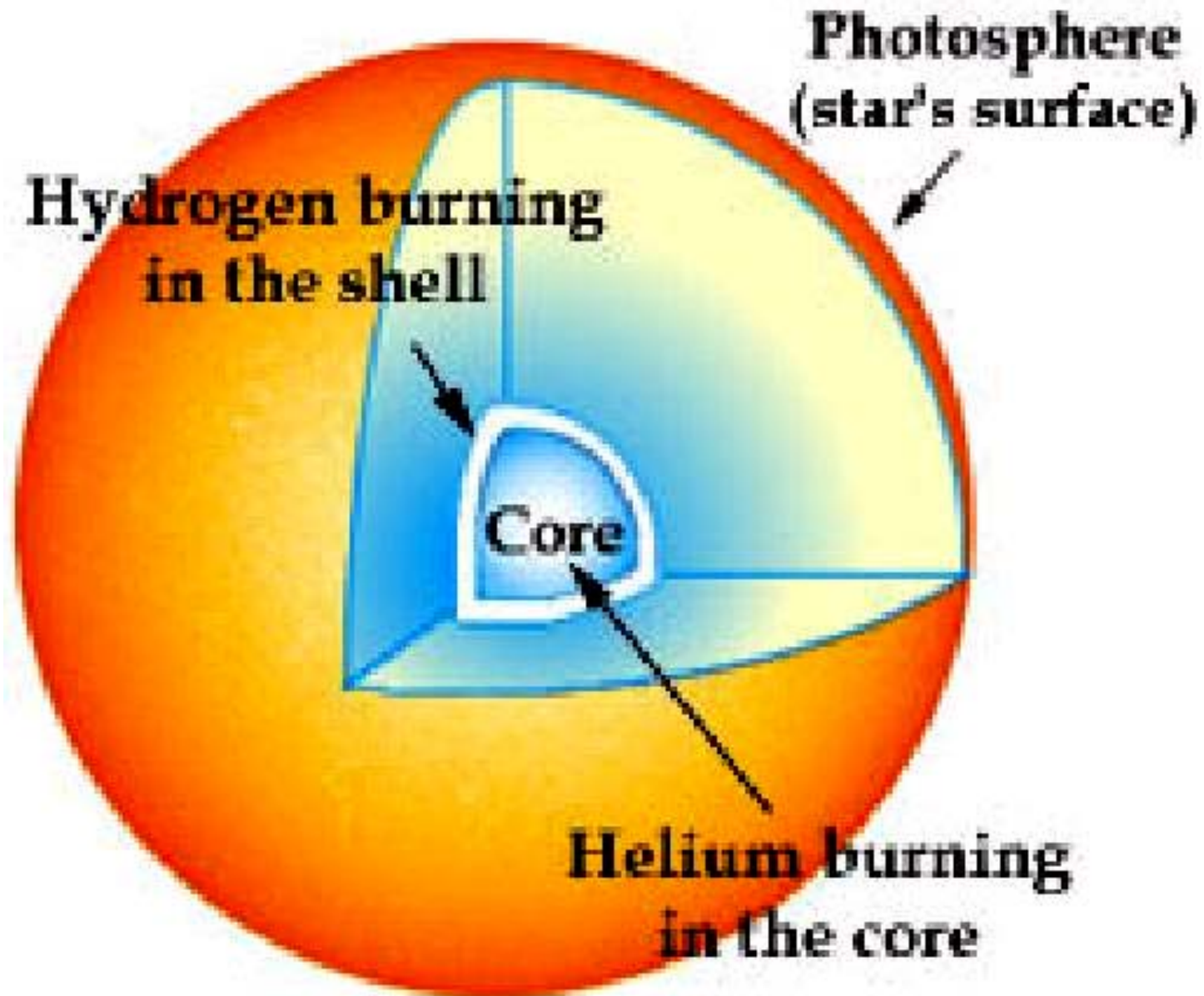
# Interior of a solar-type star



# When the fuel runs out: formation of a red giant



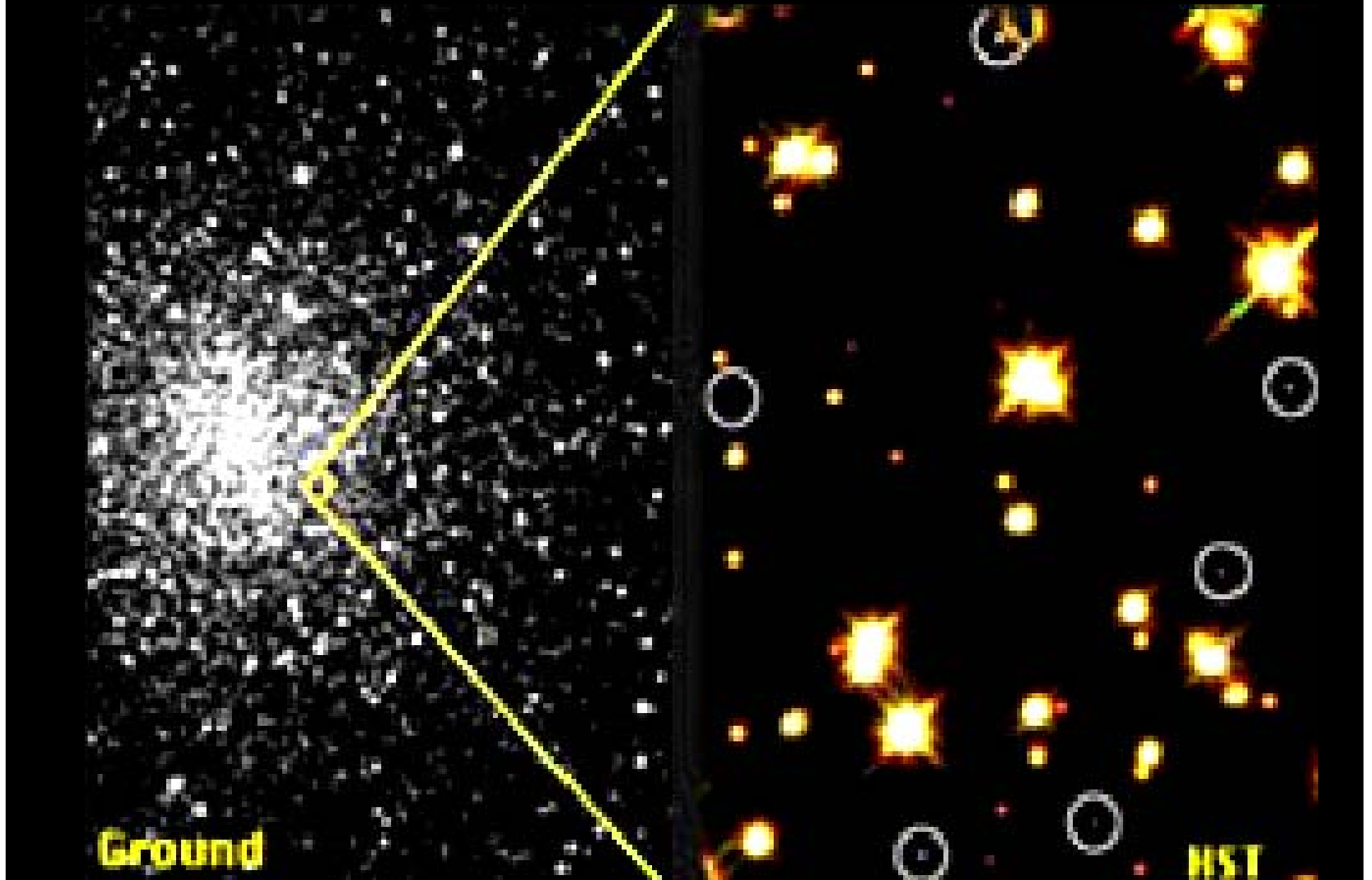
## Interior of a red giant star

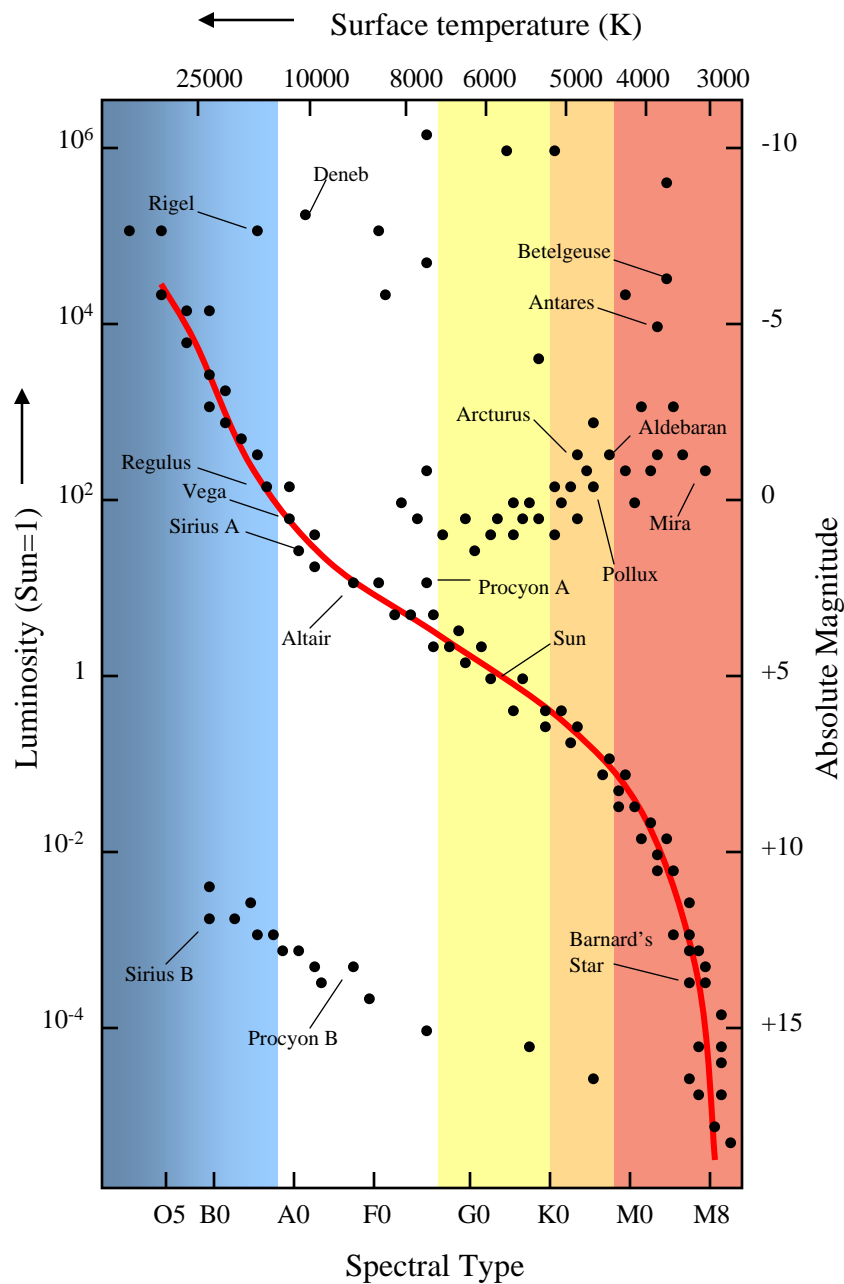


Planetary Nebula NGC 6751



# White dwarfs: earth-sized stellar relics





Stars on the **Main Sequence** turn hydrogen into helium.

Blue stars are much hotter than the Sun, and use up their hydrogen in a few million years

© Mark A. Garlick  
space-art.co.uk





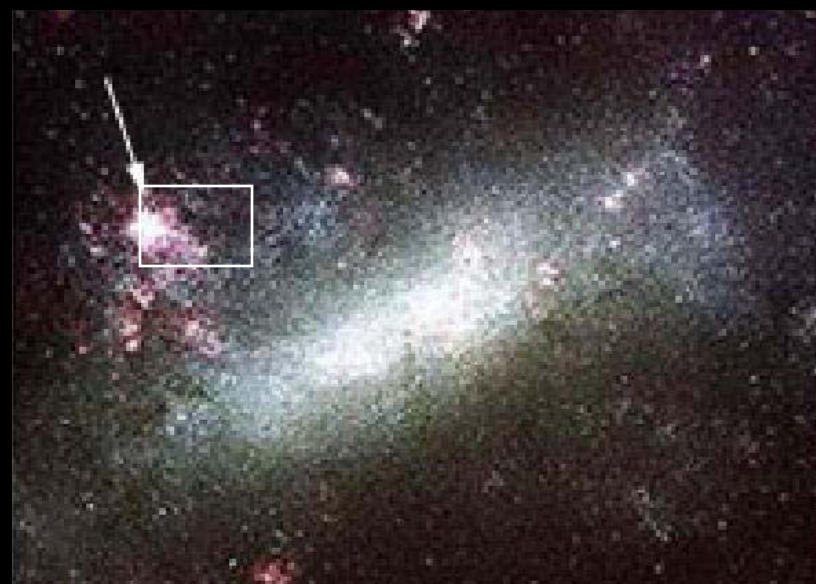
## Interior of a very massive star



# Crab Nebula: supernova of 1054

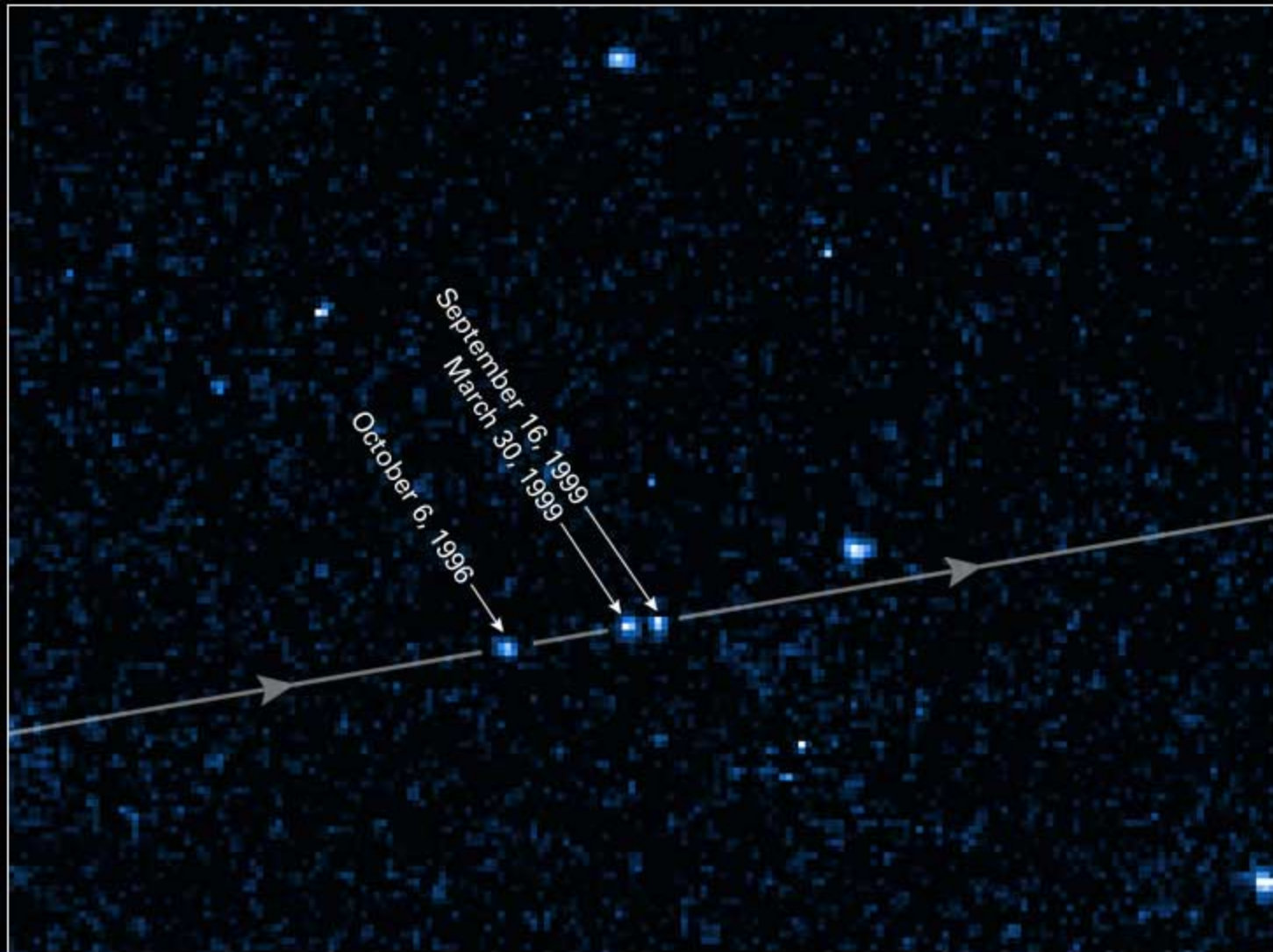


# Supernova 1987A, in the Large Magellanic Cloud



# Neutron Stars

Very much smaller: (almost) invisible at optical, but can be seen in X-Rays if their surfaces are very hot

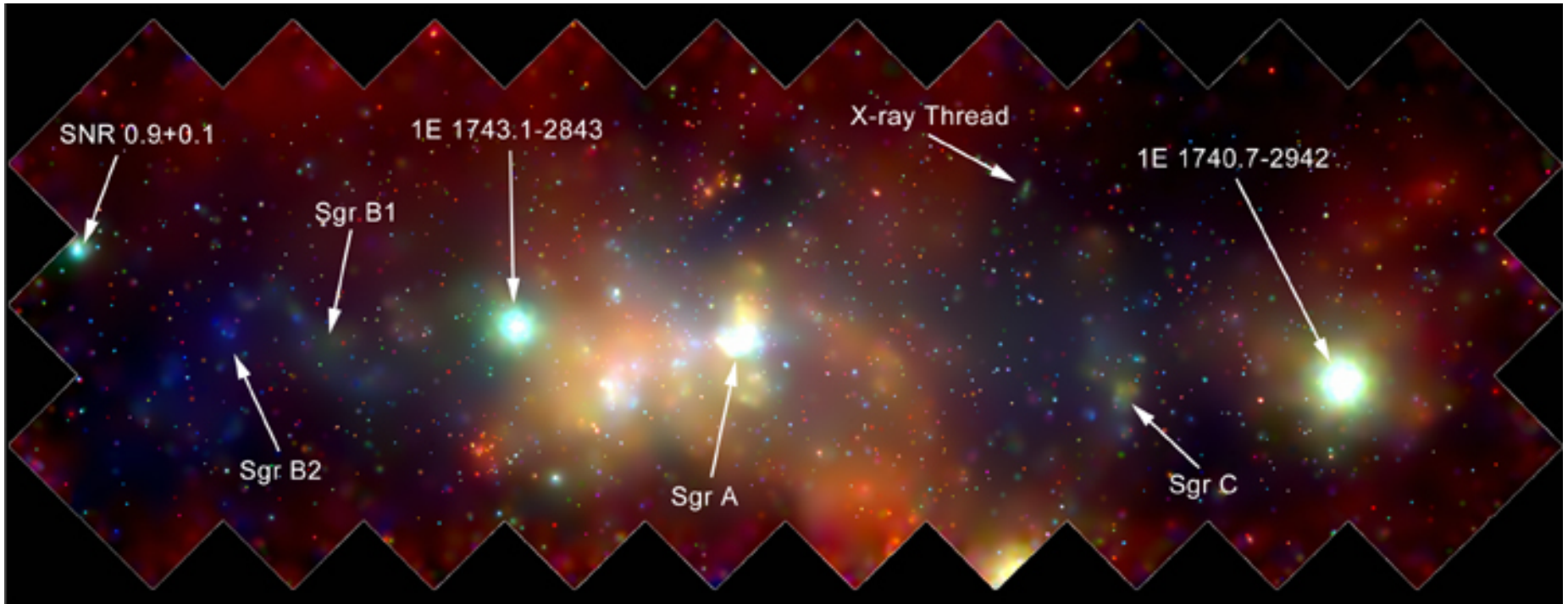


**Neutron Star RX J185635-3754**

HST • WFPC2

NASA and F. Walter (State University of New York at Stony Brook) • STScI-PRC00-35

Chandra has revealed many more X-ray binary sources in the Milky Way, globular clusters and external galaxies.



Chandra (launched 1999): high-resolution X-ray map of the Galactic Centre

# Pulsars

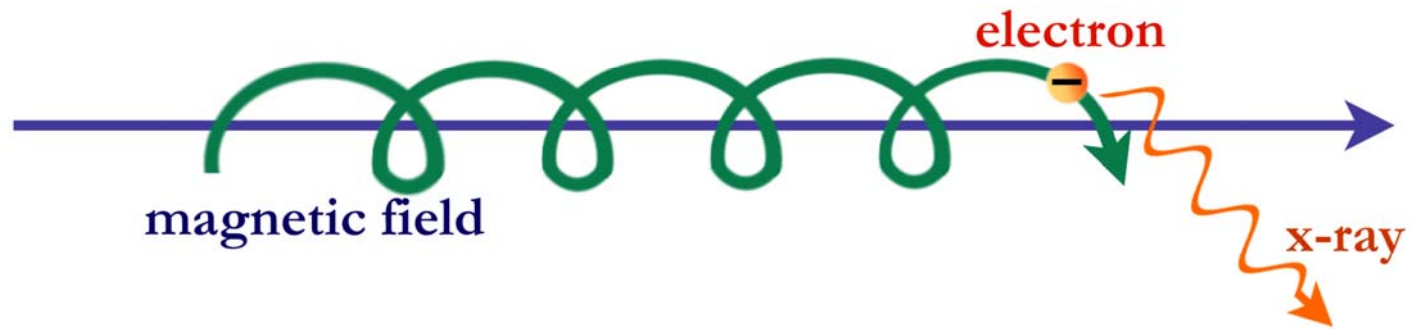
Discovered by Jocelyn Bell, in 1965.

Extremely accurate  
'clocks'

Rapidly rotating NS,  
with beams of radiation



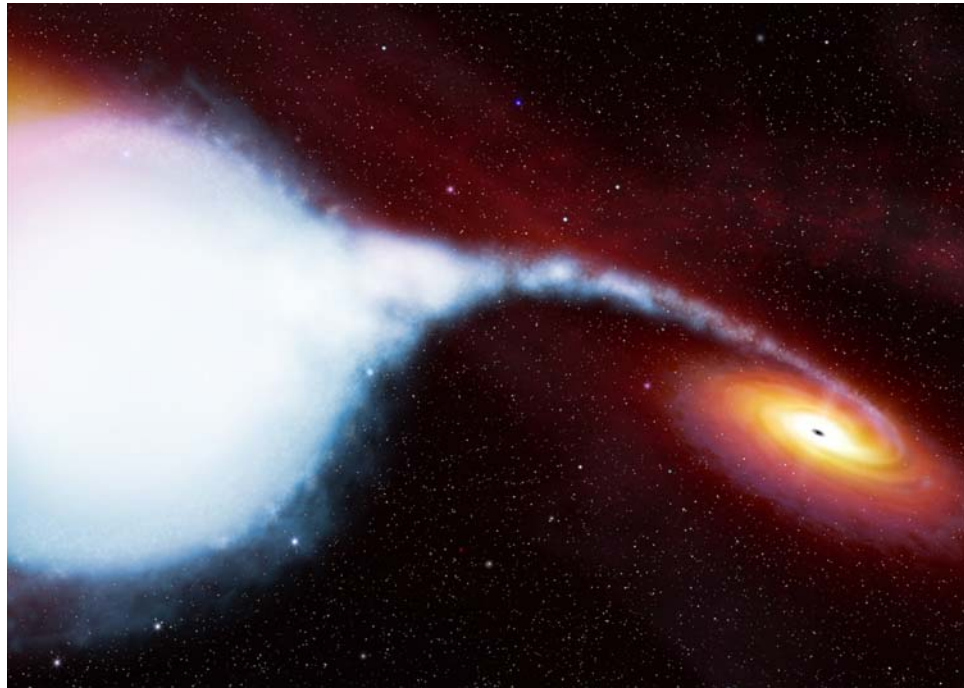
# Pulsars



Synchrotron radiation



# Evidence for stellar black holes from binary systems: e.g. Cygnus X-1

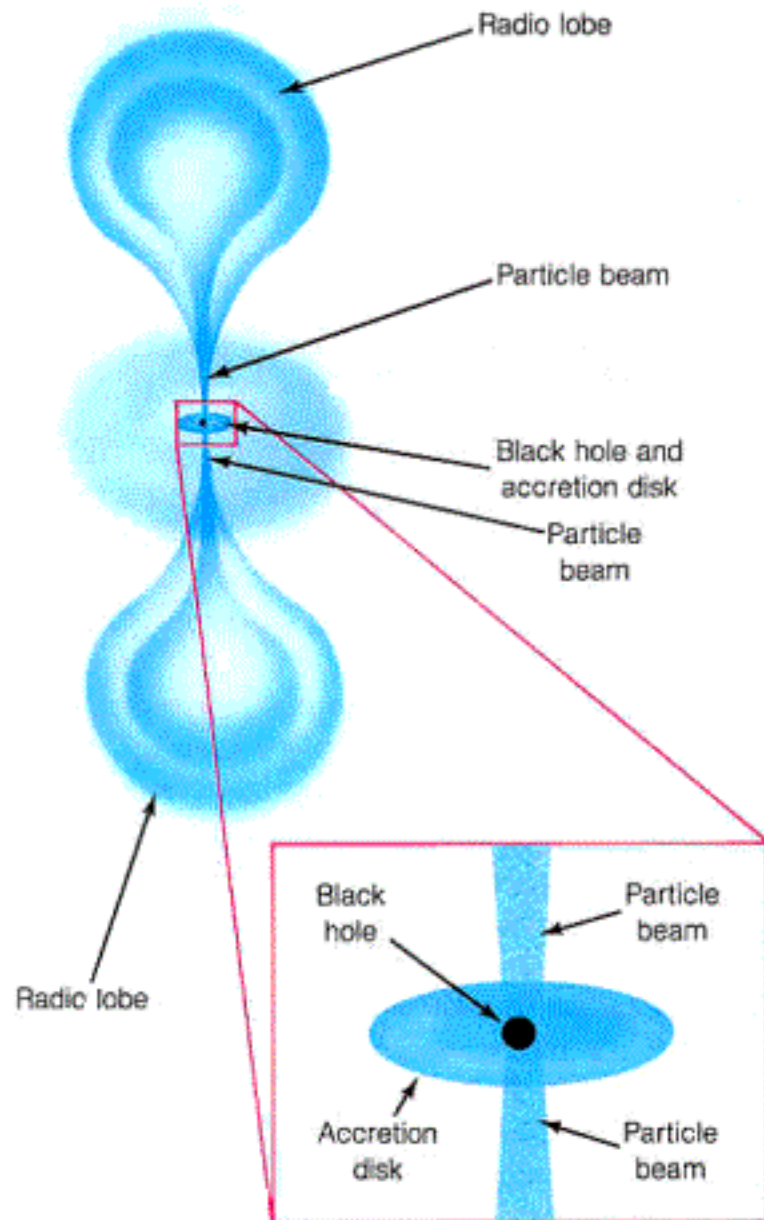


Inferred mass far exceeds the  
Oppenheimer-Volkov limit for NS

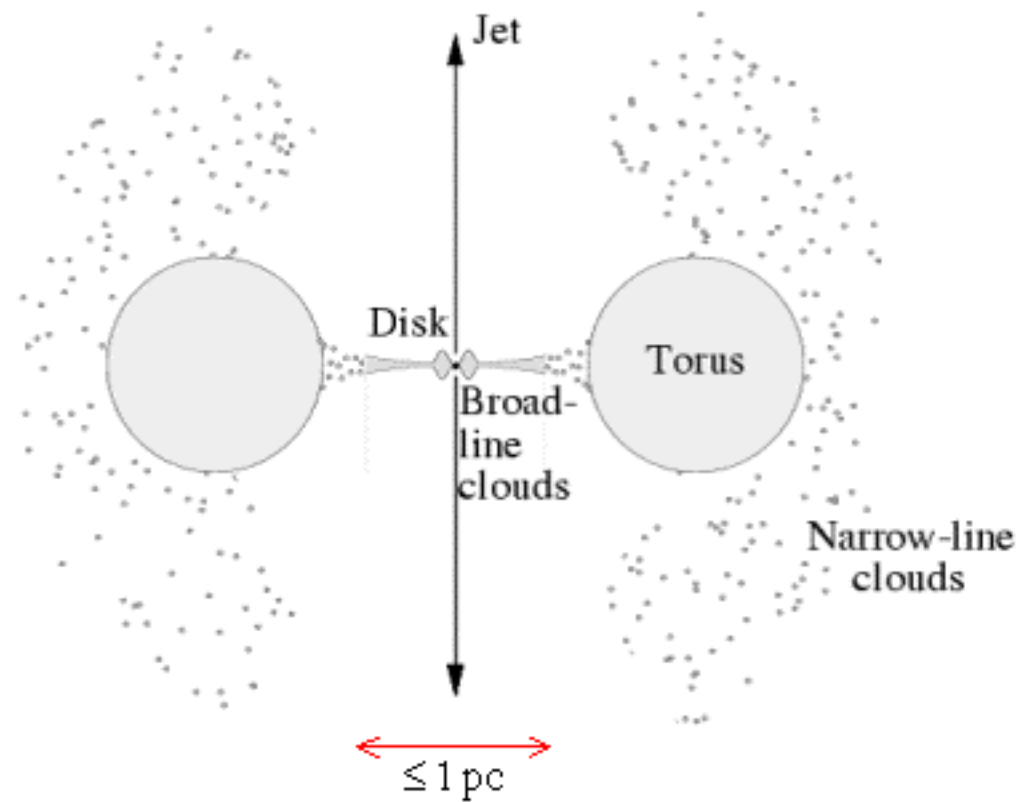
## Active Galactic Nuclei (AGN)

- Compact central region of a galaxy from which we observe substantial amounts of radiation that is *not* the light of stars, or of gas heated directly by stars.
- Active galaxies emit strongly (compared with a normal galaxy) over the whole E-M spectrum, from radio to X-rays and gamma-rays.
- The most luminous AGN easily outshine their host galaxies, and are generally found at high redshift - i.e. in the distant past

# The Unified Scheme



## Cross-section view

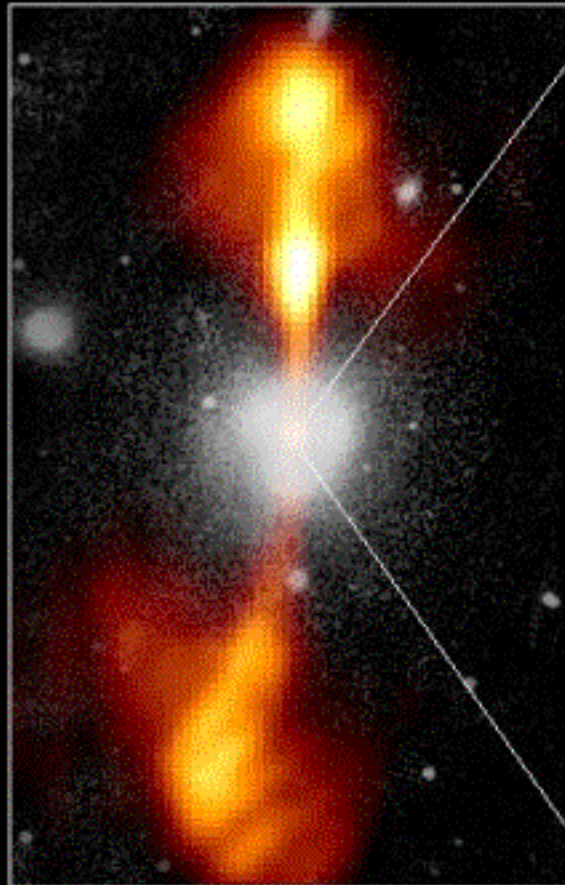


# Core of Galaxy NGC 4261

Hubble Space Telescope

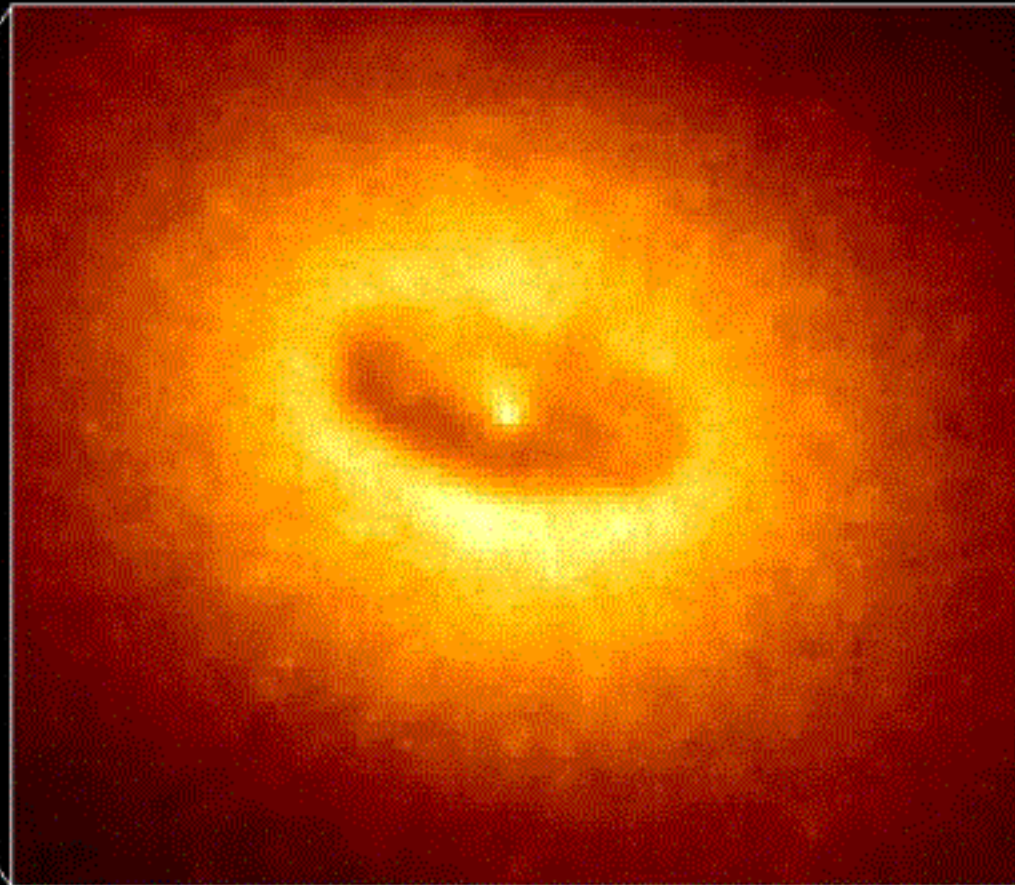
Wide Field / Planetary Camera

Ground-Based Optical/Radio Image

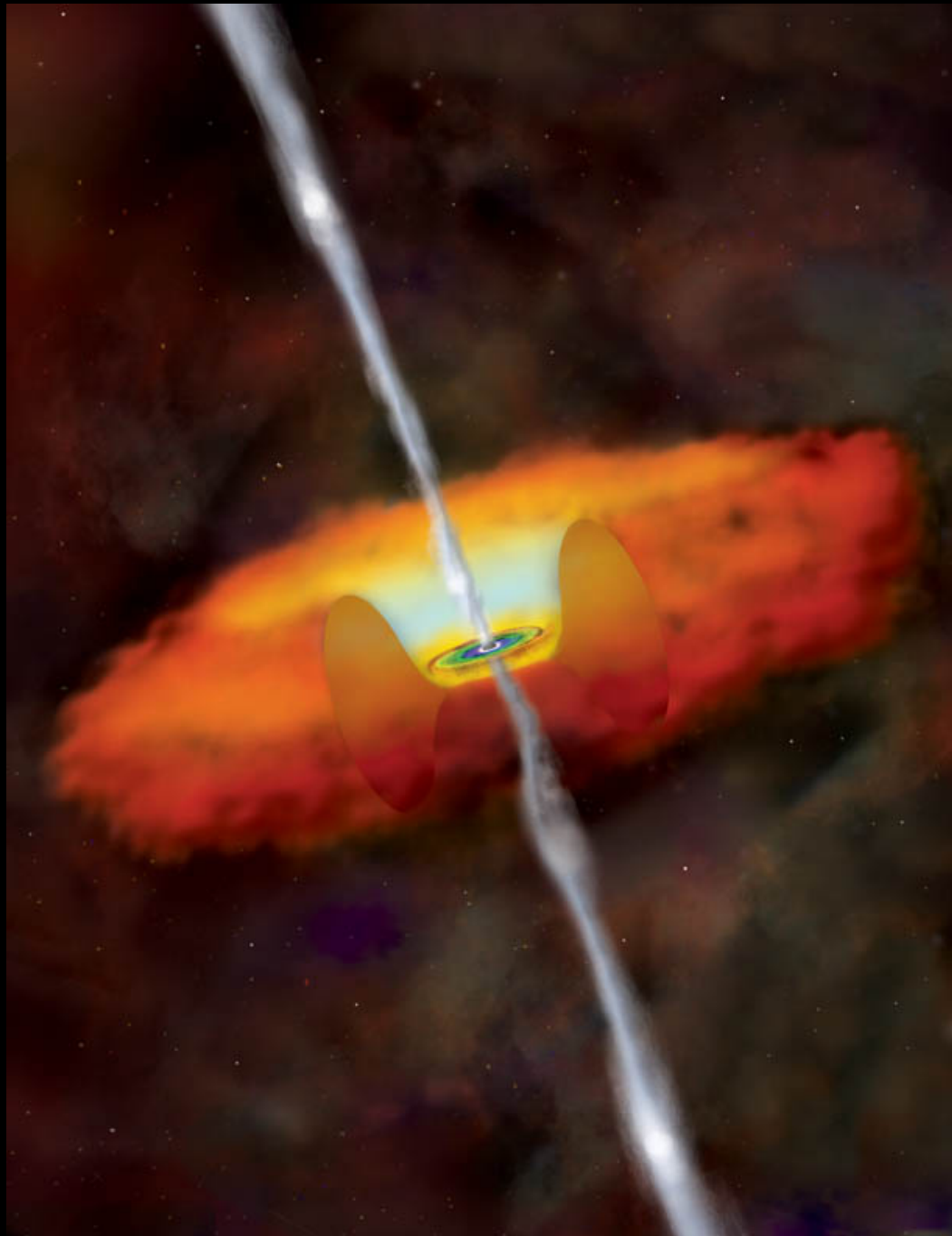


380 Arc Seconds  
88,000 LIGHTYEARS

HST Image of a Gas and Dust Disk



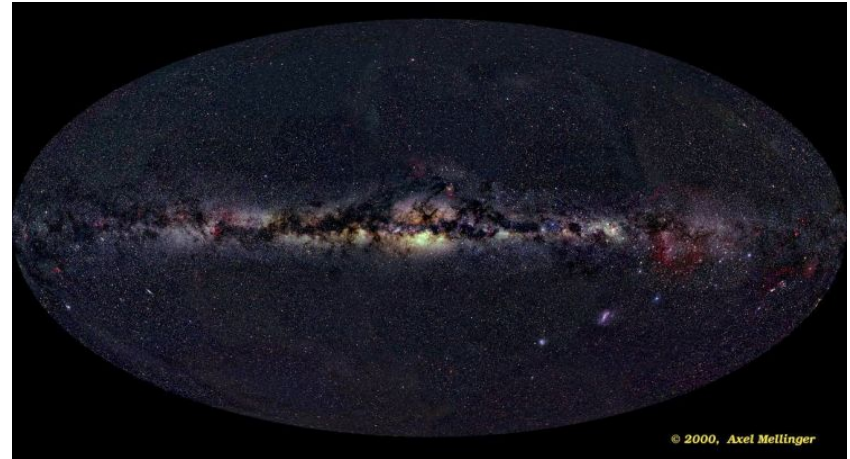
17 Arc Seconds  
400 LIGHTYEARS



## Cosmological solutions of the field equations: Einstein 1917



Einstein assumed the Milky Way to comprise the entire Universe – static and unchanging.



Introduced **cosmological constant** term in field equations to maintain static Universe.

$$\Lambda = 4\pi G \rho \quad \leftarrow \text{Mean density of matter in the Universe}$$

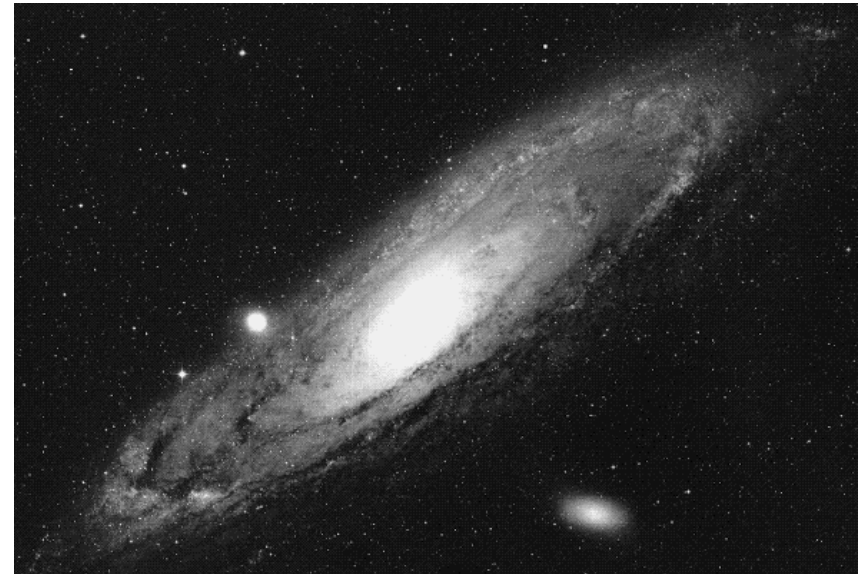
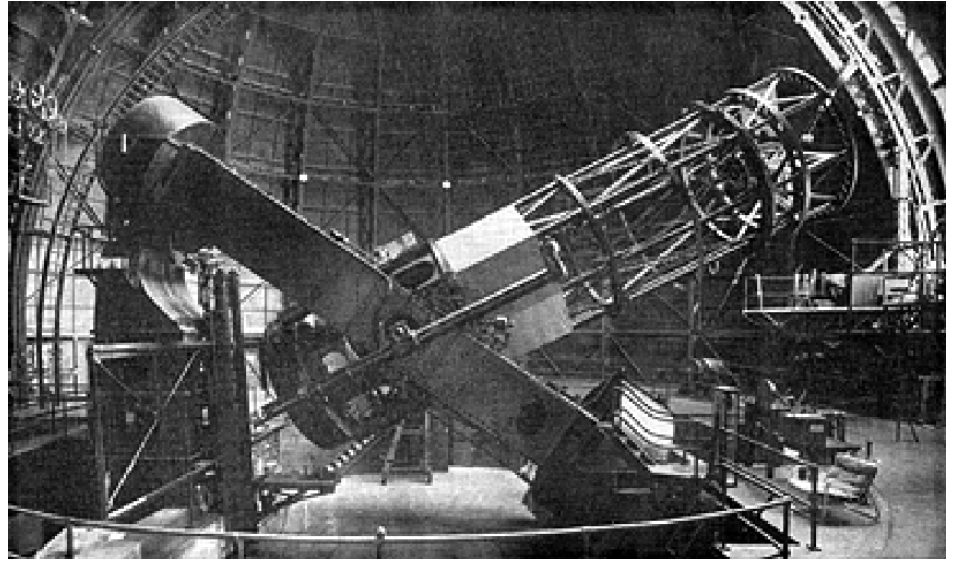
Consistent with **Mach's Principle** (Einstein 1918)

*(but de Sitter, 1917, found apparently static solution for empty Universe)*

Later shown by Eddington (1933) to be *unstable*.

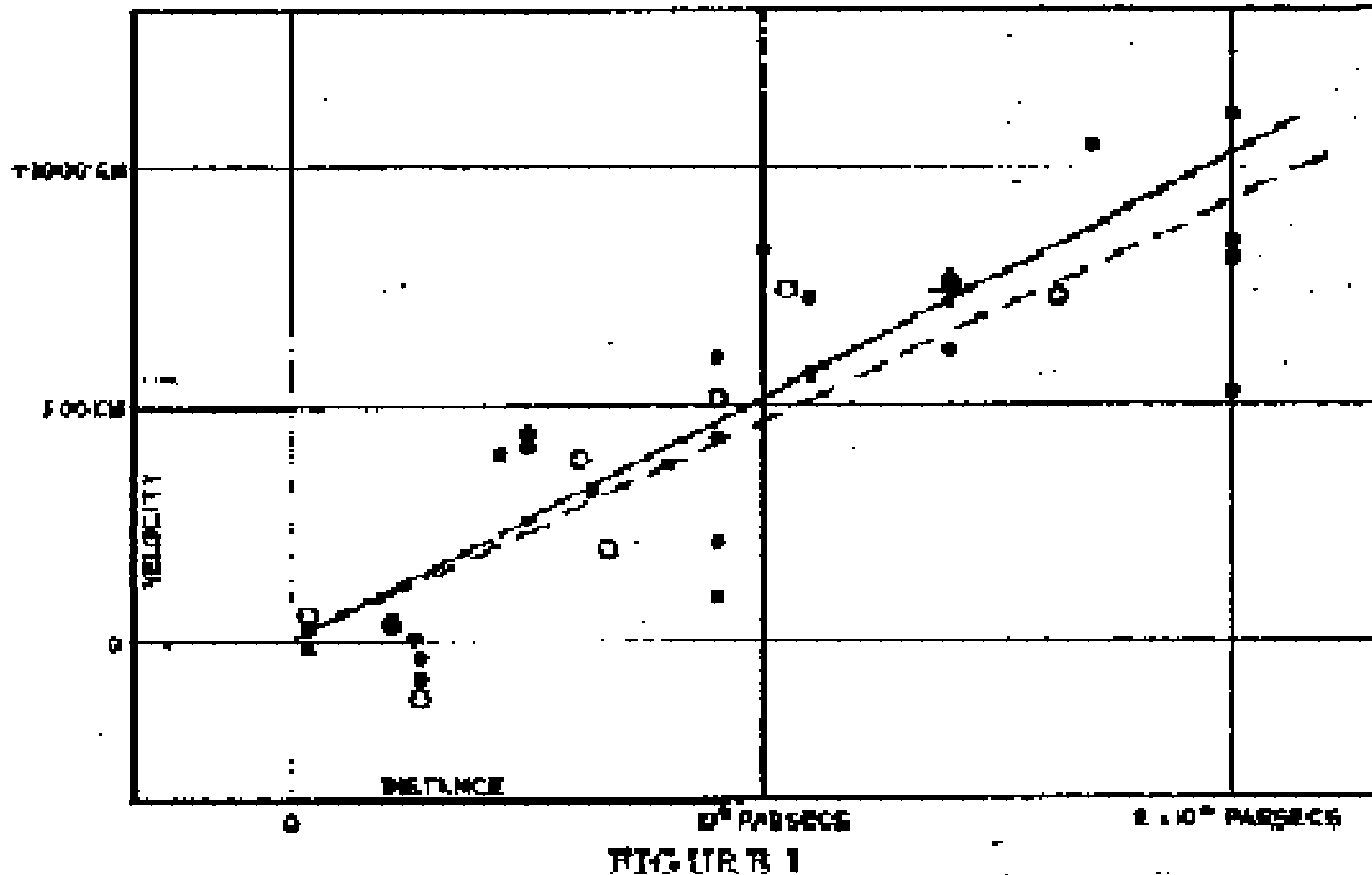


**Edwin Hubble**



Hubble (1924)

# Hubble's Law



Distant galaxies are moving away from us with a speed proportional to their distance

Hubble and Humason (1929)



***Spacetime is expanding like  
the surface of a balloon.***

***As the balloon expands,  
galaxies are carried farther  
apart***

