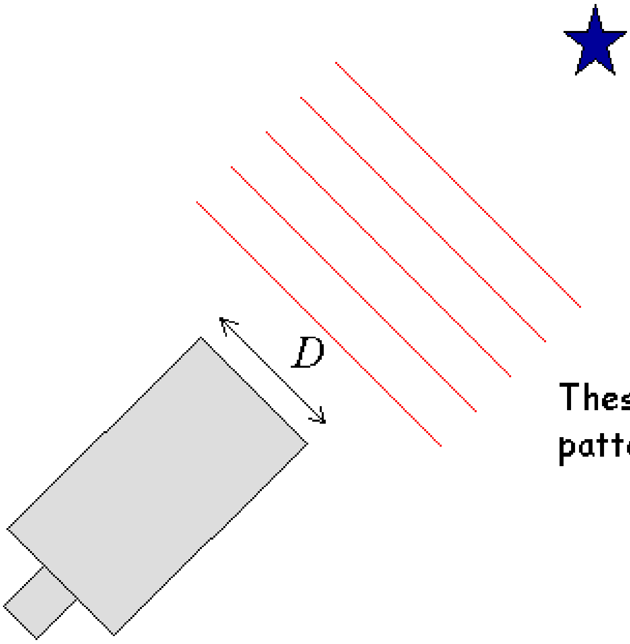
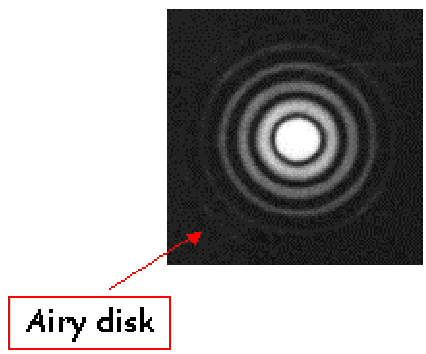
7. Resolving Power and Interferometry



Light from a point source star arrives at a telescope aperture (of diameter D) as a series of **plane waves**

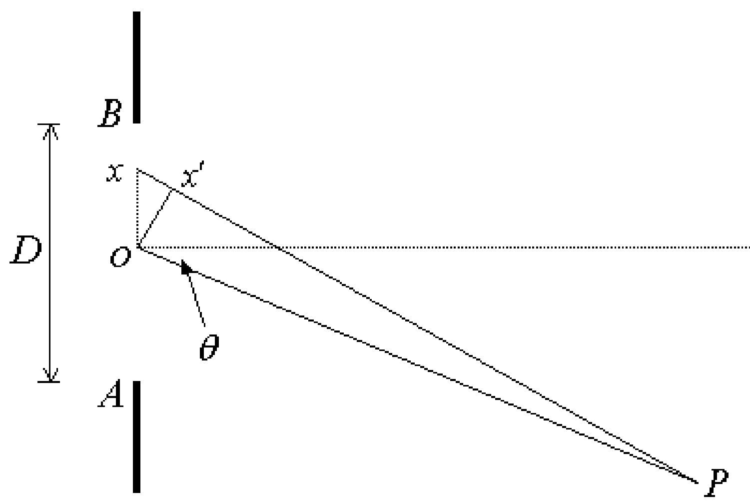
These are **diffracted**, producing an intensity pattern first analysed theoretically by **Airy**



We can work out how the width of the Airy disk pattern depends on the size of the telescope aperture and the wavelength of the incident light.

Simplified 1-d analysis:

Integrate diffraction pattern across a 'slit' of width D



Consider light observed at P, at angle θ to the axis.

Path difference between light from O and x: $xx' = x\sin\theta$

Corresponding phase difference:

$$\phi(x) = \frac{2\pi}{\lambda} x \sin\theta \quad \approx \quad \frac{2\pi\,\theta}{\lambda} x \qquad \text{for small } \theta \qquad (7.1)$$

Suppose at P wave from O has unit amplitude

Wave from x has amplitude $\psi(x) = e^{i\phi} = e^{i\frac{2\pi\theta}{\lambda}x}$ (7.2)

By the principle of superposition, **total** diffraction pattern at P obtained by integrating eq. (7.2) from $x = -D/2$ to $x = D/2$

$$\psi_{\text{tot}}(\theta) = \int_{-D/2}^{D/2} e^{i\frac{2\pi\theta}{\lambda}x}\,dx \quad (7.3)$$

Integrating gives $\psi_{\text{tot}}(\theta) = \frac{\lambda}{2\pi i\theta}\left[e^{i\pi\theta D/\lambda} - e^{-i\pi\theta D/\lambda}\right]$ (7.4)

which we can rewrite as

$$\psi_{\text{tot}}(\theta) = \frac{\lambda}{\pi\theta}\frac{1}{2i}\left[e^{i\pi\theta D/\lambda} - e^{-i\pi\theta D/\lambda}\right] \quad (7.5)$$

or as

$$\psi_{\text{tot}}(\theta) = \frac{\lambda}{\pi\theta}\sin\left(\frac{\pi\theta D}{\lambda}\right) \quad (7.6)$$

This in turn can be rewritten as $\psi_{\text{tot}}(\theta) = D\,\frac{\sin\left(\frac{\pi\theta D}{\lambda}\right)}{\frac{\pi\theta D}{\lambda}}$ (7.7)

and the intensity:

$$I(\theta) = \psi_{\text{tot}}\psi^{*}_{\text{tot}} \Rightarrow \quad I(\theta) = D^2\,\frac{\sin^2\left(\frac{\pi\theta D}{\lambda}\right)}{\left(\frac{\pi\theta D}{\lambda}\right)^2} \quad (7.8)$$

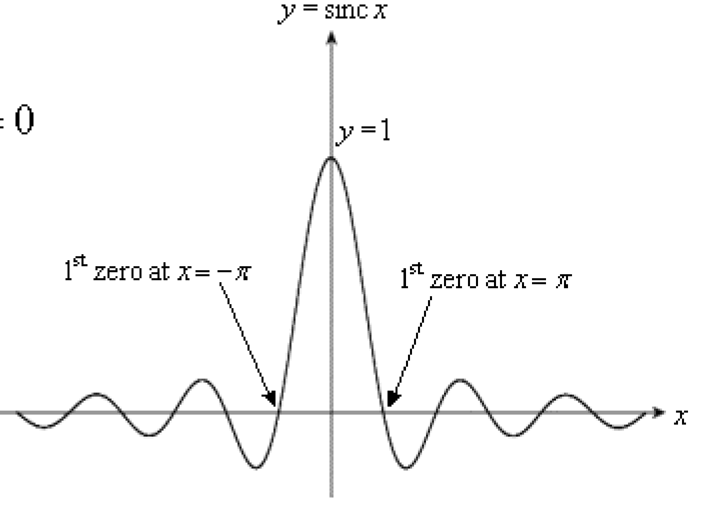
We can write eq. 7.8 as

$$I(\theta) = I_0 \,\mathrm{sinc}^2\left(\frac{\pi \theta D}{\lambda}\right) \qquad (7.9)$$

where I_0 is the intensity at $\theta = 0$ and $\mathrm{sinc}\, x \equiv \dfrac{\sin x}{x}$

The sinc function occurs frequently in optics

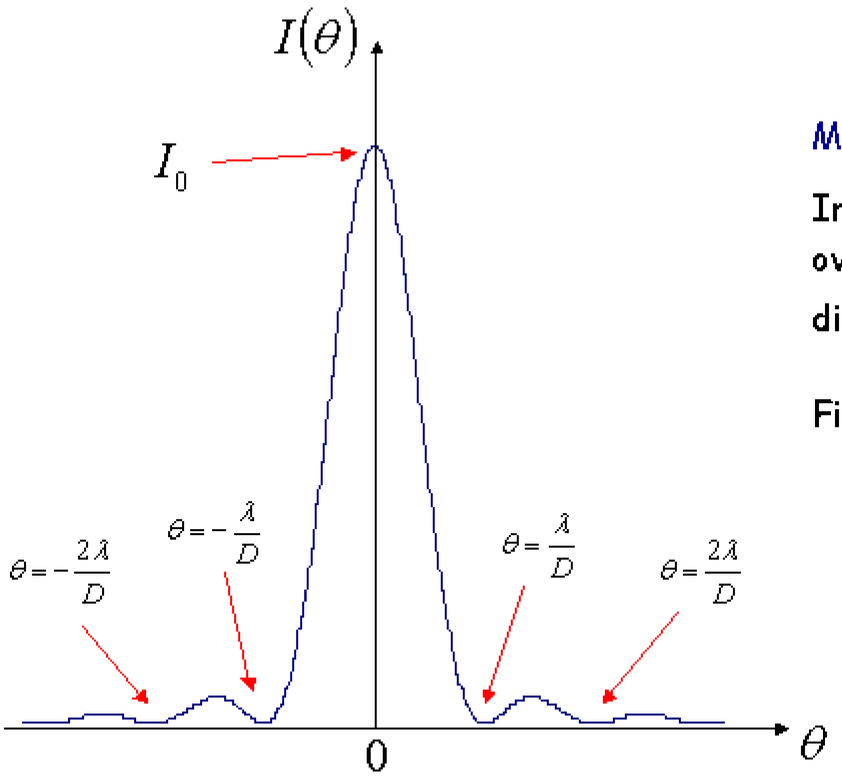
The function has a maximum at $x = 0$ and the zeros occur at $x = \pm m\pi$ for positive integer m



For our simplified 1-d analysis:

Intensity pattern has minima at $\dfrac{\pi\theta D}{\lambda} = \pm m\pi$ i.e. at

$$\theta = \pm m \frac{\lambda}{D} \qquad (7.10)$$



More exact 2-d analysis:

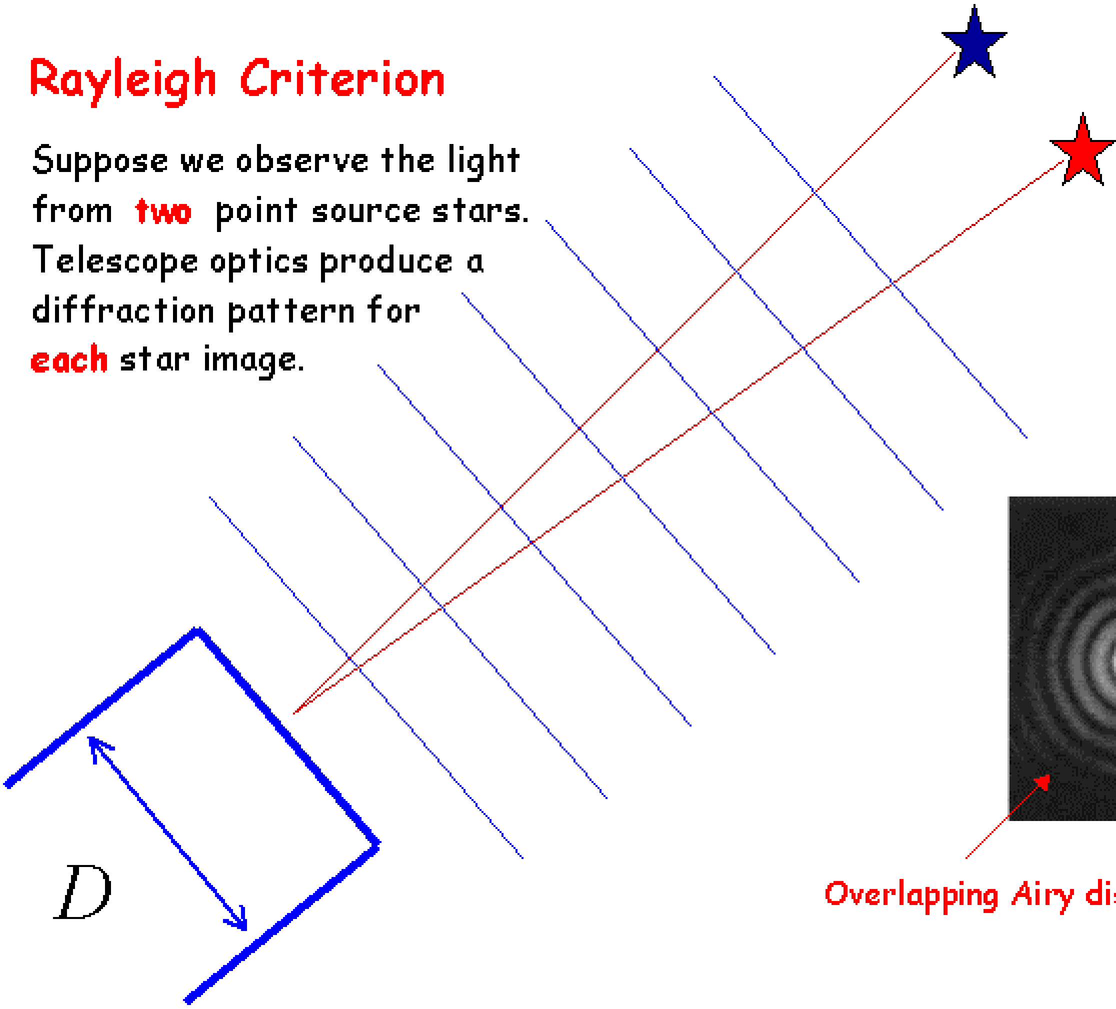
Integrate diffraction pattern over **circular aperture** of diameter D

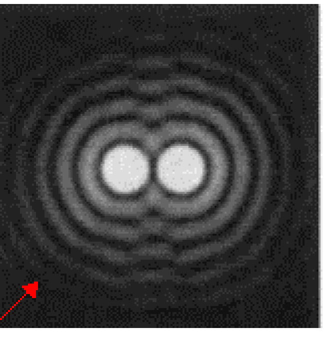
First minima at

$$\theta \approx \pm \frac{1.22\lambda}{D} \qquad (7.11)$$

Rayleigh Criterion

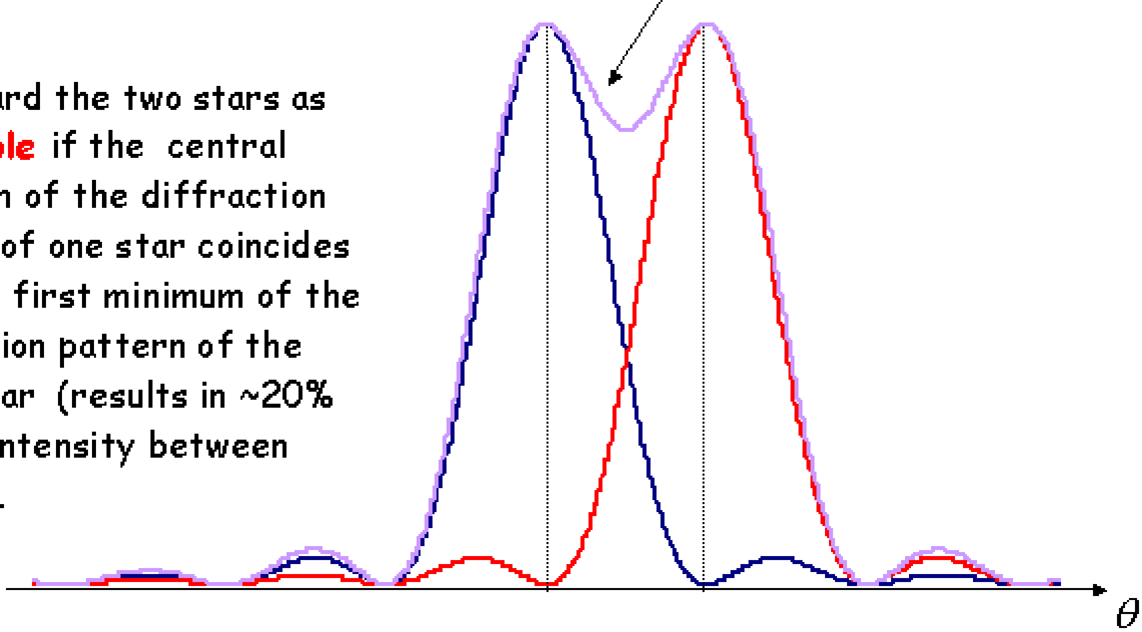
Suppose we observe the light from **two** point source stars. Telescope optics produce a diffraction pattern for **each** star image.





Overlapping Airy disks

We regard the two stars as **resolvable** if the central maximum of the diffraction pattern of one star coincides with the first minimum of the diffraction pattern of the other star (results in ~20% drop in intensity between maxima).



From eq. (7.11), the two stars are resolvable if their angular separation satisfies:

$$\theta_{\min} = \frac{1.22\,\lambda}{D} \qquad (7.12)$$

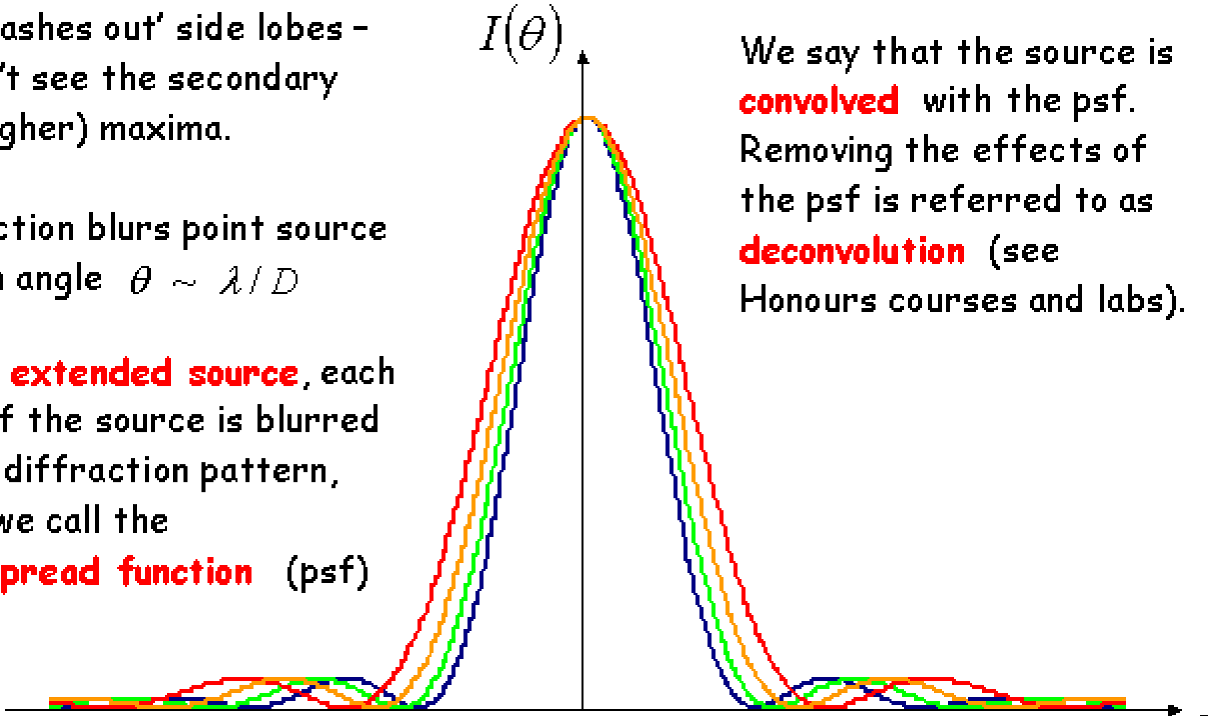
in radians

If we observe a point source which is not **monochromatic** (e.g. any star), the observed intensity is the sum (integral) of the intensity pattern at each observed wavelength

This 'washes out' side lobes – we don't see the secondary (and higher) maxima.

Diffraction blurs point source over an angle $\theta \sim \lambda / D$

For an **extended source**, each point of the source is blurred by the diffraction pattern, which we call the **point spread function** (psf)



We say that the source is **convolved** with the psf. Removing the effects of the psf is referred to as **deconvolution** (see Honours courses and labs).

Equation (7.12) defines the **theoretical angular resolving power** of a telescope.

If we can resolve features down to $\theta_{\min}$ we say that the telescope is **diffraction limited**.

e.g., for $\lambda = 550\,\mathrm{nm}$,

$$\theta_{\min} = \frac{0.14}{D[\mathrm{m}]} \text{ arcsec} \qquad (7.13)$$

For small (amateur) telescopes of aperture a few cm, $\theta_{\min}$ is even larger than the typical size of the **seeing disk** (due to atmospheric scintillation; see Section 5).

For ground-based optical telescopes with $D \geq 1\,\mathrm{m}$, on the other hand, we find that $\theta_{\min}$ is much ***smaller*** than the seeing disk. Hence, the theoretical angular resolving limit is never achieved, and we say that the telescope is **seeing limited**.

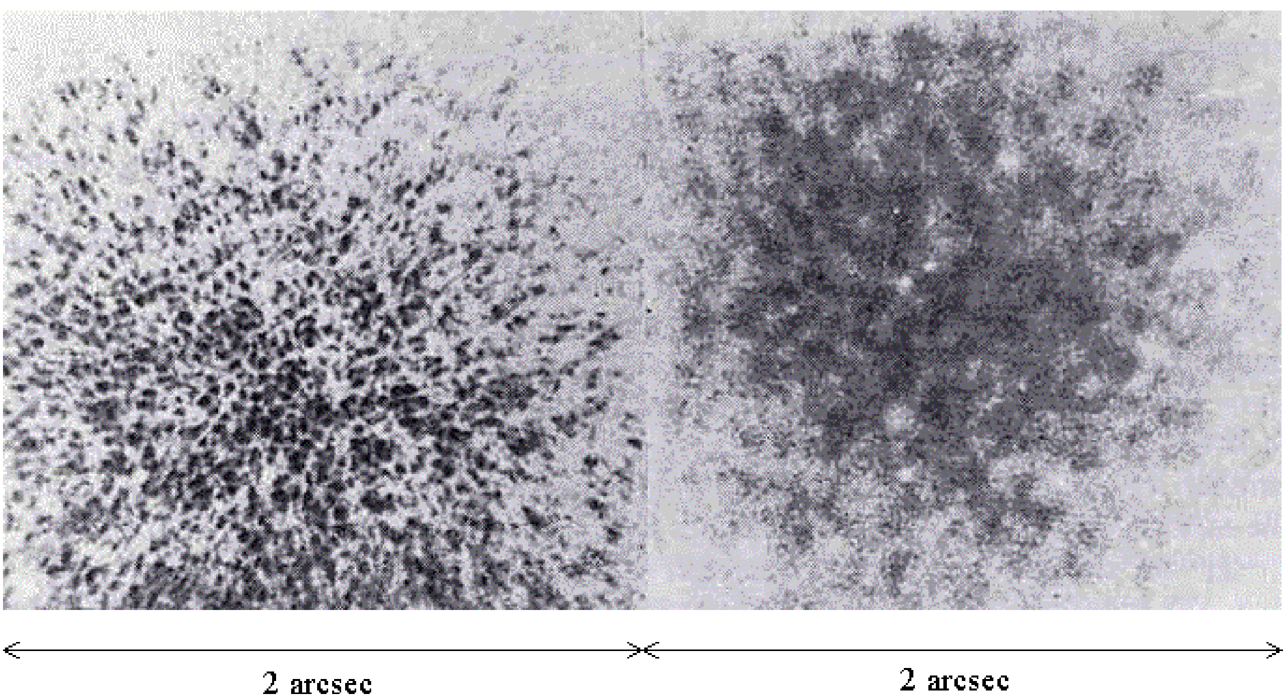
We can improve angular resolution via SPECKLE INTERFEROMETRY

- We saw in Section 5 that turbulence in the atmosphere causes **scintillation**, which smears out light into seeing disk.

- With a large telescope + a high gain detector we can collect enough photons from a bright (e.g. $m_V \sim 10$) source to get a good SNR from an exposure of only a few **milliseconds**.

- Such a short exposure time 'freezes' the atmospheric scintillation: our image is still produced by a pattern of hundreds of more/less dense cells of air – but these are **not moving**. 'Snapshot' of hundreds of light and dark spots - **SPECKLE PATTERN** over area roughly equal to the seeing disk.

- These spots are correlated - i.e. not totally random pattern. Fourier analysis of their pattern allows reconstruction of the original source intensity. **Can be used to measure stellar diameters**.



Speckle pattern from 0.02s exposure of point source star

Speckle pattern from 0.02s exposure of Betelgeuse (ang. diam. ~ 0.05 arcsec)