About the Origins of the General Theory of Relativity

by

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I was very glad to accept the invitation to say something about the history of my own scientific work. Not that I have an unduly high opinion of the importance of my own endeavours. But to write the history of the work of another requires an understanding of his mental processes which can be better achieved by ~~a~~ professional historians; while to explain one's own former way of thinking is very much easier. In this respect, one is in an incomparably more favourable position than any one else, and it would be a mistake from a sense of false modesty, to pass by an opportunity to put the story on record.

After the special theory of relativity had shown the equivalence for formulating the laws of nature of all so-called inertial systems (1905), the question whether a more general equivalence of the co-ordinate systems existed, was an obvious one. In other words, if one can only attach a relative meaning to the concept of velocity, should one nevertheless maintain the concept of acceleration as an absolute one? From the purely kinematic point of view, the relativity of any and every sort of motion was indubitable; from the physical point of view however, the inertial system seemed to have a special importance which made the use of other moving systems of co-ordinates appear artificial.

I was of course familiar with Mach's idea that inertia might not represent a resistance to acceleration as such so much as a resistance to acceleration relative to the mass of all the other bodies in the world. This idea fascinated me; but it did not provide a basis for a new theory.

I made the first step towards the solution of this problem when I endeavoured to include the law of gravity in the frame-work of the special theory of relativity. Like most physicists, at this period I endeavoured to find a " field law ", since of course the introduction of action at a distance was no longer feasible in any plausible form, once the idea of simultaneity had been abolished.

The simplest was of course to keep the Laplace scalar potential of gravity and to extend the Poisson equation by adding, in such a way as to comply with the special theory of relativity, a term differentiated with respect to the time. The law of motion of the particle in a gravitational field, of course, also had to be made to conform to the special theory of relativity. The way to do this was not unambiguously evident since clearly the inertial mass of a body might depend upon the gravitational potential. Indeed this was to be expected on account of the inertia of energy.

Investigations on these lines however led to a result that caused me grave misgivings. According to the classical mechanics, the vertical acceleration of a body in a vertical field of gravitation was independent of the horizontal component of the velocity. It follows from this that the vertical acceleration of a mechanical system (or of its centre of gravity) in such a field, should be independent of its internal kinetic energy. According to the theory I was investigating. however, the vertical acceleration was not independent of the horizontal velocity ~~resp.~~ and so also of the internal energy of the system.

and thus also

This did not agree with the old well-known empirical result that all bodies in a gravitational field were subject to the same acceleration. This principle, which can also be stated as the law of the equivalence of inertial and gravitational mass, impressed me as being of fundamental importance. I wondered how this law could exist and believed that it held the key of the real understanding of inertia and gravitation. I never seriously doubted its exact validity even though I did not know about the beautiful experiments of Eotvoes which, if I remember aright, were ~~made~~ not known to ~~the world~~ me ~~only at~~ until a later date.

I gave up, therefore, the attempt, which I have sketched above, to treat the problem of gravitation within the frame-work of the special theory of relativity; it was clearly inadequate since it failed to take into account just the most fundamental property of gravitation. The principle of the equivalence of inertial and gravitational mass could now be formulated in a very simple and intelligible manner; namely,that in a homogeneous gravitational field,all motions take place just as they would in the absence of such a field in a uniformily accelerated system. If this principle (the equivalence principle) was true for all processes,it indicated that the principle of relativity must be extended to include non-uniform motions of the co-ordinate systems if one desired to obtain an unforced and natural theory of the gravitational field. From 1908 to 1911 I concerned myself with considerations of this nature and endeavoured to extract special results which I need not describe here. Their main importance was merely that it became perfectly plain that a reasonable theory of gravitation could only be obtained by an extension of the principle of relativity.

The problem therefore was to find and to elaborate a theory expressed in equations which did not change their form for non-linear transformations of the co-ordinates. Whether this condition was to be fulfilled for all continuous transformations of the co-ordinates,or only for certain special ones,I could not say at the outset.

I saw very soon that the simple physical interpretation of the co-ordinates would vanish if,as was required by the equivalence principles,non-linear transformations were to be permissible; In other words,one could no longer demand that co-ordinate differences should be the ~~immediate~~ quantities directly) measurable, in principle (~~immediately~~ with ideal rulers and clocks. The recognition of this fact worried me a great deal, for I could not see for a long time what the co-ordinates were to represent (at all) in physics in these circumstances. The escape from this dilemma only came in 1912 as a result of the following considerations.

We were called upon to find a new formulation of the inertial principle which would become identical with Galileo's formulation in the absence of a " real " gravitational field, that is to say, if existe ~~we used~~ an inertial system (and we use it as) ~~as~~ our co-ordinate system. Galileo's formulation says; a material particle on which no forces are acting, is represented in four-dimensional space by a straight line, in other words, by a shortest line, or more exactly, by a line whose length has a stationary value. This concept presupposes the concept of the length of a line element, that is to say, of a metric structure of space ~~system of measurement (metric)~~. In the special theory of relativity as Minkowski had shown, the metric was quasi-euclidian, that is to say .the square of the length ds of a line element was a ~~definite~~ quadratic function of the differential (~~coefficients~~ of the co-ordinates) with konstant coefficients

If one introduces other co-ordinates by non-linear transformations, ds^2 remains a homogeneous function of the differential coefficients of the co-ordinates, but the coefficients ($g_{\mu\nu}$) of this function ~~$g_{\mu\nu}$~~ are no longer constant, but are functions of the co-ordinates. Mathematically, this means that the physical four-dimensional continuum has a Riemann metric. Time-like lines with an extreme value in this metric system represent the laws of motion of a material point on which, apart from gravity, no forces are acting. The coefficients $g_{\mu\nu}$ of this metric system describe the gravitational field in the co-ordinate system one has chosen. We have thus found a natural formulation of the equivalence principle; that it was permissible to extend it to all gravitational fields, was a plausible hypothesis.

The solution of the above dilemma was therefore as follows; a real physical significance attaches only to the Riemann metric, not to ~~the differential co-efficients of~~ the co-ordinates or their differences.

With this, a workable basis for the general theory of relativity had been found. Two problems however remained to be solved.

(1) How can we translate a field law given us in the terminology of the special theory of relativity into a Riemann metric?

(2) What are the differential expressions which ~~determine~~ enters into the law holding for the Riemann metric ($g_{\mu\nu}$) {law of the gravitational field)?

I worked at these problems from 1912 to 1914 with my friend Grossmann. We found that the mathematical methods for solving the first question were ready waiting for us in the ~~infinitesimal~~ absolute differential calculus of Ricci and Levi Civita.

As to the second problem, its solution obviously required that we should be able to form invariant differentials of the second order of the $g_{\mu\nu}$. We soon recognised that methods for doing this had long ago been worked out by Riemann (curvature tensor) . Already two years before the publication of the general theory of relativity, we had considered the correct field equations of gravitation, but we failed to recognise that they were physically applicable. I believed, on the contrary, that one could show that they were not compatible with experience. Further, I imagined that one could show by a general argument that a law of gravitation that was invariant for all possible transformations of co-ordinates, would not be compatible with the principle of causality. These were errors in thinking, which caused me two years of hard work before at last in 1915 I recognised them as such and returned penitently to the Riemann curvature which enabled me to find the relation to the empirical facts of astronomy.

Once ~~we have recognised~~ It ~~it was~~ the validity of this mode of thought has ~~been~~ recognised the ~~our~~ final results appear almost ~~self-evident~~ simple; any intelligent undergraduate can understand them without much trouble. But the years of searching in the dark for a truth that one feel's but cannot express, the intense desire and the alternations of confidence and misgiving, until one breaks through to clarity and understanding, is only known to him who has himself experienced it.