Astronomical Data Analysis I

Dr Martin Hendry,
Dept of Physics and Astronomy
University of Glasgow, UK

10 lectures, beginning October 2006

Dr Martin Hendry

- Room 607, Kelvin Building
- email: martin@astro.gla.ac.uk
- Tel: ext 5685
- Office hours: no formal time

Course webpages: access via A345 moodle site

http://moodle.gla.ac.uk/physics/moodle/

Course Outline

1. Introduction

- Why data analysis?
- Overview of types of astronomical data:

  Photon counts; images; spectra; time series; Fourier components.

2. A Basic Statistical Toolbox

- Sources of error: statistical versus systematic
- What is probability?
- Probability distributions and their moments
- Examples: Poisson, Uniform, Gaussian
- Bayes' Theorem and Bayesian inference

Lectures 1 - 3

(see also M-level course on Statistical Astronomy)

Course Outline

3. Model Fitting

- Least squares and maximum likelihood
- Chi-squared and goodness of fit
- Computational methods for finding maxima an minima of functions.

4. Monte Carlo Methods

- Uniform random number generators
- Transformation method
- Probability integral transform
- Rejection method

Lectures 4 - 5

Course Outline

5. Fourier Methods

- Fourier transforms: definitions and examples
- Discrete and fast Fourier transforms
- Relationship between real space and Fourier space
- Sampling of Fourier components

6. Data Acquisition

- Sampling theorems: Nyquist theorem and its applications
- Analogue to digital conversion
- Data compression for space-based data

Lectures 6 - 8

Course Outline

7. Time Series Analysis

- Beating and aliasing
- Period fitting
- Wavelets and other basis functions

8. Inverse Methods

- Ill-posedness and instability
- Smoothing and regularisation
- Deconvolution algorithms

Lectures 9 - 10

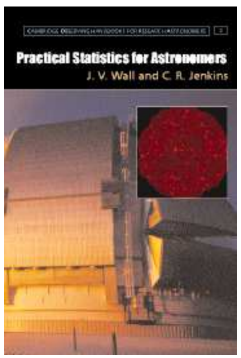
Recommended Books?

No single textbook suitable, but several may be worth a look:

Astrophysical Techniques

(IoP publishing)
C.R. Kitchin

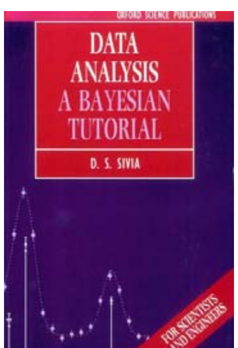
ISBN: 0750309466



Practical Statistics for Astronomers

(Cambridge Univ Press)
J.V. Wall & C.R. Jenkins

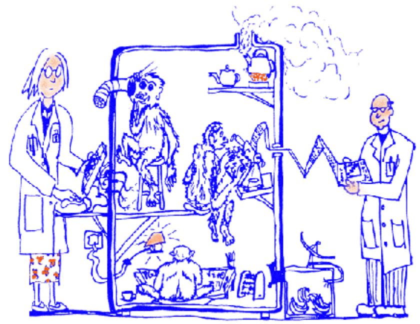
ISBN: 0521456169



Data Analysis:
A Bayesian Tutorial

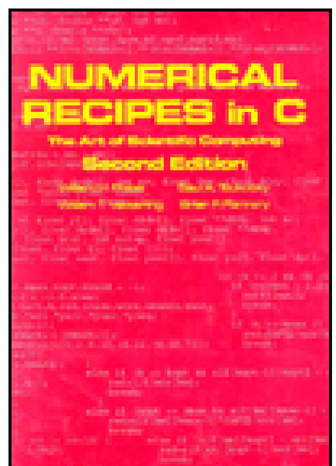
(Oxford Univ Press)
D.S. Sivia

ISBN: 0198568312

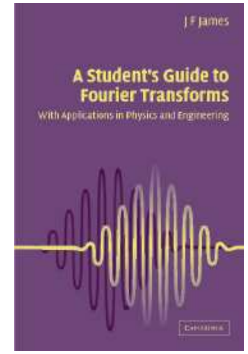


See also **free book** by Praesenjit Saha (London).

Can be downloaded via Moodle site



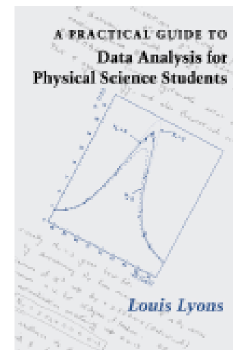
http://www.numerical-recipes.com/



A Student's Guide to Fourier Transforms

(Cambridge Univ Press)
J.F. James

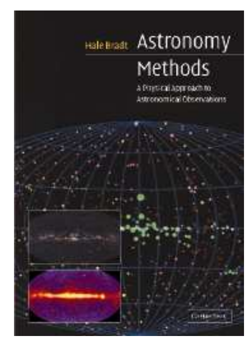
ISBN: 0521004284



A Practical Guide to Data Analysis for the Physical Sciences

(Oxford Univ Press)
Louis Lyons

ISBN: 0521424631

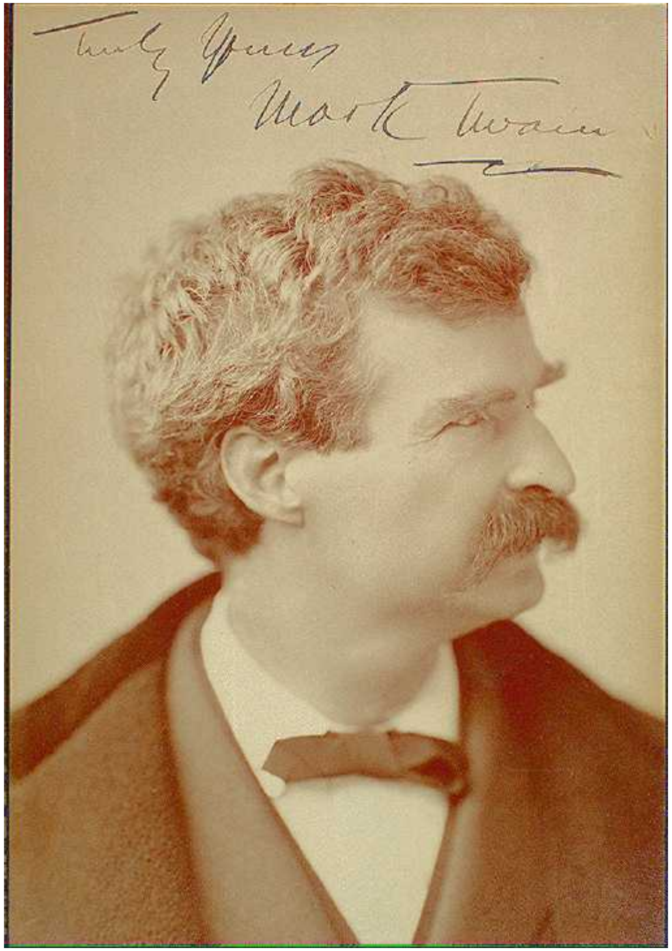


Astronomy Methods

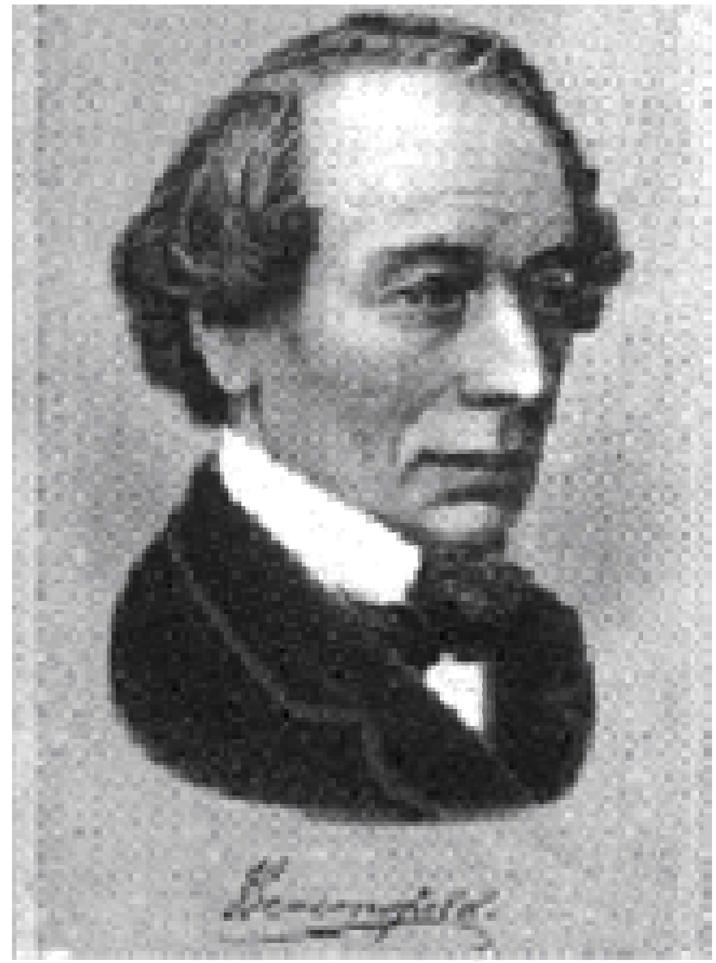
(Cambridge Univ Press)
Hale Bradt

ISBN: 052136440X

Why a course on astronomical data analysis?...



Mark Twain

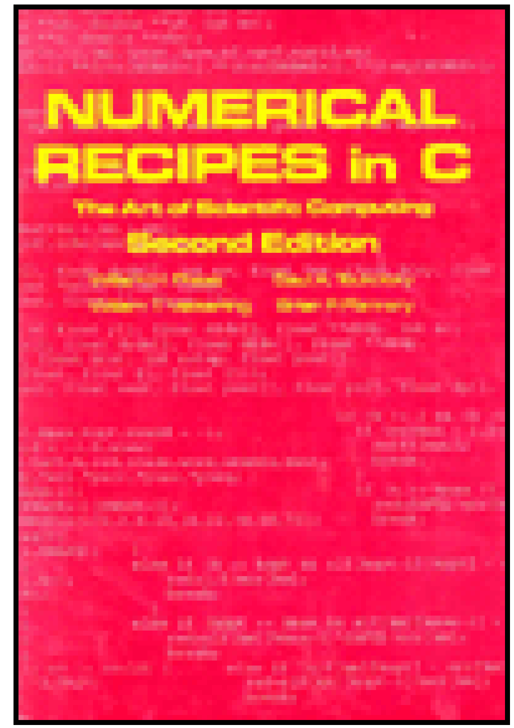


Benjamin Disraeli

There are three types of lies: lies, damned lies and statistics

Why a course on astronomical data analysis?...

Data analysis methods are often regarded as simple recipes...



http://www.numerical-recipes.com/

Why a course on astronomical data analysis?...

Data analysis methods are often regarded as simple recipes...

...but in astronomy, sometimes the recipes don't work!!!

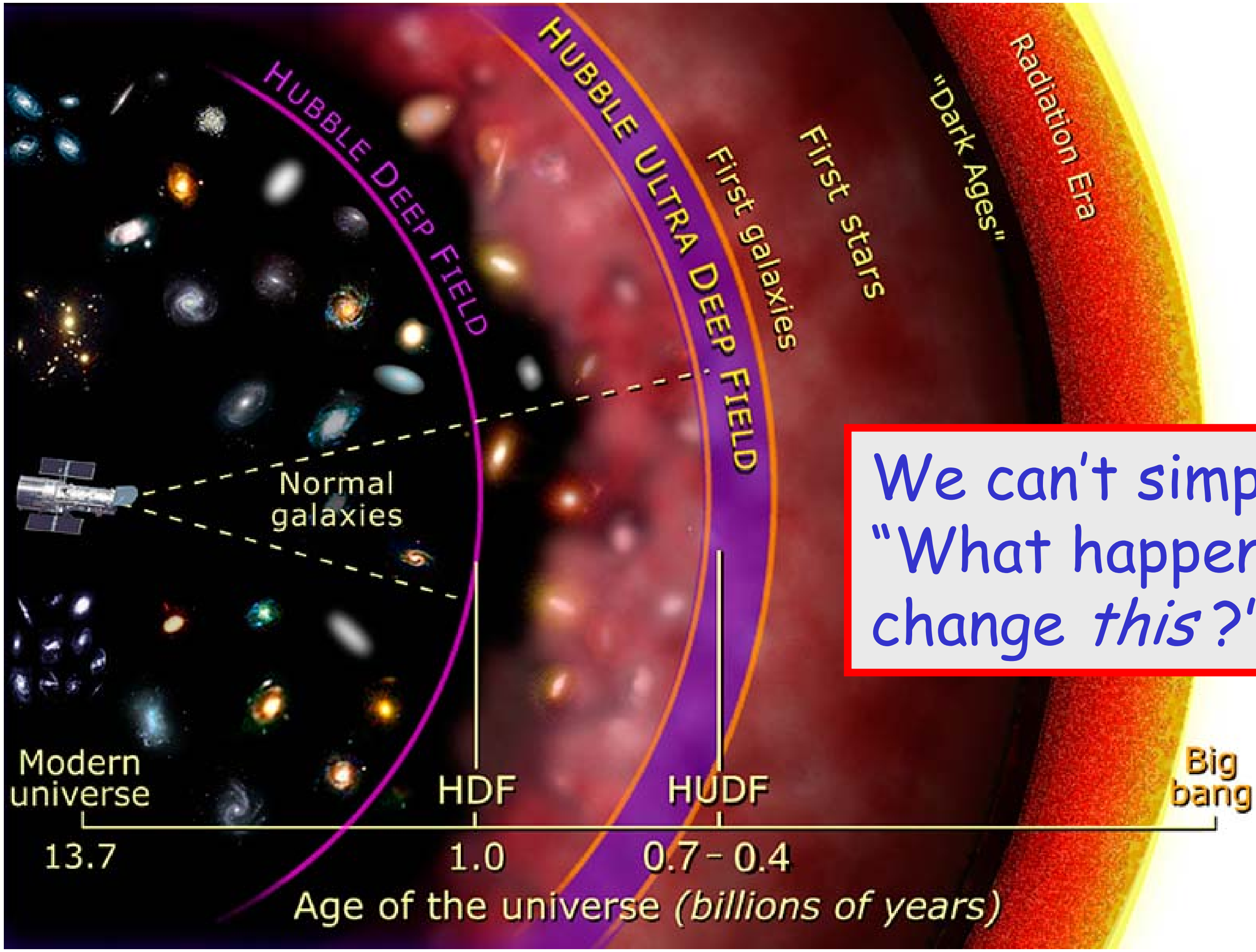
- Very weak signals
- Correlated 'residuals'
- Incorrect assumptions

↓

SYSTEMATIC ERRORS

Why a course on astronomical data analysis?...

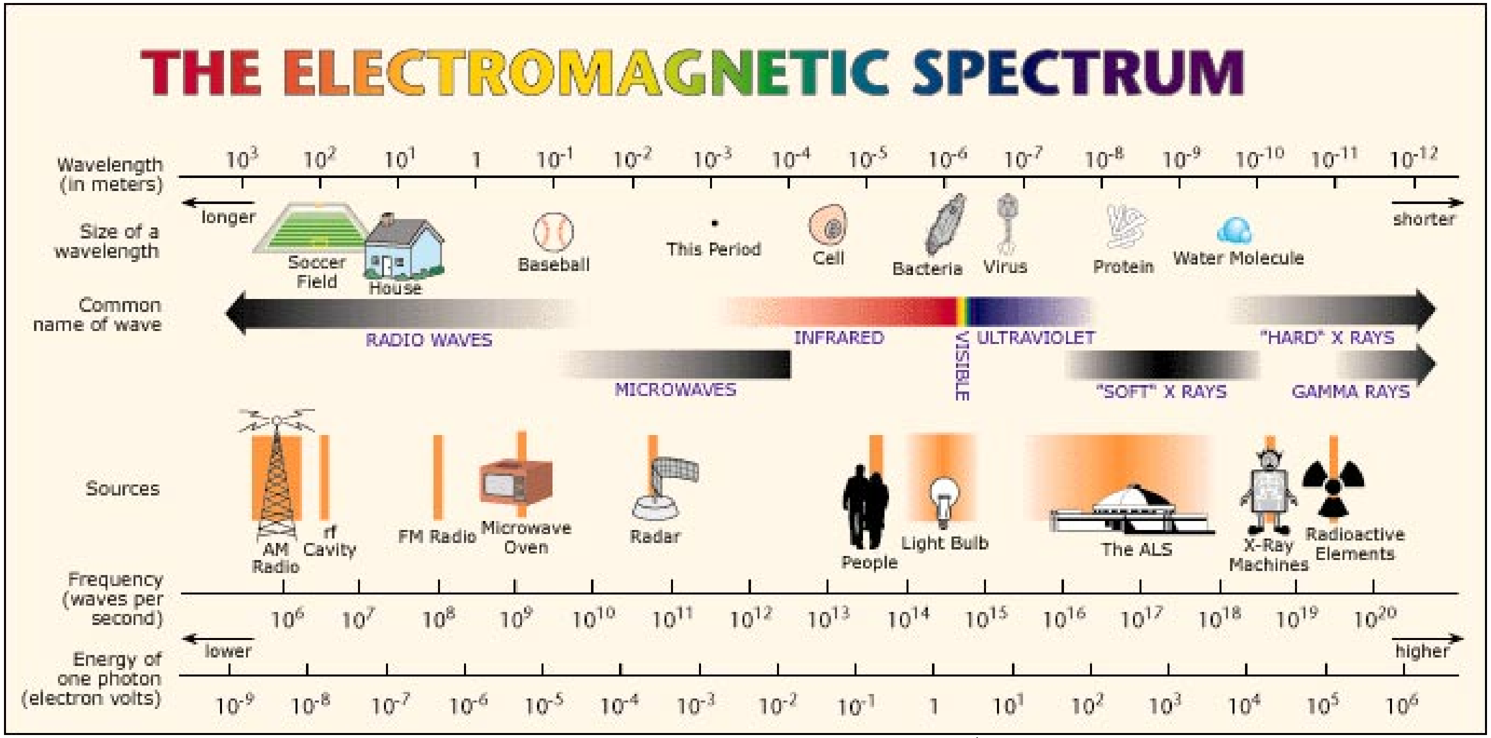
Many areas of astronomy are *Remote sensing*

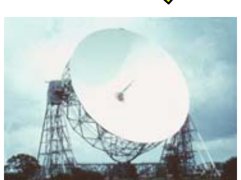


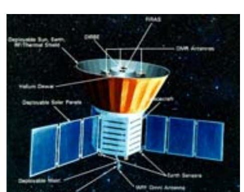
We can't simply ask "What happens if we change *this*?"

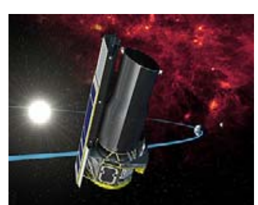
Types of Astronomical Data

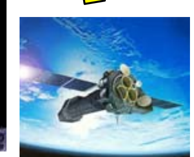
Modern observational astrophysics is **multi-wavelength** across the E-M spectrum

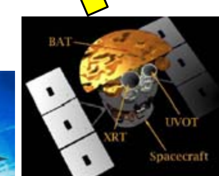












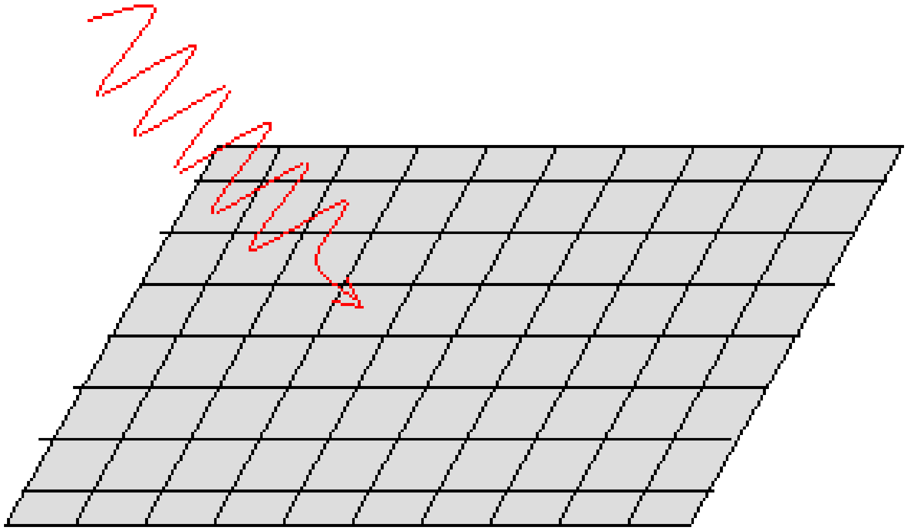
Types of Astronomical Data: CCD imaging

From near infra-red $(\lambda \sim 100\mu\text{m})$ through to UV $(\lambda \sim 10\text{nm})$ wavelengths, astronomical data arrives as photons, which trigger a CCD response.

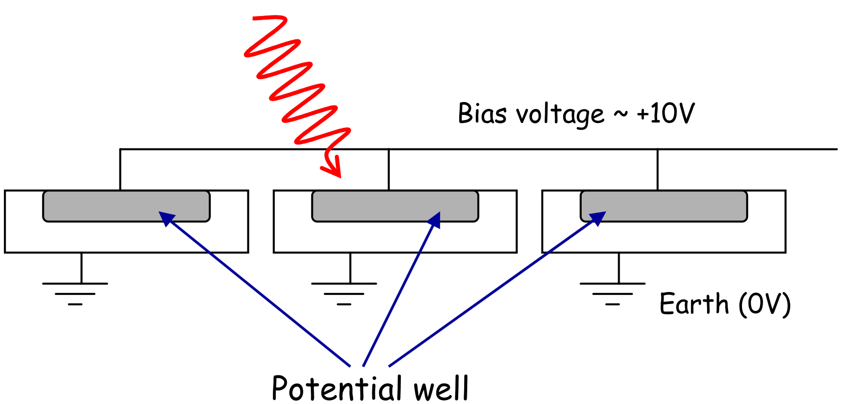
A CCD is a semiconductor array of light-sensitive pixels – typically about 20 μm across.

Image: direct 'map' of where photons arrive

Arrays of 10^7 pixels standard.



'State of the Art' – mosaics of CCDs, around 10^9 pixels in total



- Electron released when photon strikes semiconductor
- Bias voltage draws electron into potential well; stored there during exposure

Types of Astronomical Data: CCD imaging

Hence, a great deal of astronomical data consists of **counts of photons**.

These obey **Poisson statistics**.

Simeon-Denis Poisson
(1781 – 1840)

Recap of ideas from A2 Observational Astrophysics

- The number of photons arriving at our detector from a given source will **fluctuate**.
- We can treat the arrival rate of photons **statistically**, which (roughly speaking) means that we can calculate the **average** number of photons which we **expect** to arrive in a given time interval.
- We make certain assumptions (axioms):

1. Photons arrive independently in time
2. Average photon arrival rate is a constant

If our observed photons satisfy these axioms, then they are said to follow a **Poisson distribution** (See also section 2)

Suppose the (assumed constant) mean photon arrival rate is R photons per second.

If we observe for an exposure time τ seconds, then we expect to receive $R\tau$ photons in that time.

We refer to this as the **expectation value** of the number of photons, written as

$$\boxed{E(N) = \langle N \rangle = R\tau} \qquad (1.1)$$

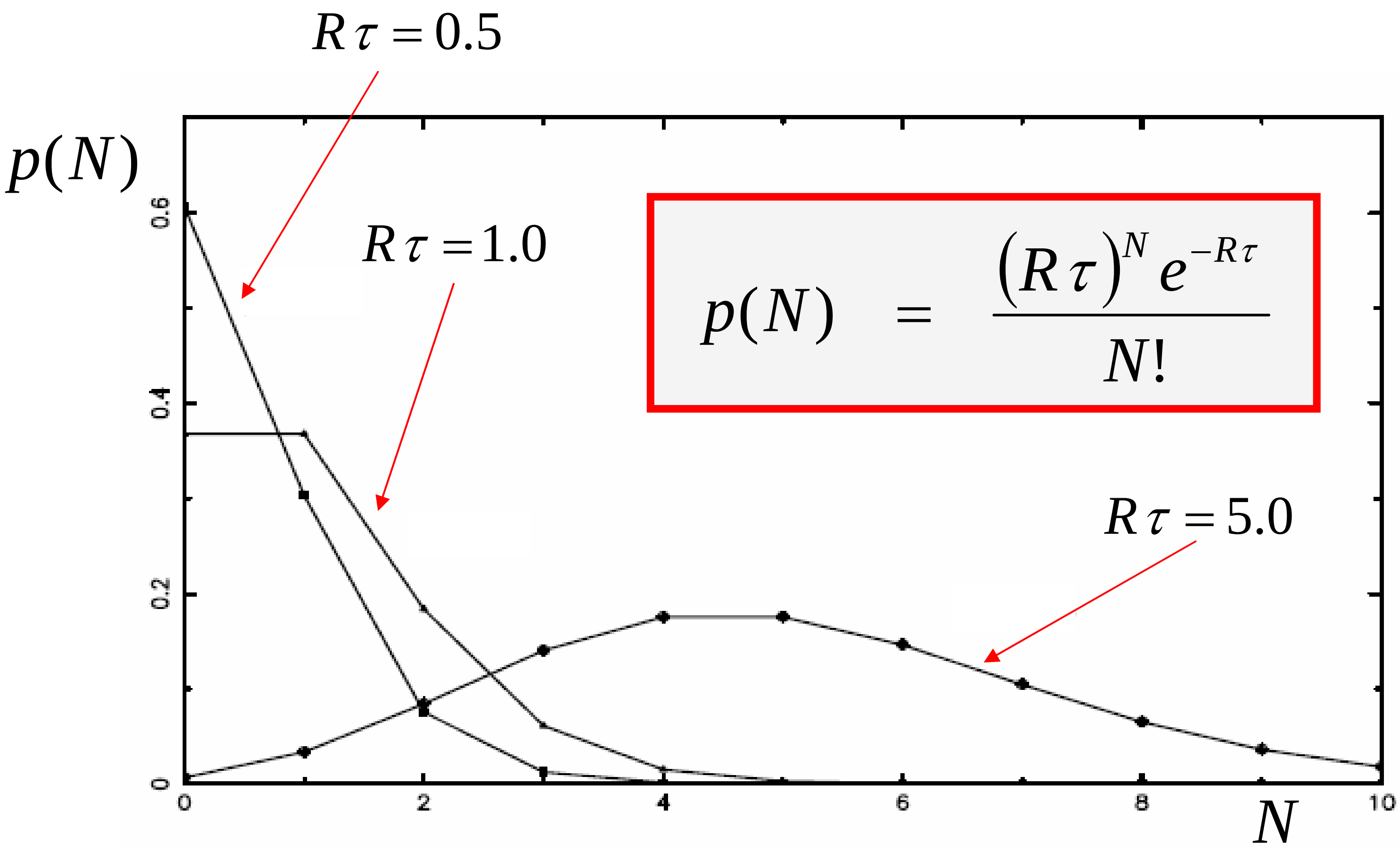
If we made a series of observations, each of time τ seconds, we wouldn't expect to receive $\langle N \rangle$ photons every time, but the average number of counts should equal $\langle N \rangle = R\tau$

(in fact this is how we can estimate the value of the rate R *)*

Given the two Poisson axioms, we can show (see Section 2) that the probability of receiving N photons in time τ is given by

$$\boxed{p(N) = \frac{(R\tau)^N e^{-R\tau}}{N!}} \qquad (1.2)$$

Poisson statistics



As $R\tau$ increases, the shape of the Poisson distribution becomes more symmetrical

(it tends to a **normal**, or **Gaussian**, distribution – see Section 2)

Poisson statistics

We can define the ***variance*** of N, which is a measure of the ***spread*** in the Poisson distribution:

$$\boxed{\mathrm{var}(N) = \sigma^2 = E\left\{\left[N - E(N)\right]^2\right\}} \qquad (1.3)$$

For a Poisson distribution, we will show in Section 2 that the variance of N is

$$\boxed{\mathrm{var}(N) = R\tau} \qquad (1.4)$$

and the ***standard deviation*** of N is $\sigma = \sqrt{R\tau}$ (1.5)

In practice we usually only observe for ***one*** period of (say) τ seconds, during which time we receive (say) a count of N_{obs} photons.

We estimate the arrival rate as

$$\boxed{\hat{R} = N_{\text{obs}} / \tau} \qquad (1.6)$$

The hat symbol here denotes that we are defining an *estimator* of R

We take N_{obs} as our 'best' estimate for $\langle N \rangle$ with error $\sqrt{N_{\text{obs}}}$

i.e we quote our experimental result for the number count of photons in time interval τ as

$$\boxed{N_{\text{obs}} \pm \sqrt{N_{\text{obs}}}} \qquad (1.7)$$

In the optical window it is still commonplace to convert photon counts to apparent magnitudes.

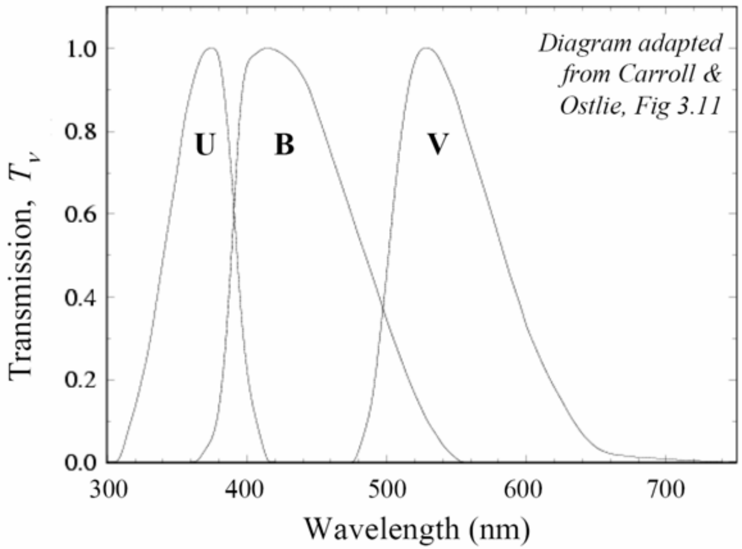
$$\boxed{m_1 - m_2 = -2.5\log_{10}\frac{N_1}{N_2}} \qquad (1.8)$$

Usually we measure magnitudes through a **filter**, which transmits only over a small range of frequencies

The **Johnson System** is a set of standard filters, from the near ultraviolet to the infrared:

U B V R I J H K L M N

$\longrightarrow$ Increasing wavelength



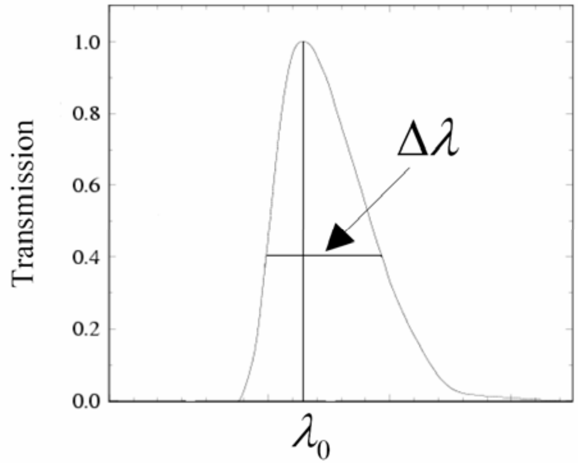
The **transmission function**, T, defines the fraction of light transmitted by the filter as a function of frequency (or wavelength)

For each Johnson filter, T peaks at some wavelength λ_0 , and has a characteristic width $\Delta\lambda$

$$m_{\mathbf{FILTER}} = -2.5\log_{10}\left(\int_0^\infty F_\nu T_{\mathbf{FILTER}}(\nu)\,d\nu\right) + C_{\mathbf{FILTER}}$$

In the optical window it is still commonplace to convert photon counts to apparent magnitudes.

$$\boxed{m_1 - m_2 = -2.5 \log_{10} \frac{N_1}{N_2}} \qquad (1.8)$$



bolometric apparent magnitude : $T(\nu) = 1$ at **all** frequencies

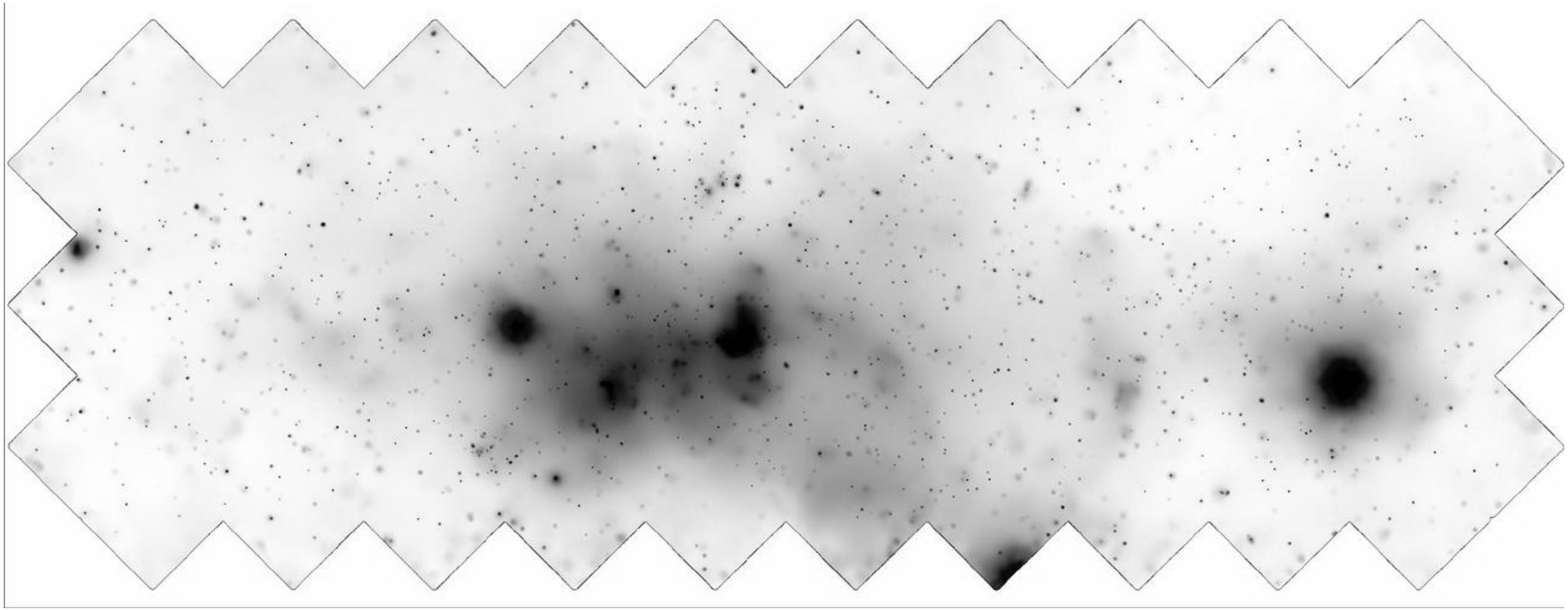
Example: **UBV** magnitudes		
Filter	λ_0 (nm)	$\Delta\lambda$ (nm)
$m_U \equiv \mathbf{U}$	365	68
$m_B \equiv \mathbf{B}$	440	98
$m_V \equiv \mathbf{V}$	550	89

Extracting accurate magnitudes (with uncertainties) requires:

- careful calibration of the CCD response function (see ADA II)

CCD imaging capability now extending into X-ray

e.g. ACIS on Chandra
CCD array with 0.5 arcsec resolution



Mosaic image of the Galactic Centre

ACIS is an imaging spectrometer.

Across the E-M spectrum we use highly *dispersed* light to probe in detail the frequency dependence of the incoming radation.

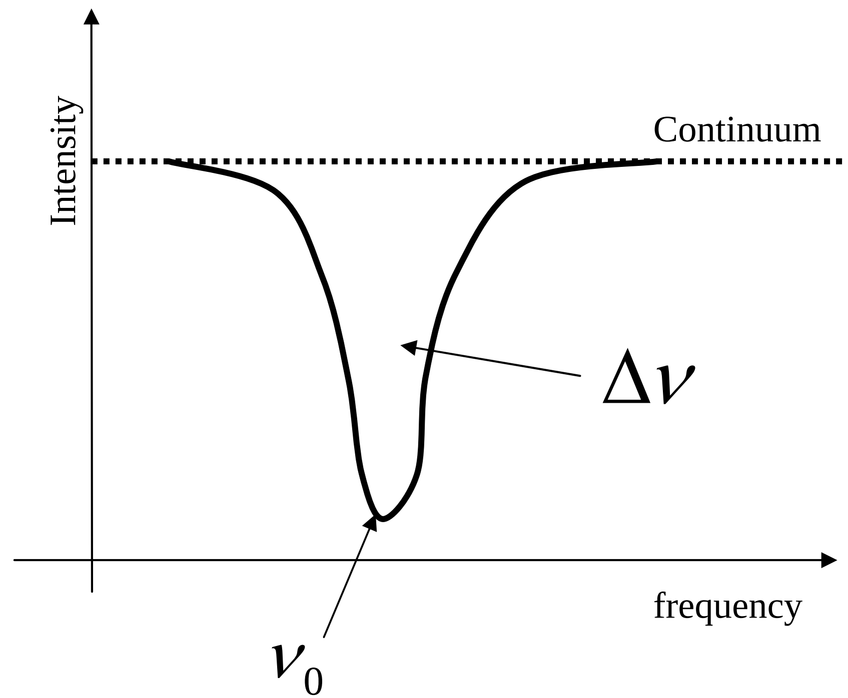
Light dispersed by a diffraction grating

Spectral Resolving Power

$$\boxed{R = \frac{\lambda}{\Delta\lambda} = N n} \qquad (1.9)$$

Number of lines

Order of diffraction pattern



Schematic diagram of an absorption line

CRIRES on VLT:

$$R \sim 10^5$$

Spectroscopy is crucial to much of astrophysics

From analysis of spectral lines we can learn about:-

Characteristic		from
1.	Chemical elements	frequency ν_0
2.	Chemical abundances	intensity
3.	Bulk velocity (i.e. velocity of atmosphere as a whole)	frequency ν_0
4.	Temperature, pressure, gravity	line width $\Delta\nu$
5.	Spread of velocities	line width $\Delta\nu$
6.	Magnetic and electric field	'fine structure' in lines (e.g. Zeeman splitting)

See ADA II for much more on *how* we extract this information

In addition to direct imaging and imaging spectroscopy, much astronomical data is collected or processed as **Fourier components**.

This is the case in radio astronomy and (interestingly) in high energy astronomy

Key elements of a radio telescope

From A2 Observational Astrophysics

Antenna (e.g. dish or dipole - see A1Y)

- chooses direction of observation
- collects radiation
- converts radiation to AC signal

Receiver (this is the 'detector')

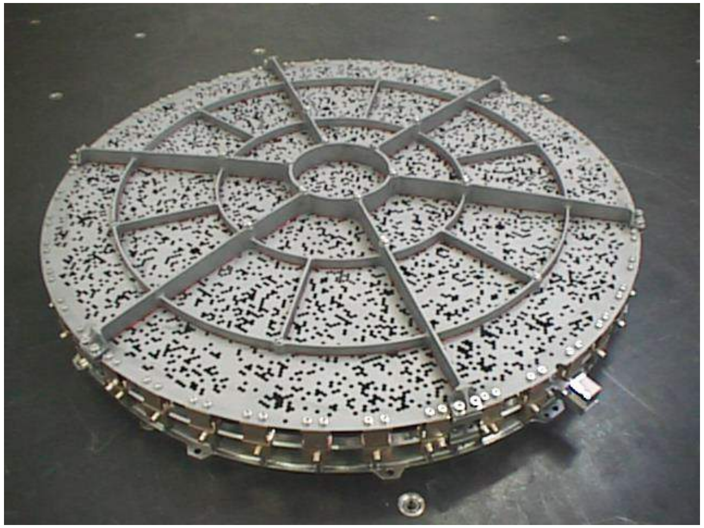
- amplifies the signal (by a factor known as the **gain**)
- selects frequency and bandwidth (compare e.g. optical filters)
- processes and records signal

So the data consist of a Fourier decomposition of the signal.

In high energy astronomy the problem is how to focus highly energetic photons.

One solution is to use a **coded mask**:

- Mask casts a 'shadow' on the detector.
- As the mask moves or rotates, the shadow pattern changes.
- Can use Fourier methods to reconstruct, from changes in the shadow pattern, where on the sky the photons came from. *(See ADA II)*



Coded mask of X-ray monitor instrument on INTEGRAL satellite

Many examples of this technology now in orbit:

e.g. RHESSI – Ramaty High Energy Solar Spectroscopic Imager

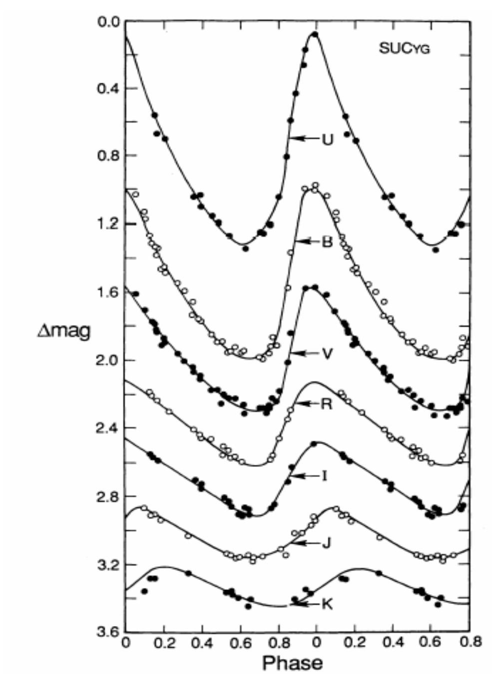
SWIFT – satellite to observe GRBs in X-rays and UV/Optical

INTEGRAL – International Gamma Ray Astrophysics Laboratory

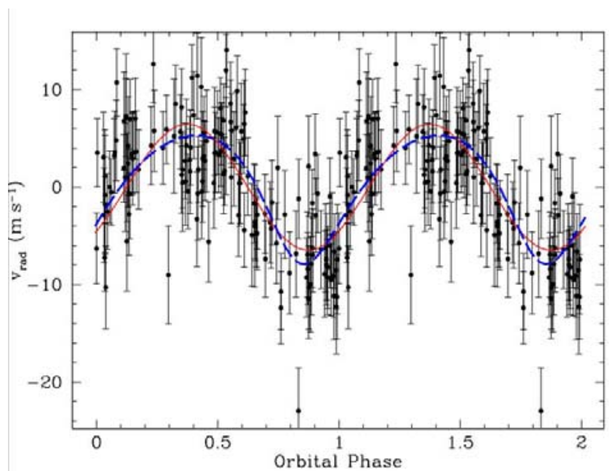
Our analysis of astronomical data (i.e. photon counts, magnitudes, spectra) often involves looking for patterns in time:

Examples

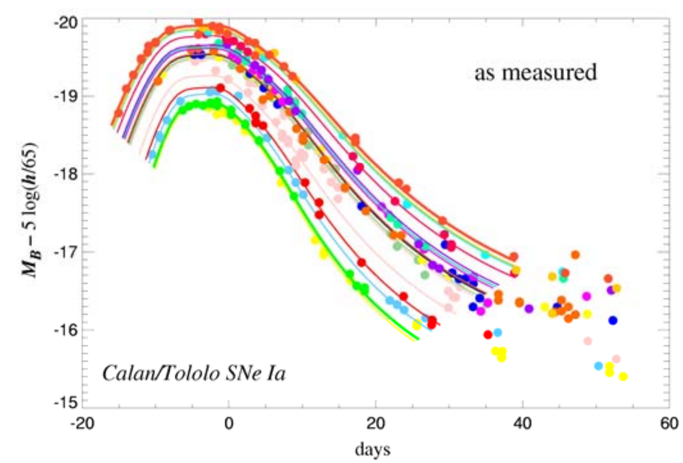
- periodic variable stars (e.g. Cepheids, RR Lyraes, eclipsing binaries)
- extra-solar planets
- transient events (e.g. GRBs, supernovae, gravitational wave sources?)



Cepheid light curves



ESP velocity curves



Type Ia supernova light curves

How do we identify and measure periodicity or transient phenomena?...

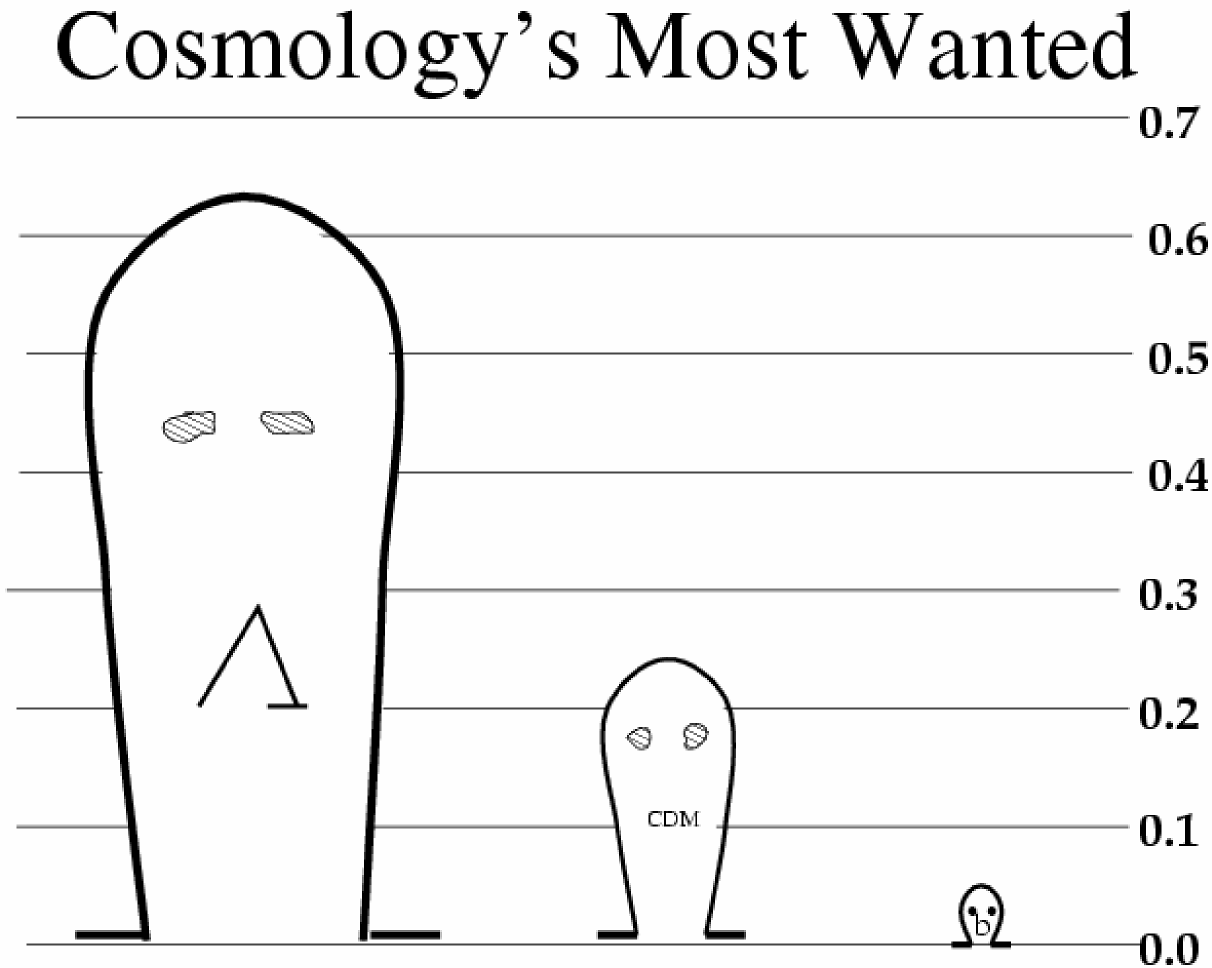
What do we do with all this astronomical data?

- We use it to test models, make inferences about parameters.
- We need good data analysis methods to make this process:
  - objective — *same data, same analysis method $\Rightarrow$ same results*
  - quantitative — *our data analysis should yield 'hard numbers' + uncertainties*
  - reliable — *not good if parameter estimates very sensitive to our assumptions; estimated uncertainties should be realistic*
  - informative — *we want to constrain physically meaningful parameters; our data analysis should help us understand "what is going on"*
  - predictive. — *the results of our data analysis should help us to make predictions with our models: i.e. future observations that could be made to better test the models.*

Some of these issues will be left to M-level Stats Astro course, but it's worth keeping them in the back of our mind.

Λ CDM

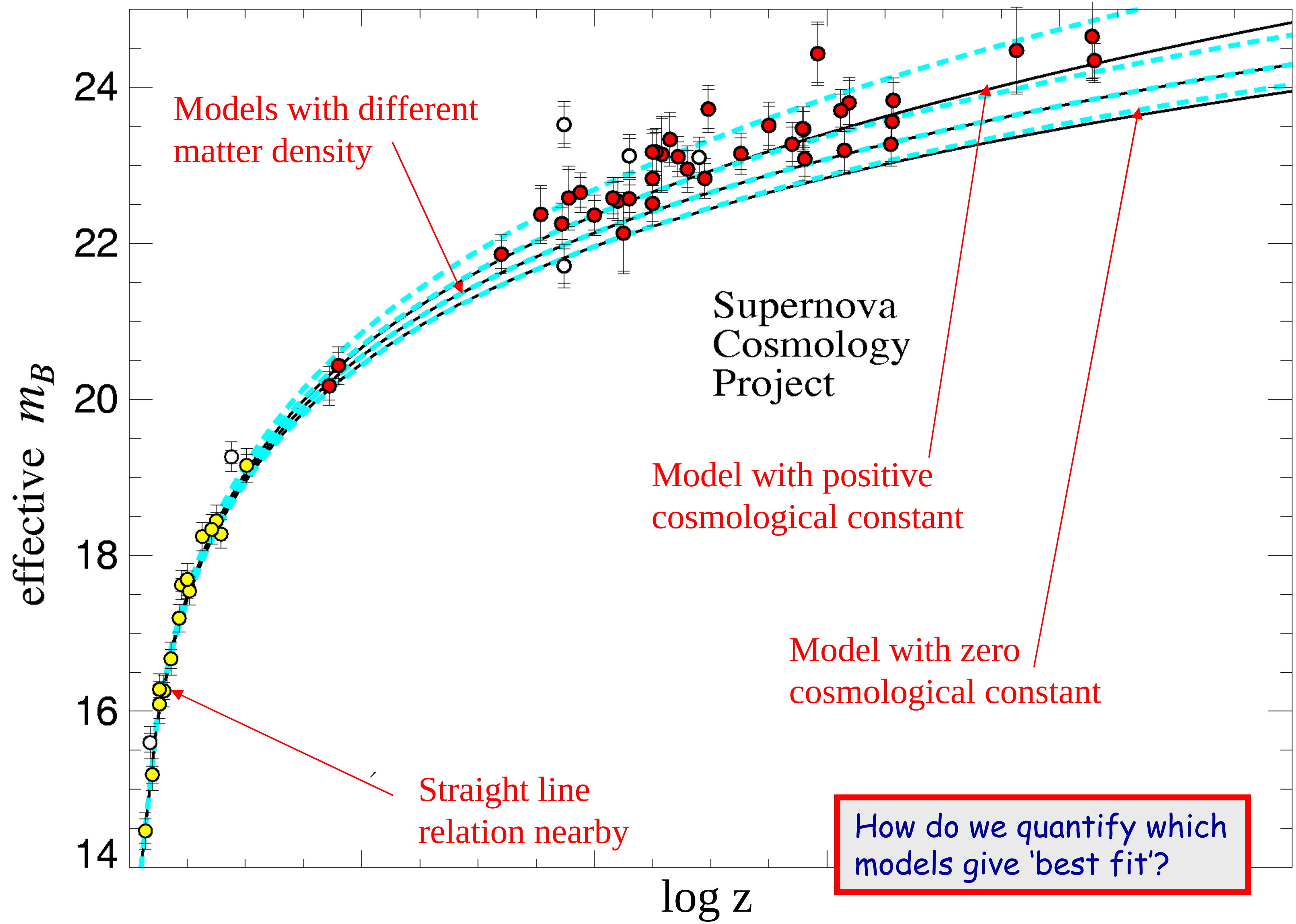
Figure 3. A line up of cosmological culprits Ω_Λ is the big shot controlling the Universe. He's going to make it blow up. Ω_{CDM} would like to make the Universe collapse but can't compete with Ω_Λ. Ω_b just follows Ω_{CDM} around. Like all dangerous criminals, one can never be sure of Ω_Λ until he is behind bars. The CMB police is being beefed up. Hundreds of heroic CMB observers are now planning his capture.



Ω_Λ	Ω_{CDM}	Ω_b
cosmological constant energy of the vacuum He never clumps His evil plan is to blow up the Universe	cold dark matter He likes to clump but has never been detected directly His evil plan is to make the Universe collapse	normal baryonic matter a pawn in the cosmic game who just follows CDM around. He thinks he's a complex life form but is really just a bunch of hydrogen

From Lineweaver (1998)

Hubble diagram of distant Type Ia supernovae



How do we quantify which models give 'best fit'?