Astronomy 345H, Session 2006-07

Astronomical Data Analysis I

Guidelines on what's examinable

<u>Section 1</u> you should be able to:

- State the axioms satisfied by a Poisson distribution
- Write down eq. 1.2 and explain the meaning of the rate parameter R
- Sketch the Poisson distribution as a function of N for different values of R
- State and prove eq. 1.3
- Know how to estimate R (via eq. 1.6)
- Explain qualitatively what we can learn from analysis of spectral lines
- List the key elements of a radio telescope
- Describe briefly how a coded mask detector works
- Give reasons why we need good data analysis methods

<u>Section 2</u> you should be able to:

- Explain, with examples, the difference between statistical and systematic errors
- Define probability as a mathematical basis for plausible reasoning
- Know the basic rules, and notation, for combining probabilities (eqs. 2.1 and 2.2)
- Understand the definition of a probability density function, and compute pdf normalisations, marginal and conditional pdfs for simple examples
- Know the definition of statistical independence (eq. 2.7) in terms of pdfs
- Define the pdf of a uniform and Gaussian pdf
- Define and compute the cumulative distribution function for simple pdfs
- Know the definition of the mean, mean square, variance and nth moment of a pdf
- Compute the mean and variance of Poisson, uniform and normal pdfs
- Define the median and mode of a distribution
- Explain qualitatively the basis of a frequentist definition of probability
- Define the sample mean, state that it is unbiased and know eq. 2.21
- State the central limit theorem and explain its importance
- Write down Bayes' formula, identifying its different terms and explaining what they represent

<u>Section 3</u> you should be able to:

- Write down the 5 parameters that characterise the bivariate normal distribution, and sketch its pdf
- Define the covariance of two variables (eq. 3.4) and its relation to the correlation coefficient
- Sketch isoprobability contours for the bivariate normal pdf for different values of ρ
- Explain the basic idea behind ordinary least squares, writing down eq. 3.8 and 3.9
- Know that LS estimators are unbiased
- Know that by careful choice of our independent variables we can make the LS estimators independent
- Explain the difference between ordinary and weighted least squares, writing down eq. 3.16
- State that we can generalise least squares to errors on both variables, general linear models, and non-linear models.
- Define what is meant by the likelihood function and explain how it is constructed
- Explain what is meant by the principle of maximum likelihood
- Show, for Gaussian independent errors, that ML and weighted LS estimators are identical

- Explain the basic ideas behind a chi-squared goodness of fit test, writing down eq. 3.33
- Sketch the chi-squared pdf and know its mean value and variance
- Carry out a simple chi-squared goodness of fit test on given data
- Explain what is meant by a P-value, and how it is interpreted
- Explain qualitatively how the method of steepest ascent/descent work

<u>Section 4</u> you should be able to:

- Explain what is meant by Monte Carlo methods
- Describe qualitatively the tests that can be applied to pseudo-random number generators
- Write down eqs. 4.3 and 4.4 and carry out simple variable transformations of pdfs
- Explain how the probability integral transform can be used to generate random numbers
- Explain how rejection sampling works, and how it can be optimised
- Describe briefly the steps involved in implementing the Metropolis algorithm, and why it is such a powerful method for exploring high-dimensional pdfs

<u>Section 5</u> you should be able to:

- Write down eq. 5.1, the Fourier series of a general function
- Write down the orthogonality properties of sin and cos
- Write down eq. 5.5, the Fourier series in complex form
- Define the Fourier transform (eq. 5.8) and its inverse (eq. 5.9)
- Prove the FT properties given in Section 5.3, including eqs. 5.13 – 5.16
- State and prove the convolution theorem, eq. 5.17
- State and prove the correlation theorem, eq. 5.20
- Define what is meant by the auto-correlation of a function, and its relation to the power spectral density
- State Parseval's theorem, eq. 5.22
- Define what is meant by the one-sided power spectral density
- Define the discrete Fourier transform, writing down eq. 5.27, and its inverse, writing down eq. 5.32
- Show that the discrete FT can be re-written in the form of eq. 5.31
- Show how the Fast Fourier Transform is defined, establishing eq. 5.37 from eq. 5.34, and explaining why the calculation of the FFT reduces an N squared process to an NlogN process

<u>Section 6</u> you should be able to:

- Explain what is meant by a signal being bandwidth limited, defining its critical (Nyquist) frequency and Nyquist rate
- State the Nyquist-Shannon sampling theorem and explain why it is important
- Write down eq. 6.5, the signal perfectly reconstructed from its discretely sampled values
- Explain what is meant by aliasing, sketching its effect (reproducing the diagrams in Sect 6.2)
- Describe qualitatively methods to combat aliasing
- Explain qualitatively how an analog to digital converter works

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March 2007