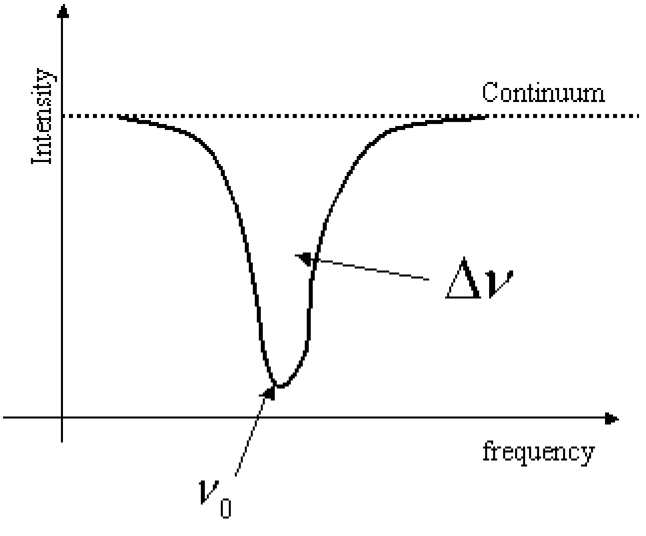
6. Spectral Techniques

Spectroscopy is the study of ***spectra*** – the quantitative measurement of **spectral lines**: their width, depth and shape.

We measure a **line profile** – a graph of the specific intensity, or flux density, of radiation received from a source as a function of frequency.



Schematic diagram of an absorption line

Often the **shape** of the line profile is simply characterised by a measure of its **width** (e.g. equivalent width – see A2 Theoretical Astrophysics).

Spectroscopy is of fundamental importance to **astrophysics** because it allows us to deduce many physical characteristics of planets, stars and galaxies – even though we observe them remotely, from enormous distances.

From analysis of spectral lines we can learn about:-

Characteristic	from
1. *Chemical elements*	frequency ν_0
2. *Chemical abundances*	intensity
3. Bulk velocity (i.e. velocity of atmosphere as a whole)	frequency ν_0
4. Temperature, pressure, gravity	line width $\Delta\nu$
5. Spread of velocities	line width $\Delta\nu$
6. *Magnetic and electric field*	'fine structure' in lines (e.g. Zeeman splitting)

We can collect light in a narrow frequency range using a **filter**

Effectiveness measured by **spectral resolving power**, R

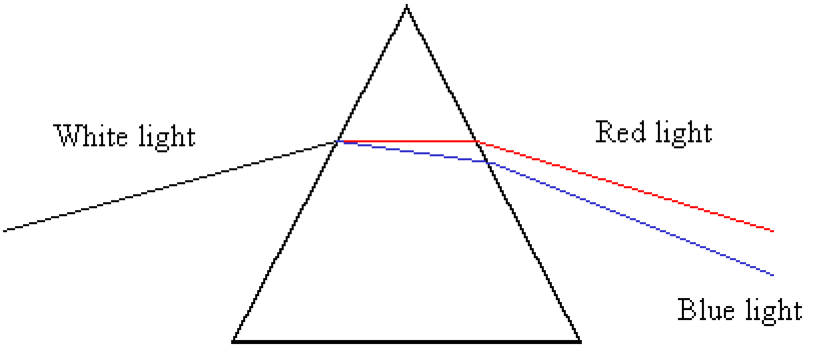
$$R = \frac{\nu_0}{\Delta\nu} = \frac{\lambda_0}{\Delta\lambda} \qquad (6.1)$$

Examples: Dye filter $R \sim 10 - 100$

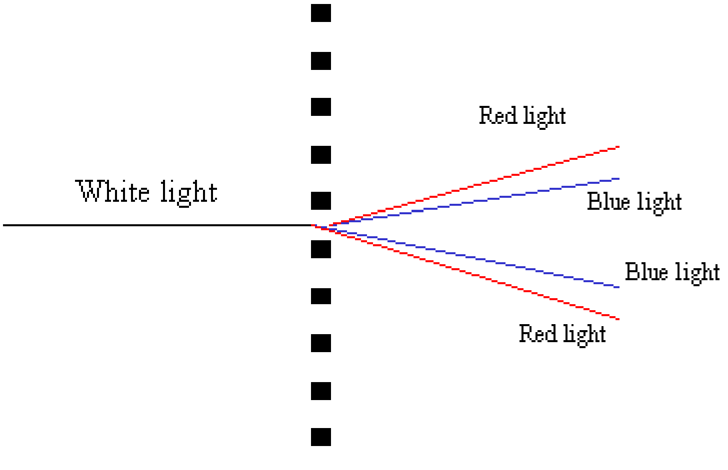
Interference filter $R \sim 10^4$

But we need $R \sim 10^5$ or higher to be useful (i.e. to be sensitive to Doppler shifts of a few km/s)

Much higher spectral resolving powers can be achieved using a **PRISM** or a **DIFFRACTION GRATING**



Note that a prism disperses blue light more strongly than red light, while for a diffraction grating red light is dispersed more strongly than blue light.



Example

The intermediate dispersion spectrograph (IDS) on the Isaac Newton Telescope in La Palma, is used to measure the rotation curve of an edge-on spiral galaxy. The IDS employs a diffraction grating with 1500 lines/mm to disperse the light, which is then focussed with a 0.5m focal length camera onto a CCD detector with a pixel size of 24 microns. When the IDS is operating at fist order, the Hα emission line profile for the spiral galaxy has a width of 20 pixels.

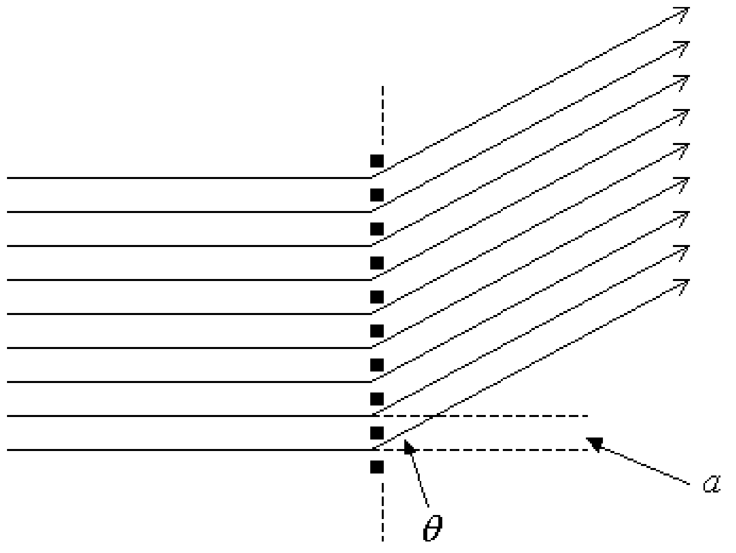
Use the above information to estimate the rotation speed of the galaxy in the outer part of its disk. (You may assume that the angle of deflection for the diffracted maxima is small)

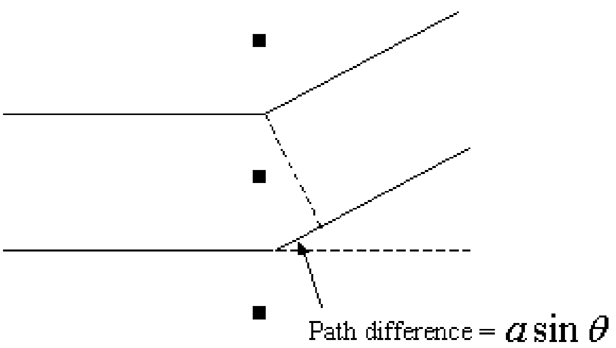
[Rest wavelength of Hα = 656.3nm]

For an **infinite diffraction grating** with light incident at right angles (normal incidence), the dispersed light has an intensity maximum when the path difference between adjacent light rays satisfies

$$a\sin\theta = n\lambda \qquad (6.2)$$

Constructive interference



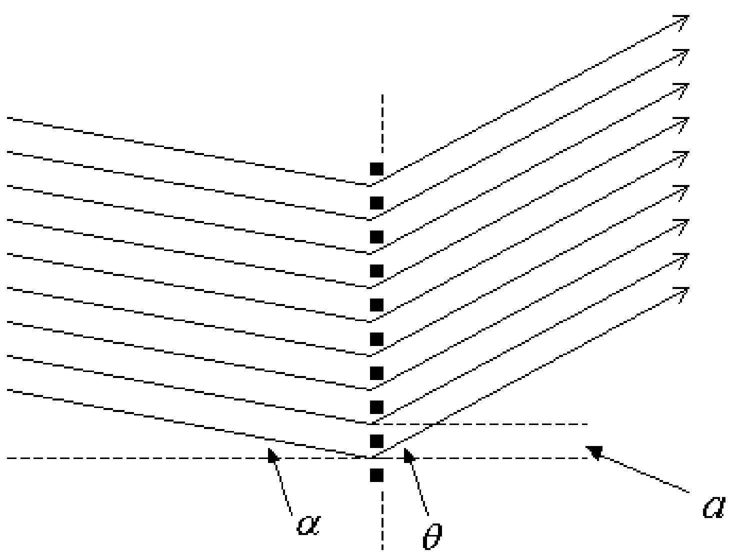


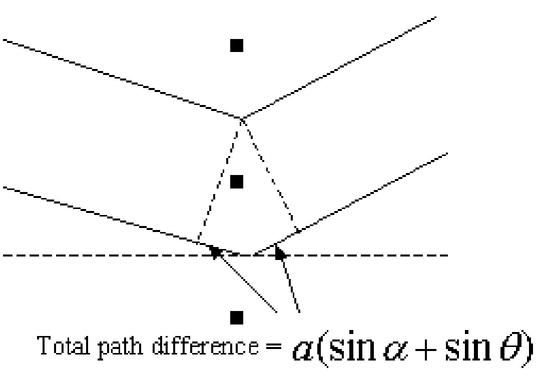
a = spacing between lines of grating

n = **order** of the maximum

If the light is incident at an oblique angle, α, maxima occur at path differences which satisfy:-

$$a(\sin\theta + \sin\alpha) = n\lambda \qquad (6.3)$$





To determine the **angular dispersion** (i.e. the spread in angular deflection corresponding to a given spread in wavelength) we differentiate (6.2) or (6.3) with respect to wavelength.

Differentiating $a\cos\theta \dfrac{d\theta}{d\lambda} = n$ (requires θ to be in radians)

Hence

$$\frac{d\theta}{d\lambda} = \frac{n}{a\cos\theta} \qquad (6.4)$$

Angular dispersion

We can achieve a high angular dispersion via:

- high order
- small a
- large θ

Suppose the dispersed light is focussed on a detector (e.g. a CCD) using a lens or mirror of focal length f

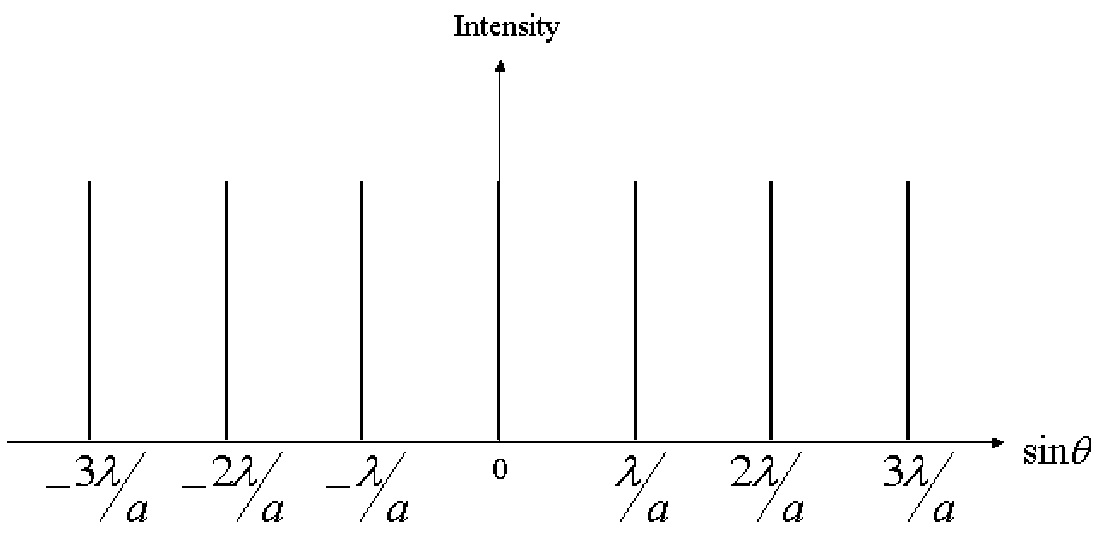
Then

$$dx = f\,d\theta = f\frac{d\theta}{d\lambda}d\lambda \qquad (6.5)$$

Linear separation of lines on detector (dx)

Actual separation of lines in spectrum ($d\lambda$)

Often also defined is the **Reciprocal Linear Dispersion (RLD)** $= \dfrac{d\lambda}{dx}$



Intensity profile from a **monochromatic** light source, of wavelength λ, diffracted by an infinite diffraction grating, is a series of infinitely thin peaks equally spaced in $\sin\theta$

In practice, of course, any grating is **finite** (i.e. only a finite number of lines is illuminated)

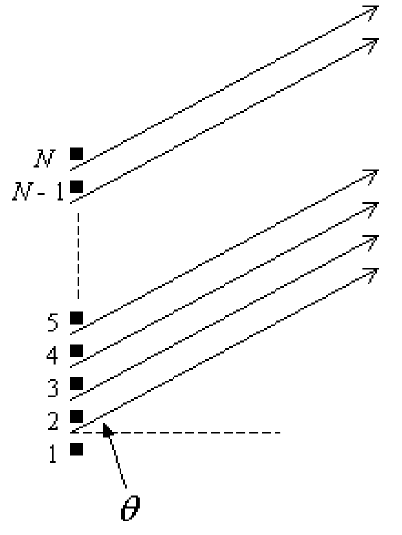
Finite diffraction grating

Consider grating with N rules.

For the m^{th} aperture,

Phase difference

$$\phi_m = m\frac{2\pi}{\lambda}a\sin\theta \quad = \quad m\phi \qquad (6.7)$$



By principle of superposition, total wave amplitude diffracted through an angle θ is (for an incident wave of unit amplitude)

$$\psi_{\text{tot}} \quad = \quad 1 + e^{i\phi} + e^{2i\phi} + \ldots + e^{i(N-1)\phi} \qquad (6.8)$$

(modelling the wave as **complex** simplifies the algebra)

Multiplying through by $e^{i\phi}$

$$e^{i\phi}\,\psi_{\text{tot}} \quad = \quad e^{i\phi} + e^{2i\phi} + \ldots + e^{iN\phi} \qquad (6.9)$$

Subtracting (6.7) from (6.6)

$$\left(1 - e^{i\phi}\right)\psi_{\text{tot}} \quad = \quad 1 - e^{iN\phi} \qquad (6.10)$$

i.e.

$$\psi_{\text{tot}} \quad = \quad \frac{1 - e^{iN\phi}}{1 - e^{i\phi}} \qquad (6.11)$$

We can rewrite the RHS as

$$\psi_{\text{tot}} = \frac{e^{iN\phi/2}\left(e^{iN\phi/2} - e^{-iN\phi/2}\right)}{e^{i\phi/2}\left(e^{i\phi/2} - e^{-i\phi/2}\right)} \qquad (6.12)$$

i.e.

$$\psi_{\text{tot}} = e^{i(N-1)\phi}\,\frac{\sin(N\phi/2)}{\sin(\phi/2)} \qquad (6.13)$$

using the result $\sin x = \frac{1}{2i}\left(e^{ix} - e^{-ix}\right)$

And the **Intensity** in direction θ is then given by $\psi\,\psi^*$

Thus

$$I(\theta) = I_0\,\frac{\sin^2[Nka\sin\theta/2]}{\sin^2[ka\sin\theta/2]} \qquad (6.14)$$

Incident intensity (I_0)

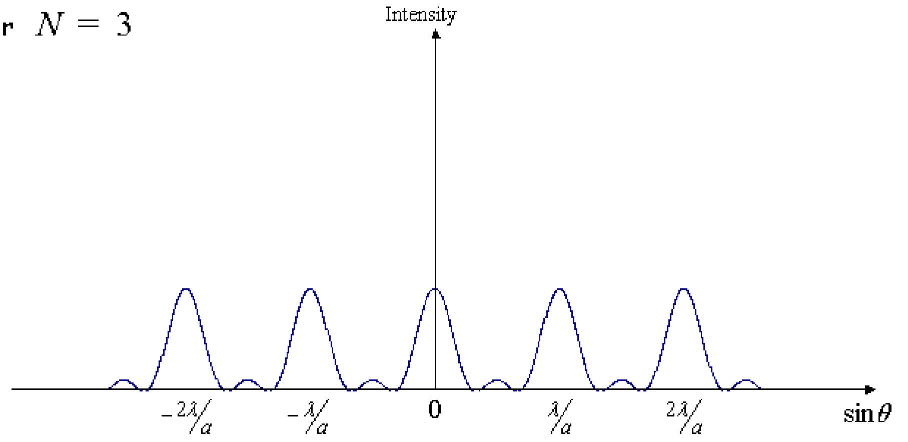
$k = \frac{2\pi}{\lambda}$ = wave number

Primary Maxima occur at $a\sin\theta = n\lambda$ (same as for infinite grating)

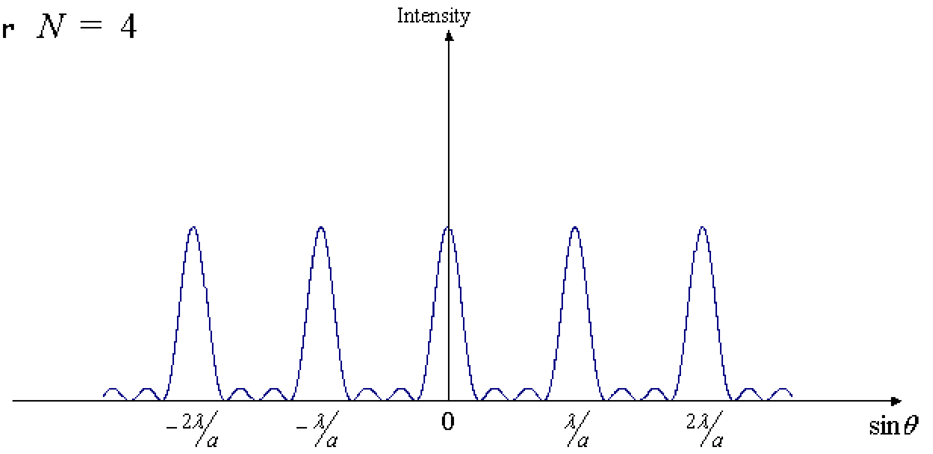
But maxima are **not** infintely narrow.

Width of maxima $W \sim \frac{\lambda}{Na}$

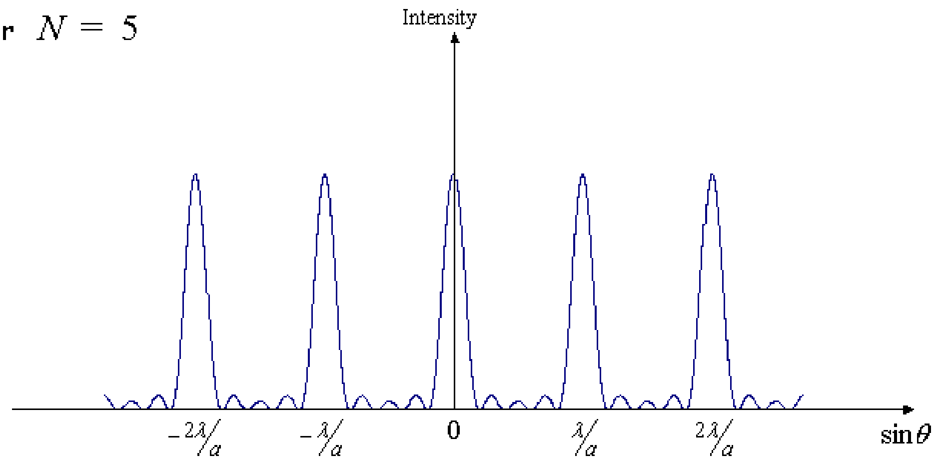
Also $N-2$ **secondary maxima** in between e.g. For $N = 3$



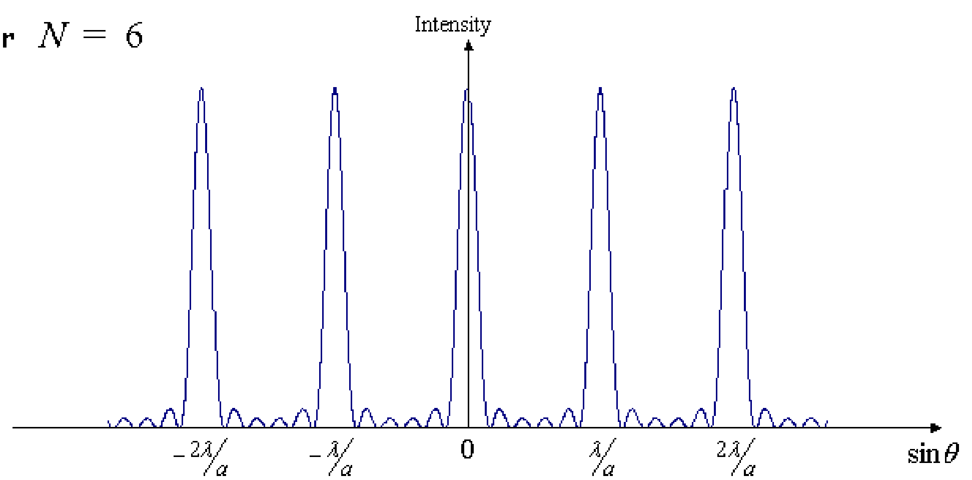
e.g. For $N = 4$



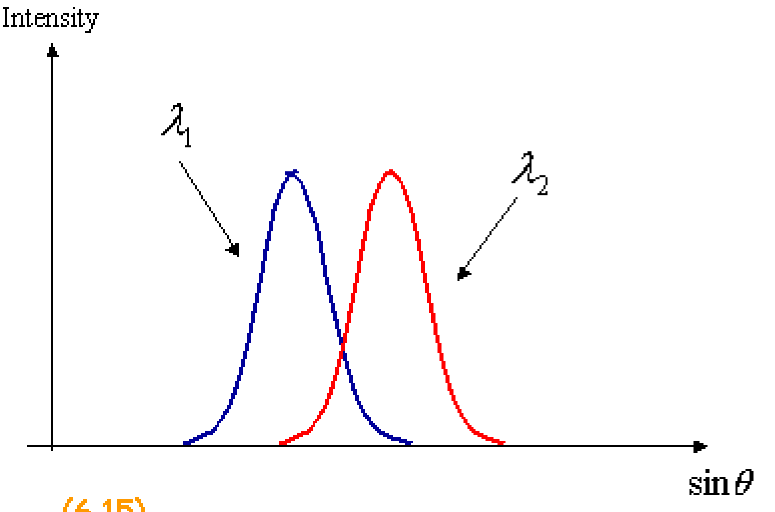
e.g. For $N = 5$



e.g. For $N = 6$



The finite width of the primary maxima limits the **resolving power** of a diffraction grating.



Consider two spectral lines, of wavelength λ_1 and λ_2

Lines are observed at order n $\Rightarrow$ light diffracted through angles satisfying

$$\sin\theta_1 = \frac{n\lambda_1}{a} \qquad \sin\theta_2 = \frac{n\lambda_2}{a} \qquad (6.15)$$

Hence

$$\sin\theta_2 - \sin\theta_1 = \frac{n(\lambda_2 - \lambda_1)}{a} = \frac{n\Delta\lambda}{a} \qquad (6.16)$$

We can only resolve the two lines provided they are separated by (at least) their width.

In other words, the resolution limit is

$$\frac{n\Delta\lambda}{a} = \frac{\lambda}{Na}$$

Or

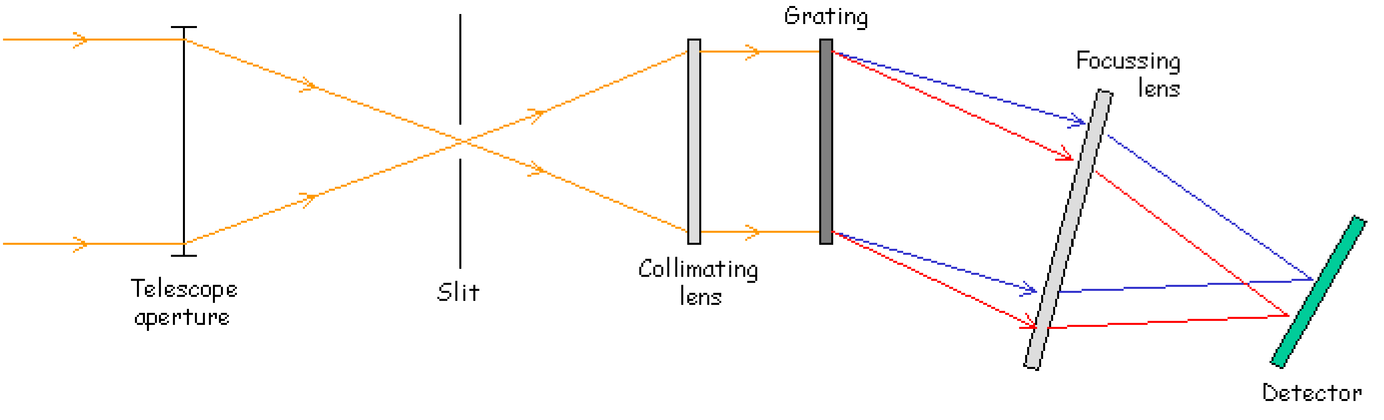
$$R = \frac{\lambda}{\Delta\lambda} = Nn \qquad (6.17)$$

Can use an **echelle grating** to achieve a high resolving power; lines are **blazed** – cut in a special pattern – to concentrate light in high orders

Spectral resolving power depends ***not*** on ruling separation, but on the **total** number of lines on the grating and the order of the maximum observed.

Design of a Slit Spectrometer

Key features of the design are summarised in the following diagram



The slit cuts out unwanted light. Its angular size at the collimating lens defines the range of angles entering the grating

Diameter of collimating lens ≅ width of grating, so that little light is lost

Grating response width = width of primary maxima = $\lambda/Na = \lambda/D_{\text{grating}}$

Focussing lens also produces a **diffraction pattern** (see next section) which 'smears out' light, with width λ/D_{focus}

Choose D_{focus} so that $\lambda/D_{\text{focus}} < \lambda/D_{\text{grating}}$ - i.e. $D_{\text{focus}} > D_{\text{grating}}$

Choose focal length of focussing lens so that width of diffraction peak at the detector ≥ width of pixel on detector.

i.e. the diffraction maxima cover several pixels