4. Sensitivity and Noise

We define:

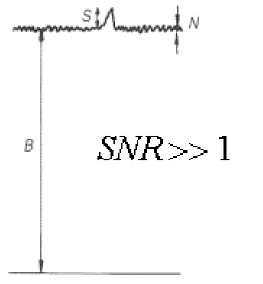
sensitivity of an instrument / detector = smallest signal that it can measure which is clearly not '**noise**' (i.e. errors from some other source).

We can measure the reliability of an observation via the **Signal-to-Noise Ratio (SNR)**.

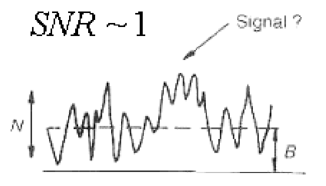
$$SNR = \frac{\text{expected signal level}}{\text{expected noise level}}$$

Generally, we don't trust observations unless the SNR is at least 3 (or preferably much greater).

Sometimes the signal might still be very weak, compared to a (removable) background.



chematic example of a signal, S, vhich is very weak, compared to he background, B (as might be he case when observing, say, a tar or planet at twilight), but is asily detected – after the ackground has been subtracted – ecause the SNR is large.



Schematic example of a signal which is comparable in strength to the background. The existence of a signal at all is in doubt, however, because the noise level is *also* comparable to the signal level – i.e. the SNR is of order unity.

Poisson statistics

For much of the E-M spectrum astronomical observations involve counting photons. However, the number of photons arriving at our telescope from a given source will **fluctuate**.

We can treat the arrival rate of photons **statistically**, which (roughly speaking) means that we can calculate the **average** number of photons which we **expect** to arrive in a given time interval.

We make certain assumptions (axioms):

1. Photons arrive independently in time
2. Average photon arrival rate is a constant

If our observed photons satisfy these axioms, then they are said to follow a **Poisson distribution**

Suppose the (assumed constant) mean photon arrival rate is R photons per second.

If we observe for an exposure time τ seconds, then we expect to receive $R\tau$ photons in that time.

We refer to this as the **expectation value** of the number of photons, written as

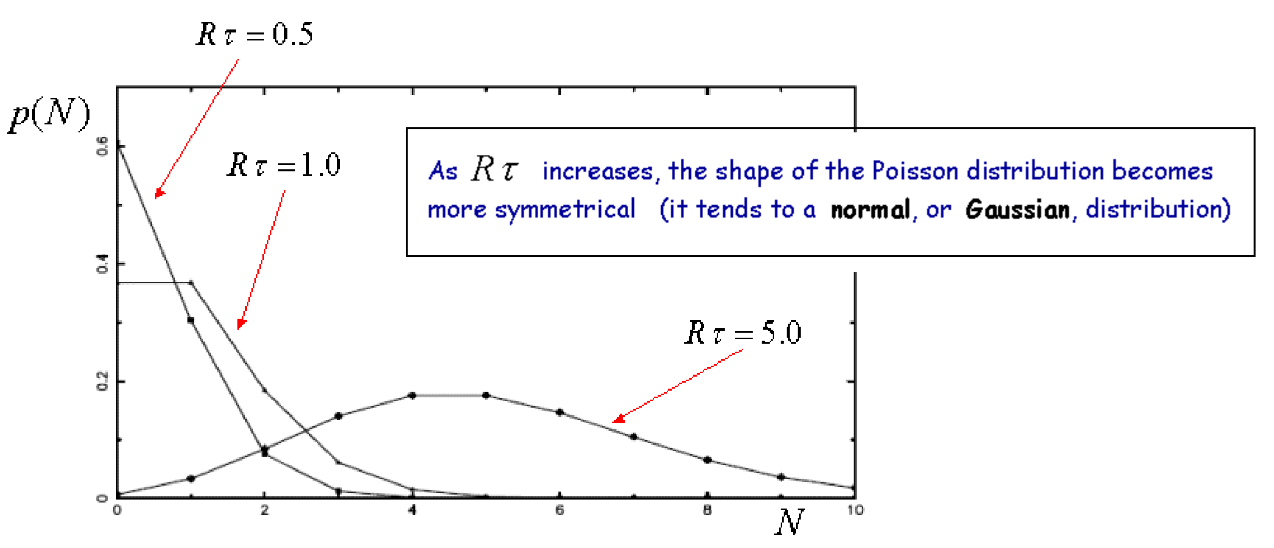
$$E(N) = \langle N \rangle = R\tau \qquad (4.1)$$

If we made a series of observations, each of time τ seconds, we wouldn't expect to receive $\langle N \rangle$ photons every time, but the average number of counts should equal $\langle N \rangle = R\tau$

(in fact this is how we can estimate the value of the rate R *)*

Given the two Poisson axioms, we can show (see non-examinable handout) that the probability of receiving N photons in time τ is given by

$$p(N) = \frac{(R\tau)^N e^{-R\tau}}{N!} \qquad (4.2)$$



For the purposes of A2, all we need to work with is the mean or expectation value of N, and its variance.

We already ***defined*** the expectation value as $E(N) = R\tau$

We can compute the mean value of N using eq. (4.2), and this ***confirms*** eq. (4.1).

We can also define the ***variance*** of N, which is a measure of the ***spread*** in the distribution:

$$\mathrm{var}(N) = \sigma^2 = E\left\{\left[N - E(N)\right]^2\right\} \qquad (4.3)$$

We can think of the variance as the ***mean squared 'error'*** in N

For a Poisson distribution, the variance of N can be shown to be

$$\mathrm{var}(N) = R\tau \qquad (4.4)$$

and the ***standard deviation*** of N is $\sigma = \sqrt{R\tau}$ (4.5)

In practice we usually only observe for ***one*** period of (say) τ seconds, during which time we receive (say) a count of N_{obs} photons.

We estimate the arrival rate as

$$\hat{R} = N_{\mathrm{obs}} / \tau \qquad (4.6)$$

We take N_{obs} as our 'best' estimate for $\langle N \rangle$ with error $\sqrt{N_{\text{obs}}}$

i.e we quote our experimental result for the number count of photons in time interval τ as

$$N_{\text{obs}} \pm \sqrt{N_{\text{obs}}} \qquad (4.7)$$

Adding Noise

Usually there are several sources of noise in our observation, each with a different variance.

Probability theory tells us that, if the sources of noise are all **independent**, then we work out the total noise by adding together the variances:

$$\sigma^2_{\text{total}} = \sigma^2_{\text{Poisson}} + \sigma^2_{\text{other}} \qquad (4.8)$$

Sources of Poisson noise:

1) fluctuations in photon count from the sky

2) dark current: thermal fluctuations in a CCD

Sources of non-Poisson noise:

1) Readout noise. e.g. a CCD can gain/lose electrons during readout. Usually $\sigma_{\text{Readout}} = \text{constant}$

Noise and Telescope / Detector Design

Suppose we observe a point source, of flux density S_ν , through a telescope with collecting area A for time τ , in bandwidth $\Delta\nu$ centred on ν_0 .

Total energy collected by detector (ignoring any absorption)

$$E_{\rm tot} = S_\nu \, A \, \Delta\nu \, \tau \qquad (4.9)$$

No. of photons collected is

$$N_{\rm tot} \approx \frac{S_\nu \, A \, \Delta\nu \, \tau}{h\nu_0} \qquad (4.10)$$

Correcting for combined quantum efficiency of telescope and detector

$$N_{\rm tot} \approx \eta \frac{S_\nu \, A \, \Delta\nu \, \tau}{h\nu_0} \qquad (4.11)$$

η: Fraction of incident photons that produce a response in the detector

Thus $\sigma_{\rm Poisson} = \sqrt{N_{\rm tot}}$ and

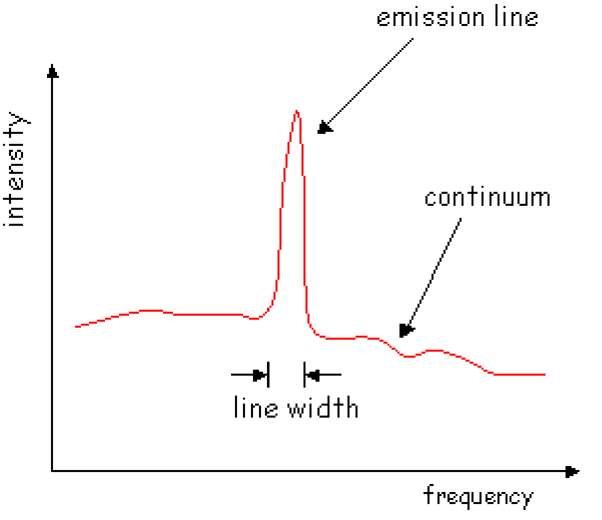
$$SNR \propto (A \, \Delta\nu \, \tau)^{1/2} \qquad (4.12)$$

Same result true for **radio observations** , even though we don't count photons. Here, noise from source and detector electronics

Line Sources

Eq. (4.12) suggests that we can increase the signal-to-noise-ratio by increasing the bandwidth of our observation.

This is **not necessarily** true if we are observing a source which emits only over a narrow frequency range – e.g. a **spectral line**.



Increasing $\Delta\nu$ beyond the line width will increase the amount of noise (from the background continuum) without further increasing the amount of signal (from the line).

See A2 Theoretical Astrophysics for more on linewidths

Example

A star is observed using a photomultiplier tube through an optical telescope of diameter 1.3 m. After 5 minutes observing, the number of counts detected through a standard B filter is 2592 and the number through a standard V filter is 3087. The combined response quantum efficiency of the telescope and detector is 0.2, and the centre of the V-band is at a wavelength of 550 nm. Estimate:

a) the flux of the star in the V-band.

b) the uncertainty in this estimate due to photon noise.

c) the apparent B-V colour index of the star.

Example

The 2-D image of a faint galaxy observed by a CCD covers 50 pixels. For an exposure of 5 seconds a total of photo-electrons are recorded by the CCD from these pixels. An adjacent section of the CCD, covering 2500 pixels, records the background sky count. During the same exposure time a total of photo-electrons are recorded from the adjacent section. Show that, after subtracting the background sky count, the signal-to-noise ratio for the detection of the galaxy is estimated to be 73.

Calculate the length of exposure required to increase the signal-to-noise ratio to 100.

Example

ESO's Very Large Telescope (VLT) comprises four telescopes each with an aperture diameter of 8.2 metres. One of these telescopes is used to observe an unresolved quasar of apparent magnitude $m_V = 17.1$ in the V-band (centre wavelength 550 nm, bandwidth 89 nm), using a cooled CCD.

Given that $m_V = 0$ corresponds to a flux density of 3670 Jy, determine the integration time necessary to obtain a signal-to-noise ratio of 100, assuming that photon arrival statistics dominate the noise and the efficiency of the combined system is 60 percent.