Astronomy A345H
Astronomical Data Analysis I: Example Sheet 1

1. The pdf of the luminosity, L, of brightest cluster galaxies can be modelled to be of exponential form, viz:
$$p(L) = A \exp[-\frac{L - L_*}{\Delta}], L > L_*, \Delta > 0$$
where A, L_* and Δ are constant parameters.

   - Determine the value of A which makes $p(L)$ a properly normalised pdf, i.e. one which integrates to unity

   - Using this pdf, determine the mean and variance of L

     Hint: use integration by parts, or use the identity
     $$\int_0^\infty x^{n-1} e^{-x} dx = (n-1)!$$

   - Derive an expression for the cdf of L, $P(L)$

   - Using the cdf, calculate the median value of L. Is the median greater than, equal to or less than the mean?

2. The random variables x and y have joint pdf given by:-
$$p(x, y) = e^{-y}, \quad 0 < x < y < \infty, \quad \text{zero elsewhere}$$

   (a) Determine the marginal pdf of x and the marginal pdf of y

   (b) Determine the conditional pdf of x given y and y given x

   (c) Are x and y independent random variables?

3. The random variables, x and y, have joint pdf given by:-
$$p(x, y) = 12xy(1 - y), \quad 0 < x < 1, 0 < y < 1, \quad \text{zero elsewhere}$$

   Show that x and y are statistically independent.

4. The fraction, x, of the surface of a star covered in starspots is modelled as a random variable with pdf (with k constant)
$$p(x) = \frac{k}{\sqrt{x(1 - x)}}, \qquad 0 < x < 1$$

   (a) Determine k so that $p(x)$ is properly normalised

   (b) Find the expected fraction of the surface covered in starspots

   (c) What is the probability that the fraction covered is less than 25%?

5. By writing down the explicit integral expressions for the expectation and variance of a random variable x, in terms of its pdf $p(x)$, show that the variance of x satisfies

$$\text{var}(x) = E(x^2) - [E(x)]^2$$

6. Prove the results given in the table of Section 2, which lists the mean and variance of the Poisson, uniform and Normal pdfs.