Astronomy A345H

Astronomical Data Analysis I: Example Sheet 4

Verifying properties of Fourier Series and Fourier Transforms

1. Verify the orthogonality properties of sine and cosine functions, as stated in Section 5.1:

$$\int_{-\pi}^{\pi} \sin mx \sin nx dx = \pi\delta_{mn} \qquad \int_{-\pi}^{\pi} \cos mx \cos nx dx = \pi\delta_{mn} \qquad \int_{-\pi}^{\pi} \sin mx \cos nx dx = 0$$

2. Consider the time domain function $h(t)$ with Fourier transform $H(f)$. Show that

   a) if $h(t)$ is real, then $H(-f) = [H(f)]^*$

   b) if $h(t)$ is imaginary, then $H(-f) = -[H(f)]^*$

   c) if $h(t)$ is an even function, then $H(f)$ is even

   d) if $h(t)$ is an odd function, then $H(f)$ is odd

   e) if $h(t)$ is real and even, then $H(f)$ is real and even

   f) if $h(t)$ is real and odd, then $H(f)$ is imaginary and odd

   g) if $h(t)$ is imaginary and even, then $H(f)$ is imaginary and even

   h) if $h(t)$ is imaginary and odd, then $H(f)$ is real and odd

3. Show that $h(at)$ has Fourier transform $\frac{1}{|a|} H(f/a)$

4. Show that $\frac{1}{|b|} h(t/b)$ has Fourier transform $H(bf)$

5. Show that $h(t - t_0)$ has Fourier transform $H(f)e^{2\pi i f t_0}$

6. Show that $h(t)e^{-2\pi i f_0 t}$ has Fourier transform $H(f - f_0)$

7. We define the **convolution function** $(g * h)(t) = \int_{-\infty}^{\infty} g(s)\, h(t-s)\, ds$. Show from first principles that the Fourier transform of $(g * h)(t)$ is equal to the product of the Fourier transform of $g(t)$ and $h(t)$.

8. We define the **correlation function** $\mathrm{Corr}(g,h) = \int_{-\infty}^{\infty} g(s+t)\, h(s)\, ds$. Show from first principles that the Fourier transform of the correlation function is equal to the Fourier transform of $g(t)$ multiplied by the complex conjugate of the Fourier transform of $h(t)$.